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ALBERTUS MAGNUS AND MATHEMATICS:

A TRANSLATION WITH ANNOTATIONS OF THOSE PORTIONS OF THE COMMENTARY ON EUCLID'S *ELEMENTS* PUBLISHED BY BERNHARD GEYER

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SUMMARIES

This article consists of an English translation, with mathematical and philosophical notes, of three sections of the commentary by Albertus Magnus on Euclid's Elements*: (1) the Prologue, (2) the question "Is an angle a quantity?" and (3) Book I, Proposition 11. The critical apparatus which Bernhard Geyer provided for (1) and (2) is also translated into English and updated by references to the new Cologne edition of Albert's* Opera Omnia.

Cette oeuvre contient une traduction en anglais, avec des notes mathématiques et philosophiques, de trois parties du commentaire d'Albertus Magnus sur les Éléments *d'Euclide: (1) le Prologue, (2) la question "Un angle est-ce une quantité?" et (3) le Livre I, Proposition 11. On présente aussi une traduction en anglais de l'apparatus criticus fourni par Bernhard Geyer pour (1) et (2), en y ajoutant les références à la nouvelle édition-Cologne des oeuvres complètes d'Albert.*

Dieser Artikel enthält eine englische Übersetzung dreier Abschnitte aus Albertus Magnus Kommentar zu den Elementen *des Euklid nämlich (1) den Prolog, dann (2) die Frage, ob ein Winkel eine Quantität sei, und schließlich (3) den elften Lehrsatz des Ersten Buches. Ein mathematischer und philosophischer Kommentar begleitet diese Übersetzung. Der Kritische Apparat, womit Bernhard Geyer (1) und (2) des Textes versehen hat, ist ebenfalls übersetzt und mit Hinweisen auf die neue Kölner Ausgabe der Werke Alberts des Großen ausgestaltet worden.*

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INTRODUCTION

J. L. Heiberg's critical edition of the Greek text of Euclid's *Elements of Geometry* was published by Teubner between 1883 and 1888. It was followed in 1908 by Heath's English translation, with introduction and commentary, which was published by the Cambridge University Press. Before the time of Heiberg, the commentary on the *Elements* by Albertus Magnus (1193-1280) had not yet been discovered, although the tradition that he had written one was noted in the historical and philosophical literature.

> *In mathematics he commented on and explained Euclid.*
> [Milman 1861, viii, 265]

> *L'habile interprète des théorèmes d'Euclide.*
> [Hauréau 1872-1880, ii, 103]

In 1932, Bernhard Geyer of Bonn, Director of the Albertus Magnus Institute, examined a manuscript (MS 80/45) in the Dominican Library of Vienna, the second part of which contained a commentary on the first four books of Euclid's *Elements*. Its author was identified by the rubricator to be an Albert. The existence of this manuscript had been noted twenty-seven years earlier by M. Weiss [1905, n. 260]. Geyer afterward (1944) wrote a paper (not published until 1958) in which, having first shown that Albert had indeed written a commentary on the *Elements,* he argued that the Vienna manuscript contained that commentary and was even an autograph of Albert. His article ended with two excerpts from the manuscript, the "Proöemium" and the question "Utrum angulus sit quantitas." E. Bessel-Hagen of Bonn undertook to prepare the manuscript for publication in the Cologne edition of the *Opera Omnia* of Albert, and after his death the work was entrusted to J. E. Hofmann of Berlin. Hofmann [1960] presented a report on the commentary to the 1958 International Congress of Mathematicians but was unable to edit the text. The enterprise was then handed over to and completed by P. M. J. E. Tummers of Nijmegen, who prepared an article on his progress for the commemorative volume published in 1980 by the Pontifical Institute of Mediaeval Studies in Toronto in honor of the 700th anniversary of the death of Albert. (This article is preceded by a more general one by G. A. Molland on "Mathematics in the Thought of Albertus Magnus.") All these developments were summarized in one sentence by J. Murdoch of Harvard University, whose article in the *Dictionary of Scientific Biography* may be consulted for the progress, since Heath, in the study of the reception of Euclid's *Elements:*

> *Indeed, there seems to be but one "original" commentary proper in Latin, questionably ascribed to the thirteenth-century Dominican philosopher Albertus Magnus and, in any event, greatly dependent upon earlier translated material (notably the commentary of al-Nayrīzī).* [Murdoch 1971, 448a]

The present article contains an English translation, with mathematical and philosophical commentary, of those portions of Albert's manuscript published by Geyer and also of Book I, Proposition 11, the text of which is given in a footnote by Tummers. Geyer added 21 footnotes to the text of Albert, which he referred to as MS. A. For the most part, these are verifications of passages and excerpts from other works of Albert which ought to be compared with statements made in the commentary. I have translated his notes, collected them before my own, and updated them by including the reference to the new Cologne edition of Albert's works for each source he gives. I refer to these notes in the translation by lowercase letters.

TRANSLATION

1. PROLOGUE

Just as philosophy, as Aristotle [1] says in the sixth book of his *Metaphysics* [a], is divided into three parts [2] (viz., physics [3], where the object of study is one whose form [4] as well as whose matter is of interest; mathematics [5], which considers the form that is in movable matter [6] but is nonetheless not conceived together with it by definition [b, 7]; and a third part which deals with the divine separate things [8], which are neither in movable matter nor conceived together with it by definition), so the mind of man is capable of being perfected in three respects, one corresponding to each of these three branches of study. The first branch causes the mind to turn to sense perception, while the second equips it for dealing with mental images [9], but the third makes it divine and renders [10] it somehow like an intelligence, in which process some part of the light of the intelligences, which are joined to the first principle [11], is set flowing into it [12]. Ptolemy [c], great in all divisions of learning, bears witness, however, that the first of these branches is unable to lead [d] man to a firm understanding of itself because of the mobility and changeableness of its subject [13]. That this is so is clear from the differences of opinion among the natural philosophers, which even to this day cannot be brought into harmony [14]. What is more, the third part is so high above us that, as the Prince of the Philosophers [15] himself says [e], our understanding is to it as the eyes of a bat are to the light of the sun [16]. The middle one, though, truly receives [f] the name of knowledge [g, 17], because it is both accessible to us and extracted [h], by definition, from the changeableness of sensible matter. Anyone who seeks to reach the truth of knowledge must therefore give it the greatest attention [18].

The Pythagoreans [19], we know, handed down that mathematics can be naturally divided in turn into two parts, namely, that which concerns discrete quantity and that which concerns continuous quantity [20]. What is more, discrete mathematics is itself twofold, consisting of absolute discrete mathematics, which is the theory of numbers and their properties, and related discrete mathematics, which, as it is applied to harmonies, music appropriated to itself [21]. Continuous mathematics is likewise divided into two parts, geometry, which deals with configurations or shapes of a continuum at rest, and astrology [22], which studies the configurations which are perceived in the distances, eclipses, conjunctions, and trine, square, and sextile aspects [23] of the mobile continuum, in so far as these configurations can be observed in the contemplation of motion taking place on one or more circles [24].

The study of geometry, therefore, comes after the treatment of arithmetic and music. The name itself means measurement of land [25], either because in Egypt [26], where mathematics had its origins [i], the land and fields were divided up geometrically, or because man takes possession of land differently from other animals, for the human race, as Aristotle says [j], uses art and reasoning [27], wherefore whoever is a man refers everything he does to the standard of reason. Those who do otherwise [28] are no different from wild beasts except in the shape of their bodies. For this reason, Aristippus [k, 29], the Socratic philosopher, when he was shipwrecked on the shores of Rhodes and saw geometrical figures drawn in the port [30], cried out to the others, who were terrified lest the island be inhabited by savages, that there was a good chance that they would find friends there, because he had seen the signs of human beings in the port. And indeed, this turned out to be the case, for once the inhabitants had heard the philosopher, they honored, on his account, all who had been shipwrecked with him, and provided them munificently with clothes, ships, and money.

Since now, as we have said, geometry is about immobile quantity, it is necessary to see what the principle [31] of this quantity is and what kinds of continuous quantity arise from this principle. As Alfarabius [32] says in his commentary on the theorems of Euclid, there are only three primary kinds [33] of continuous quantity [ℓ], line [34], surface, and solid. To see this, note that if we consider any straight motion whatsoever of a point (and this is its only simple [35] motion), we shall observe only length without breadth, that is, a line [36]. If, however, the point moves in a circle, it describes a perimeter, which is a circular line. This is the case with the moving leg of a compass. But this motion makes two forms, a convex and a concave, and therefore is not primary, so that the circular line it produces is not primary either, whereas a straight line is, because there is only one kind of straight line. A point, then, by its primary motion, does not give rise to more than one kind of quantity. Now if a line moves, it will do so either in the manner of a point [37] or in the manner of a quantity with length [38]. If it moves in the first way, its motion will not differ from that of a point and will therefore describe nothing other than an extension of the line. If, however, it moves in the second way, into itself or into a part of itself, it necessarily describes a surface, which is the second kind of continuous quantity. If, furthermore, we consider a moving surface, we see that it moves according to the form of a line [39], lengthwise, or in its own way, according to breadth [40]. If it moves in the first way, it extends itself; if in the second way, it produces a solid, since it is necessary that breadth, by its motion, be formed into depth. We can proceed in this way no further, since it is impossible, as Aristotle proves in the frist book of *On the Heavens,* for a fourth dimension [41] to be added to a solid, no matter how it moves; the reason for this is that one cannot imagine more than three mutually perpendicular lines intersecting at a point [m].

Now if anyone should object and say that there are more than these three kinds of continuous quantity, namely, that locus, motion, and time, for example, are also such, let it suffice to refute him, for the moment, to say that there would be no locus without motion, and therefore, mathematically speaking, locus is a surface [42]. Moreover, motion would not be continuous were it not for the space over which it occurs, and time gets its continuity from motion, so both motion and time are continuous on account of space [43]. Moreover, a bit of motion [44] and an instant of time [45] are indivisible because of the indivisible element of space, which is the point [n]. For these reasons, locus, motion, and time are not true primary kinds of continuous quantity.

It is also to be observed that continuous quantity is most closely connected with matter, and so we are justified in making the analogy that just as a potential ingredient is not actualized in one material substance and is in another [o, 46], as the simple is in the mixed, and the mixed in the combined, and the combined in the heterogeneous [p, 47], so is the line in the surface, and the surface and the line in the solid. From this it follows that the point is ultimately [48] the principle of the continuous, and if compared to the line, it is the immediate principle [49]. If the line is compared to the surface, the line is the immediate principle. The solid, however, is the principle of nothing [50]. The reason for all this is that every principle is simple and remains indivisible in the division of that whose principle it is. The point, therefore, because it is in no way [51] divisible, is the ultimate principle of the continuous. The line is the principle according to that with respect to which it is indivisible, namely, according to breadth, and it has the point for its principle according to that with respect to which it is divisible, that is, according to length [52]. Corresponding statements can be made about the surface in comparison with the line and the solid. Moreover, just as we said that a surface, in turn, by its motion, necessarily generates a solid, even so, if this motion is of a particular sort [53], the surface will produce regular kinds of solids, which are the only ones geometry deals with, for a sure account cannot be given of the irregular ones like the figures of animals [54]. If, for example, one angle of a triangle, which is a regular figure, is kept fixed while another [55] is raised little by little by the continuous motion of a rotation until the side they have in common is lifted directly above the fixed point, a pyramid will necessarily be produced, a solid which is the subject of many proofs in geometry [56]. If the figure of a circle is lifted by however much equally and continuously, it will produce a cylinder; if a concave surface is rotated, a sphere will be generated, about which zealous geometers have discovered wonders. Because, therefore, the principle of all these [whether immediately or ultimately] is the point, we shall begin with it as we take up the definitions, the principles, one could say, of the proofs.

2. WHETHER OR NOT AN ANGLE IS A QUANTITY [57]

Many are in doubt as to whether an angle [58] is a quantity [59] and, if it is, as to what kind of quantity it is [q]. For there are those who say that an angle is a relation, because it is called an application [60], and they give four reasons in support of their opinion [61]:

(1) It is not a line, because it has breadth [62], nor a body, because it does not necessarily have depth [63], nor a surface, because it cannot be divided up in a way that a surface can be divided, for no angle is divided breadthwise, but only lengthwise [r, 64], as is proved below in Theorem 9 [65].

(2) Every quantity remains a quantity when it is doubled [66], but a certain angle does not remain an angle when it is doubled, namely, a right angle [67]. Therefore, this angle, when doubled, does not remain a quantity [68]. Therefore, an angle, by its very nature, does not seem to be a quanity [s].

(3) No quantity is an accident [59] of another quantity. But an angle is an accident of another quantity; therefore it is not a quantity. For it is an accident of a surface or a body to be angular.

(4) Nothing that is in itself a kind of quality is also a quantity. But an angle is a kind of quality. Therefore, it is not a quantity. For an angle, by its very nature, cuts a figure up, and this ability to divide is a kind of quality.

We also find four reasons given in favor of the opposite point of view.

(1) Whatever can be increased and decreased [70] is a quantity, but an angle can be increased and decreased. Therefore, it is a quantity. For an obtuse angle is greater than a right angle, and an acute angle is less [t].

(2) The subject of an attribute belongs to the same category as that of which the attribute is descriptive [71]. Acuteness and obtuseness [72] are conditions of quantity and conditions that describe an angle. Therefore an angle is a quantity.

(3) That which divisiblity necessarily suits is a quantity. But that divisibility is suitable for an angle is shown below in Theorem 9 [73]. Therefore, an angle is a quantity, for an angle is divisible lengthwise.

(4) Everything having dimension or dimensions is a quantity. But an angle has dimensions, length, and breadth. Therefore it is a quantity.

It seems, then, that we must say that an *angle* is a quantity [74], but that *angularity* is a quality accidental to quantity [u, 75]. For Sambelichyus, on account of the first reason given above [76], said that a "surface angle" [77] is midway between a line and a surface because it has breadth like a surface but cannot be divided breadthwise and has length like a line and can be divided lengthwise. Furthermore, a solid angle [78] is midway between a surface and a solid because it may be said to have depth but cannot be divided depthwise, only breadthwise and lengthwise. This he supports by referring to the great Apollonius [79], who defines an angle as a sort of intermediate quantity. He says that an angle is the contraction [80] of a surface or solid to one point; the contraction is enclosed by lines that intersect one another, though not directly [81], or by lines that come to an end on the surface [82]. He says "not directly" as Euclid also does, because if they intersected directly, they would not determine an angle, but a line instead. Even Aganyz seems to agree in this, when he says that an angle is a quantity having two or three dimensions whose extremities come together at a point. Most felicitous, however, is the definition of Yrynus, who says that an angle is a quantity which a simpler, related quantity encloses when it comes to a point. For a surface angle is enclosed by lines, since it is midway between a quantity that has one dimension and another that has two, while it itself has two. A solid angle, though, terminates in surfaces [83], being midway between a surface, which has two dimensions, and a solid, which has three.

3. BOOK I, PROPOSITION 11

Given a straight line, construct a line perpendicular to it at a given point on it (Fig. 1). Let *AB* be the given straight line and *C* the given point on it. Then use the second theorem [84] to mark off on *AB* equal distances on both sides of *C* [85]. Then construct, by Theorem 1 [86], an equilateral triangle on that part of the line containing *C* [87, 88]. Or, if better, use the preceding theorem to divide the straight line into equal parts at *C*, and construct, by Theorem 1, an equilateral triangle on the two pieces of the line taken together [88]. Then divide [89] $\angle ADB$ [90] of the equilateral Δ *ABD* by means of Theorem 9 [91]. Draw line *DC* [92]. We shall now show that *DC* is the desired perpendicular. We have two triangles, Δ *DCB* and Δ *DCA*, with two sides of one equal to two sides of the other, for *DB* [93] = *DA* (the big triangle is equilateral) and *DC* is a common side. Also the angle $\angle CDB$ contained between the equal sides is equal to the angle $\angle CDA$ contained between the equal sides. Therefore, by Theorem 4 [94], the bases are equal [95], and the remaining angles are equal. Therefore $\angle DCB = \angle DCA$, and the straight line *DC* makes two equal angles $\angle DCB$ and $\angle DCA$ with the straight line *AB*. Thus, by the definition of perpendicular, *DC* is a perpendicular, which is what we wanted to show.

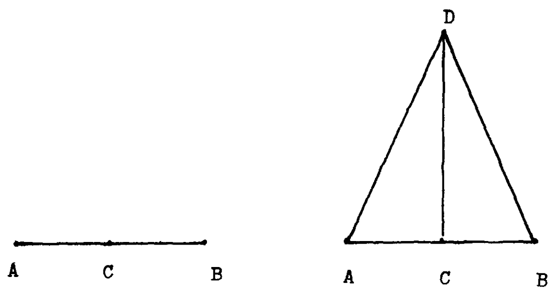

Figure 1

GEYER'S NOTES

a. Aristotle, *Metaphysics*, Book 6, Chapter 1 (1026 a 18-19). Cf. Albert, *Metaphysics*, Book 6, Tract 1, Chapter 2 (Borgnet ed. VI, 384 ff., Cologne ed. XVI/2 (1964), pp. 303-305).

b. The expression *diffinitiva ratio* is favoured by Albert. (Compare *loc. cit.* 384-386.) It is the translation of Aristotle's ὁριστικὸς λόγος. (*On the Soul,* Book 2, Chapter 2 (413 a 14), *Metaphysics,* Book 8, Chapter 3 (1043 b 31)).

c. *The Almagest of Ptolemy of Pelusium,* at the press of Peter Liechtenstein of Cologne, Venice, 1515, First Book, Chapter 1: The two remaining parts of the theoretical division are understood by conjecture alone, not grasped in terms of true knowledge. The theoretical part, indeed, is not grasped because it is never seen; however, the natural one is not grasped because of the motion of matter, the fickleness of its course, the speed at which it changes, and the brevity of time it endures. The agreement of the wise is therefore never expected in these matters.

d. The first leaves of the manuscript (A) are damaged in this place, so that some words are completely or partially illegible.

e. Aristotle, *Metaphysics,* Book 2, Chapter 1 (993 b 9-10). Albert, *Metaphysics,* Book 2, Tract 1, Chapter 2 (Borgnet ed. VI, 117 b, Cologne ed. XVI/1 (1960), p. 92, ℓ. 71).

f. *Optinebit* has been conjectured for *op.* .... MS. A.

g. Albert, *Metaphysics,* Book 1, Treatise 1, Chapter 1 (Borgnet ed. VI, 2 a, Cologne ed. XVI/1 (1960), p. 2, ℓℓ. 31 sqq.): And therefore those qualities reached through the speculative intellect have acquired the name of science. *On the Intellect and the Intelligible,* Book 1, Tract 3, Chapter 2 (Borgnet ed. IX, 500 a, Cologne ed. VII (to appear)): Mathematics is very much the object of the intellect.

h. *Extracta* has been conjectured for *ext.* .... MS. A.

i. Aristotle, *Metaphysics,* Book 1, Chapter 1 (981 b 23). Albert, *Metaphysics,* Book 1, Tract 1, Chapter 10 (Borgnet ed. VI, 20 a, Cologne ed. XVI/1 (1960), p. 15, ℓ. 76).

j. Aristotle, *Metaphysics,* Book 1, Chapter 1 (980 b 27-28). Albert, *Metaphysics,* Book 1, Tract 1, Chapter 6 (Borgnet ed. VI, 13 a, Cologne ed. XVI/1 (1960), p. 10, ℓℓ. 18-22).

k. Vitruvius, *On Architecture,* Book 6, Chapter 1.

ℓ. Compare Albert, *On the Heavens,* Book 1, Tract 1, Chapter 2 (Borgnet ed. IV, 4 a, Cologne ed. V/1 (1971), p. 3, ℓℓ. 10-31). The solid, surface, and line have thus in turn been accepted as the species of the continuous.... But among these species of the continuous, the solid is more divisible than the others, because it is divided with respect to all dimensions as it has all dimensions. And, therefore, since it cannot be enlarged by another dimension, it is perfect among those things that have quantity.... Lines are either simple or composite. The simple ones are those like the straight line and the circle. But the straight line is simple both according to form and according to essence (*essentiam*), since on both sides it has only the form of the straight. The circle, however, is not absolutely simple because on one side it has the concave form and on the other the convex.

m. Compare Albert, *loc. cit.*, 4b (Cologne ed., *ibid.* ℓℓ. 65-67): The reason for this is mathematical, since it cannot be understood how more than three diameters can intersect one another at right angles. Compare Albert, *On the Categories,* Tract 3, Chapter 8 (Borgnet ed. I, 208 b, Cologne ed. I (to appear)).

n. Compare Albert, *Metaphysics,* Book 5, Tract 3, Chapter 1 (Borgnet ed. VI. 325 a, Cologne ed., XVI/1 (1960), p. 257, ℓℓ. 68-82): Other things are indeed called divisible quantities (*quanta*) because they are incidentally subject to division (*quanto*) in so far as they are related to division by the numbering in some way of the parts of that incidentally divisible quantity, and not by themselves, such as motion and time.... I do not say that motion itself which moves and is carried is a divisible quantity, but rather that it is a divisible quantity through that by which or in which there is motion, e.g., space. For by the same token that space is a divisible quantity, so also is motion. Time, further, is a divisible quantity in this, that it is an attribute of motion, according to which attribute (time), motion is a divisible quantity. [N.B. In the Cologne edition, read *refertur* for *referuntur,* and thus alter the translation given above (3rd line of note n) to read "subject to a divisible quantity (*quanto*), in so far as it is related...."]

o. Compare *On the Categories,* Tract 3, Chapter 1 (Borgnet ed. I, 194 a, Cologne ed. I (to appear)): Since matter, as Avicenna says in his *Sufficientia* [*Kitab al-*

Shifa], is subject to generation and in general to change, it becomes three-dimensional through generation according to how much it has the potential of three dimensionality and according to how much that potentiality is not actually formed in some other being.

p. "as ... heterogeneous" is in the margin of MS. A.

q. Compare Albert, *Metaphysics*, Book 5, Tract 3, Chapter 1 (Borgnet ed. VI, 326 a, Cologne ed. XVI/1 (1960), p. 258, *ll.* 40-41): There is, moreover, a quite reasonable doubt about the angle itself, as to whether or not it is a special kind of quantum or quantity.

r. *Ibid. ll.* 43-55: It stands to reason that an angle is not a quantum as a line is (*ad modum lineae*), because an angle is contained between two lines; it is the intersection of two lines extended over a surface, and it is not a direct application. Therefore, it is not like a line with respect to quantity. Similarly, it is not like a surface, because every surface is divisible breadthwise, but an angle cannot be divided breadthwise, but only lengthwise, otherwise it would not be the indivisible intersection of two lines. The fact that it may not be a solid is clear from the fact that it does not have depth, for between two lines there can be no depth. [N.B. The Cologne edition reads "*ex hoc patet*" for "*et hoc patet*" (3rd line up from the bottom of note r).]

s. *Ibid. ll.* 56-61: What is more, whatever is a kind of continuous quantity remains, when doubled, the same kind and mode (*modus*) of quantity. But there is a certain angle which, when doubled, does not remain the same kind and mode of quantity, namely, a right angle, which, if doubled, does not remain the intersection of lines indirectly applied.

t. *Ibid. ll.* 62-68: An angle, therefore, seems to be a special mode of quantity. For it cannot be said that that is not a quantity, which can be increased and decreased in quantity, which is finite in quantity, and, especially, which arises out of measurements of quantities. An angle, though, is something to which all these things apply.

u. *Ibid. ll.* 69-75, 96—p. 259, *l.* 1: We shall say that an angle is not a special mode of quantum or quantity, but rather that an angle tells us a quality about a certain quantity. Whence an angle has to do with the figuration of a quantity. Wherefore, just as to come to an end is a quality about a quantity, so to form an angle and, indeed, the angle itself are qualities about a quantity.... We have said that an angle has to do with how a continuous quantity comes to an end. Now there are those who have used this type of reasoning to argue that an angle is midway between a line and a surface, but it is more appropriate to accept the former explanation because it does not posit the existence of a new entity (*quantum*). [N.B. The Cologne edition reads "*qualitas circa quantitatem dicta, ita...*" instead of "*qualitas circa quantitatem, ita...*," so that we should translate "a quality said about a quantity" in line 4 of note u above.]

COMMENTARY

1. Albert (1193-1280) was the first of the Scholastics (i.e., Mediaeval Schoolmen) to comment on all the known works of Aristotle (384 B.C.-322 B.C.) and to attempt to harmonize them with the teachings of the Church. His work and that of his student Thomas Aquinas (1225-1274) did not at once meet with universal approbation; in fact, the novelties of Thomas were censured at Paris by the bishop, Stephen Tempier (1277), and at Oxford by Kilwardby (1277) and Peckham (1284), Archbishops of Canterbury [Überweg-Geyer 1953, 484]. It would, no doubt, have proven impossible to adopt Aristotle at all, due to his denial of a creator-God and his silence on the fate of the individual soul, if the 13th century had not been an age that had come to look on philosophy as the "handmaiden of theology."

In view of Albert's project to give Aristotle to the Latins, why, we may ask, did he choose to comment on mathematics, for Atistotle himself did not write a book on that subject. Indeed, the source of our Vienna MS. A was not an Aristotelian work, but the commentary of al-Nayrīzī mentioned by Murdoch above. (See also [76] below.) Albert himself has anticipated our question, and we may find the answer in the first paragraph of this prologue, particularly the last sentence.

2. Aristotle divided philosophy into theoretical philosophy and practical philosophy. It was later much debated whether he viewed logic as a third constituent division or as the body of rules of method employed by the other two [Taylor 1912,

21]. Theoretical philosophy he further divided into metaphysics, or first philosophy, natural philosophy, or second philosophy, and mathematics. His students considered practical philosophy to consist of ethics, economics, and politics. The Stoics (who, together with the Platonists, Aristotelians, and Epicureans, formed the four "heresies" or possibilities of ancient philosophy) had also divided philosophy into three parts, namely, logic, physics, and ethics.

Albert is elliptic here, for he knows very well that Aristotle is referring to theoretical philosophy alone when he makes the tripartite division. See Albert, *Metaphysics,* Book VI, Tract 1, Chapter 2 (Cologne ed. XVI/2, p. 305, ℓℓ. 25-28), as well as the translation of Aristotle's *Metaphysics,* Book VI, Chapter 1, which Albert used. A discussion of this translation may be found in the introduction to the Cologne edition of Albert's *Metaphysics* (XVI/1: X-XIII).

3. "The study of nature" would be a more exact translation of *physica,* since physics did not have in those days the special meaning it enjoys today. "Natural philosophy" is also a translation often used for the mediaeval *physica.*

4. *forma.* The ideas (ἰδέαι) of Plato were called by Aristotle μορφαί and εἴδη. From μορφή comes the Latin *forma* or form. "Form" is conventionally used in translating mediaeval texts to indicate the influence of or adherence to Aristotelian philosophy as opposed to Platonism and its "ideas."

5. Cf. Albert, *In Evangelium Matthaei,* II, 1 (Borgnet ed. XX, 61b, Cologne ed., XXI (to appear)):

> *Mathematicus autem duplex est. Mathesis enim idem est quod scientia de separatis et abstractis, quae licet secundum esse suum naturale sint in rebus motui subjectis, tamen diffinitione abstracta considerantur, sicut est tota quadrivii scientia.*
>
> Mathematician *has two meanings. One has to do with the fact that mathesis is the same as the science of separated and abstracted aspects, which by their very nature are in things subject to motion; nevertheless they are by definition considered apart from those things, as is the whole science of the quadrivium.*

Mathematics comes from the Greek μάθεσις, learning. For *abstracta,* translated "abstracted," cf. Albert's *Metaphysics,* Book XII, Tract 1, Chapter 3 (Cologne ed. XVI/2, p. 550, ℓℓ. 47-52).

6. Contrary to what Albert says here, Aristotle writes (*Metaphysics* 1076 32 seqq.) that the objects of mathematics (e.g., numbers) are not "in" sensible things. It was Avicenna (980-1037), physician and philosopher, who said that the ideas are in the matter in which they appear or are exemplified.

For Aristotle, mathematics arises because of the need for the measurement of motion. The type of existence enjoyed by the objects of mathematics was much disputed among the ancients, as was the type of existence enjoyed by the ideas. For Plato, the ideas were independent of the human intellect and were beyond existence. Aristotle misunderstood Plato and thought he claimed that the ideas existed (as if there were somewhere an ideal horse), whence arose the interminable controversies between "nominalists" and "realists" [Cherniss 1944, *passim*]. The doctrine of Avicenna on this issue held particular authority during the Middle Ages. He taught that the ideas exist

> *(1)* ante res, *"before the things," in the mind of God as Platonic exemplars according to which the things are made; (2)* in rebus, *"in the things" in which they appear or are exemplified; and (3)* post res, *"after the things," as abstract(ed) ideas in the human mind; but universals* [*ideas*] *do not exist in the natural world apart from individual things.* [Durant 1950, 255]

7. "By definition" is the meaning of the technical term *ratio diffinitiva,* which literally translates Aristotle's ὁριστικὸς λόγος. The ὁριστικὸς is merely explicative. Since the teminology of Aristotle had made no impression on Latin, the Scholastic philosophers translated his phrases literally

> *so that many a strange-sounding Latin phrase in the writings of the Schoolmen would be very good Aristotelian Greek, if rendered word for word into that language.* [Turner 1912, 550 b]

8. Albert means, for example, the intelligences, who are mentioned by name in the next sentence. An intelligence is a being which performs in the natural order functions corresponding to those that an angel performs in the spiritual order. For example, the intelligences keep the planets in their orbits. In German one calls these beings *Ferngeiste*. They are the progeny of the divine intellect. See Albert, *On the Heavens,* Book 1, Tract 3, Chapter 9, and Book 2, Tract 3, Chapters 4 and 14 (Cologne ed. V/1 (1971), p. 74, *ℓℓ.* 11-12; p. 149, *ℓ.* 96—p. 150, *ℓ.* 3; and p. 174, *ℓ.* 67.

9. Albert has *imaginatio,* which is the Greek φαντασία (Aristotle, *De Anima*, 432 a 15 - 434 21), best rendered by the German *Einbildungskraft;* it is the ability of the mind to "see" the objects of mathematics just as the eyes see physical objects.

10. The text has here, as frequently, the enclitic *-que*. Geyer points out [1958, 169-170] that the use of *-que* is rare in the undisputed works of Albert but quite common in this commentary, and this he cites as the major argument from style against Albert's authorship of the manuscript. His observation that the mathematical nature of the work might account for a certain difference of style has no relevance to the frequent appearance of *-que*. In truth, Geyer works with the assumption that all is by Albert's hand [pp. 167-169], but the *Doctor Universalis* must have had many disciples who were honoured to write up his utterances. This particular manuscript might have been written by some *famulus* or secretary who was a "*-que* person." What is more, with regard to Geyer's discussion of the unusual formation of some letters, to which he gives special attention because of his belief that the manuscript is an autograph of Albert, it cannot be ruled out, as he himself admits [p. 169, 3 ff], that a single person may write differently at different times. One just does not know for sure whether Albert is the author of the commentary, much less whether he personally wrote the manuscript. Paul Hossfeld has an as yet unpublished article in which he argues that Albert dictated the commentary to an amanuensis.

11. God. The first principle is that to which all things can ultimately be traced. Principle is the Latin *principium,* the Greek ἀρχή, which translates as "beginning." Se also note [31] below.

12. Cf. Psalm XXXVI, 10: *In lumine tuo videbimus lumen* (In your light we shall see the light) and the Nicene Creed: *lumen de lumine* (light from light).

13. We have here a Platonic notion. This mingling of Platonic and Aristotelian thought is due to the fact that Albert's source at the moment is Ptolemy (fl. 2nd century A.D.), whose ideas on the certainty of knowledge are in some respects like those of Plato [Pedersen 1974, 28], while his primary source for his division of knowledge is Aristotle. It was the major accomplishment of Pomponatius (1462-1525) to present the unadulterated doctrine of Aristotle, just as Ficino (1433-1499) re-introduced pure Platonic thought to Europe. However, Pomponatius was not so independent in his interpretation of Aristotle that he did not owe some debt to the Scholastics. See [Mahoney 1980, *passim;* Pine 1975, *passim*].

14. William James (1842-1910) says the same thing as did Ptolemy about theology and theologians. Theology cannot be called a science, he argues, because there are so many diverse opinions among the theologians, each doing what he must to salvage his assumptions [James 1902, especially Lecture 18, "Philosophy"].

15. The prince is Aristotle, whom Dante (1265-1321) called *il maestro di color che sanno,* the teacher of them that know (*Inferno,* IV, 131). He was considered the philosopher par excellence, so that although Proclus (410-485) "considered himself to be simply a Platonist" [Kullmann 1950, 141], his theology was later called Aristotelian, perhaps because it was so learned [Dodds 1933, xxix-xxx]. Avicenna referred to Aristotle as "the Philosopher," and so he was called for the remainder of the Middle Ages.

16. In the story of the cave in the seventh book of Plato's *Republic* (514a-517c), the sun in the visible world is likened to the idea of the good; light is compared to truth (508E). See note [12] above.

17. *Scientia* is the Latin word for knowledge. It should not be understood here in the special restricted sense in which it is used today. It translates the Greek ἐπιστήμη, which was the Platonic technical term for the state of mind of those who meditate on the ideas. According to Plato, the mental state of those who concern themselves with the sensible, material objects of nature was the inferior δόξα, the Latin *opinio,* opinion, for which see the "differences of opinion" above.

18. Pius II (Aeneas Silvius Piccolomini, 1406-1464) wrote of man being able, by assiduous study, to arrive at the pearl of knowledge (*adipisci valet scientiae margaritam*) in his bull for the establishment of the University of Basel.

19. For the Pythagoreans, see [Diels 1903; Frank 1923; Heath 1908, Excursus I; Taylor 1928].

20. The division of mathematics into the realms of the discrete and the continuous can be traced back to Boëthius (*Arithmetica* I, 1). Tradition, though, traces the distinction back to the Pythagoreans or even to Pythagoras himself. See [Smith 1923, II, 26] and the references cited by Tummers [1980, note 27].

21. The existence of incommensurables was known to the ancients. Even today, though, when we speak of "Number Theory," we usually mean the study of positive whole numbers. For the observations of Dominicus Gundissalinus (d. 1151) on these matters, see [Baur 1903, 111, 1-4] and its English translation in [Grant 1974, 72].

22. Astronomy and astrology were distinct sciences in the Middle Ages. *Astrologia* was often reserved for the more mathematical of the two. *Astronomia* was either the generic term for the whole study of the heavens or the term for that study which investigates the influence of the heavenly bodies in terrestrial affairs. For Albert's definition of *astrologia*, cf. *Metaphysics*, Book III, Tract 2, Chapter 3 (Cologne ed. XVI/1, p. 117, *ll.* 40-44). For Albert on *astronomia*, cf. *In Post. Anal.*, Book I, Tract 1, Chapter 3 (Borgnet ed. II 8 a, Cologne ed. to appear), and *De Fato*, a 4 (Cologne ed. XVII/1, p. 73, *ll.* 36-44).

23. Two heavenly bodies are in the trine, square, or sextile aspect if they are one-third (120°), one-fourth (90°), or one-sixth (60°) part of the zodiac distant from each other, respectively.

24. Given the earlier reference to Ptolemy, it is quite likely that Albert has in mind here his epicycles, whereby all motions in the heavens were explained in terms of the compounding of uniform circular motions.

The Pythagorean division of mathematics into arithmetic, music, geometry, and astrology or astronomy was known as the quadrivium (a phrase coined by Boëthius (475-524)), in contradistinction to the trivium of logic, rhetoric, and grammar. In Albert's time, some of these subjects encompassed more than we might expect from their names; e.g., geometry included geography, rhetoric included law, and grammar included literature. This division of the most important subject matter into the "seven liberal arts" goes back to the *Institutiones Divinarum et Saecularium Litterarum* of Cassiodorus (c. 490-583), who borrowed it from the *De Nuptiis* of Martinus Capella (5th century A.D.), whence come the figures of the seven liberal arts that can be discovered on the façades of mediaeval cathedrals (e.g., Chartres, west portal).

25. γῆ = Earth, μετρεῖν = to measure.

26. The Platonists had a special predilection for Egypt, whence geometry arose, as Herodotus (484?-425) says, because of the Nile:

> *Sesostris ... made a division of the soil of Egypt among the inhabitants.... If the river carried away any portion of a man's lot, ... the king sent persons to examine and determine by measurement the exact content of the loss.... From this practice, I think, geometry first came to be known in Egypt, whence it passed into Greece.* (II, 109)

27. This is his definition of man (*Metaphysics* 980 b 27).

28. The rabble, barbarians, savages.

29. This story of Aristippus (435?-356?) comes from the *De Architectura* I, 6, 1, of Vitruvius (fl. 1st century B.C.). The philosopher's life by Diogenes Laertius (fl. 2nd century A.D.) contains, as do all the biographies by that author, many anecdotes in addition to this one which, if not true, are, as the Italians say, good stories.

30. Teachers and students would draw mathematical diagrams in the sand with sticks, since paper and pencil were unknown. Archimedes (287?-212) was doing this when he was slain by the Roman soldier, who was provoked by his rebuke, *Noli tangere circulos meos!* (Keep off my cirlces!) See [Plutarch, *Marcellus*, XIX].

31. According to Aristotle (*Metaphysics* IV, 1 (1013 a 15)), the *principium* is that in virtue of which something exists or is done or known. By the *principium* of continuous quantity, Albert means not the number line, which we accept as revealing the essence of continuity, but the point, because he takes the line to be generated by the movement of a point.

32. For the life and works of Mohammed Abu Nasr of Farab (880-950), see [Mahdi 1971]. The passage Albert refers to cannot, according to Tummers [1980, 492], be located in any of the works of Alfarabius that have survived.

33. *tres species primae.*

> *For Aristotle, there is a distinction between two kinds of geometrical objects: on the one side the line, surface, and body together with their principle, the point; on the other side the geometrical figures [such] as the circle and square. The first ones are the base, "the underlying matter" of the second ones, and the nature of the first ones is "quantitas," extentionality in one, two, and three [dimensions], as Aristotle says in the* Metaphysics *(VI 1 1061).* [Tummers, to appear, I, 1]

34. Albert has *linea,* but *linea* is more general than our straight line, which in Latin is *linea recta.* A circle or conchoid is also a *linea.* There does not seem to be any one modern term which can take on all the meanings of the Latin *linea.*

35. *simplex secundum formam. Simplex* is the quality by virtue of which a substance has neither constitutive nor quantitative parts. Albert means here that one cannot, with respect to any criterion, distinguish between the lines this sort of motion generates. One can, though, when a curved line is generated, distinguish two types of circular arcs, concave up and concave down.

36. Euclid's (fl. ca. 300 B.C.) definition of a line is "breadthless length." The ancients recognized the necessity for postulates and axioms, but not for undefined terms.

37. That is, it will merely describe a line, something a moving point can do. This happens when a line moves along itself or when a circular arc is rotated along itself.

38. That is, it will make use of its one dimension, as when a line sweeps out a plane, or when a circle sweeps out a cylinder or a sphere.

39. That is, it will simply extend itself, as when a hemisphere is revolved about a diameter of its base and produces a sphere, something a moving arc can do.

40. That is, it will make use of its second dimension, as when a square is moved so as to sweep out a cube.

41. Cayley (1821-1895) and Grassmann (1809-1877) were the first to introduce the fourth dimension into geometry (in 1843 and 1844, respectively).

42. *Locus* (τόπος) means place, which Aristotle defines as the boundary or inner surface of the containing body at which it is in contact with the contained body; cf. *Physics* IV, 4 (212 a 5-6). The objection might be raised that place ought also to be considered a continuous quantity, for it had been defined by some as the extension between the bounding surfaces of a containing body; cf. *ibid.* (212 b 6-9, 13-29) for Aristotle's refutations.

43. *esse motus et temporis continuum est a spatio.*

44. *momentum.*

45. "An instant of time" is the translation of *nunc,* Aristotle's νῦν, the "now" (*Physics* 219a-220ℓ *et passim*).

46. So we might say that although all lumber has the potential (possibility) of being made into chairs, only some of it actually is so formed, and in that of it that has been fashioned into tables, there remains the potential for making chairs. For the Aristotelian doctrine of the potential and the actual, see [Taylor 1912, 47-49]. The terminology is more suited to biology than to mathematics.

47. Mixed (*mixtum*), combined (*complexionatum*), and heterogeneous (*eterogenium*) are technical terms for progressively impurer substances.

48. *simpliciter.* See note [35]. The idea is that since the motion of the point produces the line, the point is the principle of the line, but it is ultimately, though not immediately, the principle of the surface too, because by its motion it produces the line whose motion produces the surface. By the same reasoning, the point is ultimately the principle of the solid also. It is therefore the principle, either immediately (of the line) or ultimately (of the surface and solid), of the three principle kinds of continuous quantity.

49. Literally, the line is "principled" (*principiatum*) by the point.

50. It is always a derived quantity because there is, according to Albert, no fourth dimension into which it can be moved to produce something whose principle it might be.

51. *simpliciter.*

52. The line is therefore not *simpliciter* indivisible, because it *can* be divided, though only lengthwise, not breadthwise.

53. *si secundum speciem accipiatur.*

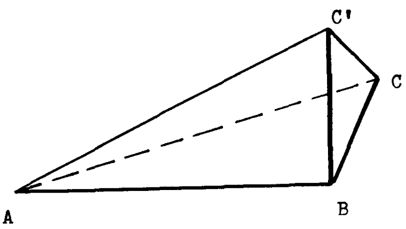

Figure 2

54. Albert calls lines, surfaces, and solids that can be defined mathematically (e.g., circle, sphere, pyramid) regular. The solid determined, however, by a cow's body cannot be so defined, and Albert calls such a figure irregular.

> *Many of the geometers' figures are in no way found in natural bodies, and many natural figures, and particularly those of animals and plants, are not determinable by the art of geometry. (Albert,* Physica, *Book III, Tract 2, Chapter 17, Borgnet ed. III 235 b, Cologne ed. IV (to appear), translated in* [Molland 1980]*.)*

55. We should say that the vertex is what is kept fixed or what is moved. $\angle CAB$ (or $\angle CBA$) is the fixed angle, while $\angle ACB$ is the moved angle. *AB* is the axis of rotation. (See Fig. 2.)

56. Rotate ΔABC around *AB* until *C* reaches a point *C'* so that the plane of $\Delta ABC'$ is perpendicular to the plane of ΔABC. The pyramid Albert is talking about has vertices *A, B, C,* and *C'*.

57. See [Tummers, to appear] for many valuable notes and texts relevant to this section.

58. The ancients had a very general notion of angle. If *AB* and *CB* were any two curves (not necessarily plane curves) intersecting at *B,* it was agreed that they formed an angle there (Fig. 3). What is more, if a solid "came to a point" somewhere, e.g., as a cone does at its apex, it was said to determine an "angle with depth" or "solid angle" there. Albert quotes below the remarks of Simplicius about solid angles, but in the *Metaphysics* he takes no notice of them. (See note r.)

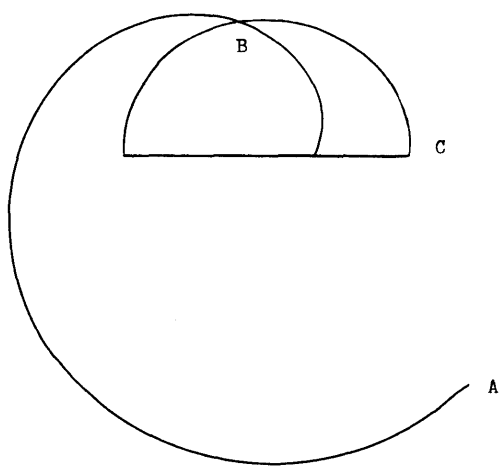

Figure 3

59. The categories (κατηγορίαι, Latin: *praedicamenta*) of Aristotle, whereby the ancients attempted to explain in how many senses the copula is used when we say x is (a) y, are:

	Greek		*Latin*	*English*
I.	οὐσία	(being)	substantia	substance
II.	ούσια	(how much?)	quantum	quantity
III.	ποῖον	(how?)	quale	quality
IV.	πρός τι	(in what way?)	relatio	relation
V.	ποῦ	(where?)	locus	place
VI.	πότε	(when?)	tempus	time
VII.	κεῖσθαι	(to lie)	situs	position
VIII.	ἔχειν	(to have)	habitus	possession
IX.	ποιεῖν	(to do)	actus	activity
X.	πάσχειν	(to have done to one)	passio	passivity

Concepts belonging to categories II-X are called accidents (συμβεβηκότα) or things capable of being said about or predicated of (*accidunt, conveniunt*) concepts in I.

> *There was much debate among philosophers as to the particular category (according to the Aristotelian scheme) in which an angle should be placed; is it namely a quantum (ποσόν), quale (ποιόν), or relation (πρός τι)?* [Heath 1908, I, 177]

This very great controversy was due to the fact that no definition of the angle can be found in the extant works of Aristotle. Mathematical examples of quality are straightness and curvature; examples of relation are similarity, equality, and congruence.

60. To apply (Euclid's παραβάλλειν) a line *AB* to a line *CD* is to place *A* on *C* (Fig. 4). The application is indirect if *B* does not lie on *CD* or its extension; if it does so lie, the application is called direct. The conic sections take their names from the terminology of application, but the application in that case is one of rectangles to lines rather than of lines to lines as here. See [Heath 1908, I, 343-345].

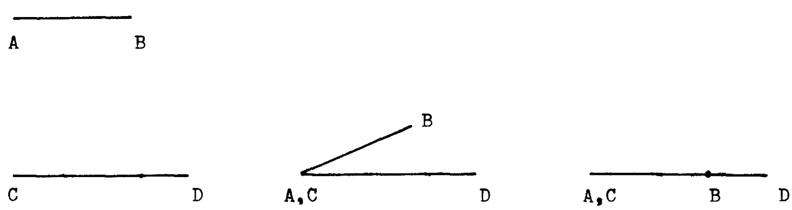

Figure 4

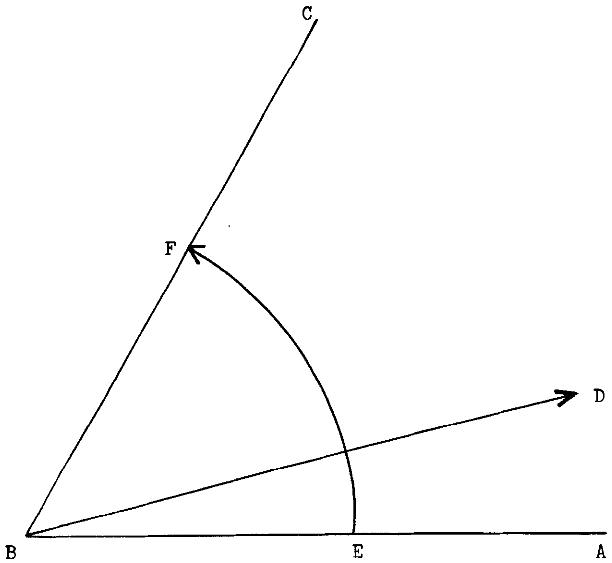

Figure 5

61. We observe here the standard method of the Scholastic doctors, who introduce all noteworthy arguments for or against a proposition, the latter being stated first. The method originated with the *Sic et Non (Yes and No)* of Abélard (1079-1142), written about 1120.

62. The arrow *BD* indicates the direction of increasing "length" of $\angle ABC$. The arrow on the arc *EF* indicates the direction of increasing "breadth." *BD* divides the angle into two angles, but *EF* does not. (See Fig. 5.)

63. Only in some instances, such as at a vertex of a solid (for example, a cone, cube, or pyramid), does one have an "angle with depth."

64. See the example in note [62]. In Geyer's note r, "indivisible" refers to the point at which the two lines intersect.

65. Any rectilineal angle (an angle formed by two intersecting straight lines) can be bisected [Heath 1908, I, 264].

66. To double a quantity is to produce the same sort of quantity with twice the measure appropriate to it. To double a line, then, is to extend it to a line with twice the length of the original, whereas to double a cube is to produce a new cube with twice the volume. (To do this with straightedge and compass alone was the Delian Problem, one of the three classic problems of the Greeks.)

67. Euclid's definition of the angle excluded the "straight angle."

68. But it then becomes two lines or two quantities; what it does not remain is a quantity of the same kind, if one does not allow straight angles.

69. *accidit*. For *A* to be an accident of *B* means that *A* can be predicated of *B*; e.g., obtuseness is an accident of an angle, but a point is not an accident of a line. An accident is that which cannot exist and which cannot be conceived of except as dependent upon some presupposed being.

70. *cui accidit esse maius et minus.*

71. *cui convenit passio, convenit et subiectum.*

72. *acuitas* (sharpness) *et hebetudo* (dullness).

73. See note [65].

74. Because of the last four reasons given.

75. Because of the first four reasons given. Avicenna had reached a similar conclusion. See [Tummers, to appear, III, 1]. At the end of Geyer's note u, we see that Albert uses "Ockham's Razor" a half-century before the *Venerabilis Inceptor*.

76. That is, an angle is not a line, solid, or surface, because it has breadth but not depth and cannot be divided according to breadth.

Sambelichyus, Aganyz, and Yrynus are Latin transliterations of Arabic transliterations of the Greek names which we know as Simplicius (fl. 500 A.D.), Aganis (evidently a contemporary of Simplicius, see [Tummers, to appear, note 39]), and Heron (fl. 3rd century A.D.). We are reminded of the observation of Renan [1852, 52] that the Mediaeval edition of the commentaries of Averroës (1126-1198) was a "Latin translation of a Hebrew translation of a commentary made upon an Arabic translation of a Syriac translation of a Greek text." Albert made full use of a Latin translation by Gerard of Cremona (12th century) of the commentary on Euclid's *Elements* by the Persian Abu' l- ʿAbbās Al-Faḍl ibn Hātim Al-Nayrīzī, whom he calls Anaritius (d. ca. 922). For more on these mathematicians, consult [Heath 1908] and the *Dictionary of Scientific Biography*.

77. An angle determined by the intersection of any arcs.

78. For "angles with depth," see note [58].

79. Apollonius of Perga (262-190 B.C.), author of the *Conic Sections*.

80. συναγωγή, which in Latin is *coniunctio*.

81. See notes [60] and [67].

82. As, for example, the generators of a cone come to an end at the apex.

83. The solid angle at the apex of a cone is enclosed by the surface of the cone and may be said to terminate there.

84. To place at a given point (as an extremity) a straight line equal to a given straight line [Heath 1908, I, 244].

85. Let A' and B' be the points taken on opposite sides of C such that $\overline{A'C} = \overline{CB'}$. Albert, without explicitly saying so, renames A' and B' as A and B, respectively, and makes no further reference to the original points A and B (see Fig. 6).

86. On a given finite straight line to construct an equilateral triangle [Heath 1908, I, 241].

87. That is, on the segment which I call $\overline{A'B'}$.

88 ... 88. *vel si maius, ..., deinde super utramque partem simul equilaterum triangulum per primum theorema statue.* "Better" is for *maius*, which means "greater," but *maius* is neuter, and the only neuter noun thus far is *theorema*, which it cannot modify. Is it possible that we have here a mistake for *melius*? Tummers thinks that this difficult clause is a bit of practical advice for those having trouble drawing the figure. As regards *maius*, he suggests:

> *maius could refer to "the whole thing." It is also possible that* maius *refers to "line," because one often sees that a feminine as well as a neuter refers to line (*linea or latus *being omitted).* [private communication]

The "preceding theorem" is Theorem 10: To bisect a given finite straight line [Heath 1908, I, 267].

89. Albert means "bisect."

90. D must be the third vertex (after A and B) of the equilateral triangle that has been constructed.

91. See note [65].

92. Albert assumes without proof that the bisector of $\angle ADB$ meets AB at C. This is true, but what he should have done is denote by C' the point where the bisector meets AB and then, after proving triangles $DC'B$ and $DC'A$ congruent, observe that $\overline{AC'} = \overline{C'B}$ so $C' = C$. Albert does not use the fact that C bisects AB in the following proof that ΔDCB and ΔDCA are congruent.

93. The manuscript has, by mistake, DC instead of DB.

94. If two triangles have the two sides equal to two sides, respectively, and have the angles contained by the equal straight lines equal, they will also have the base equal to the base, the triangle will be equal to the triangle, and the remaining angles will be equal to the remaining angles, respectively, namely, those which the equal sides subtend [Heath 1908, I, 247-250].

Figure 6

95. By assuming that the bisector of $\angle ADB$ meets AB at C, Albert already had established that the bases were equal. (See note [92].) Can the confusion indicated by this statement be explained by supposing that whoever wrote this manuscript did not accurately recall the utterances of the master regarding this proposition, or must we suspect that the Christian Aristotle was no more sophisticated in matters mathematical than his pagan mentor?

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