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2013 Class. Quantum Grav. 30 165020

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New limits on the violation of local position invariance of gravity

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Received 10 June 2013, in final form 4 July 2013

Published 26 July 2013

Online at stacks.iop.org/CQG/30/165020

Abstract

Within the parameterized post-Newtonian (PPN) formalism, there could be an anisotropy of local gravity induced by an external matter distribution, even for a fully conservative metric theory of gravity. It reflects the breakdown of the local position invariance of gravity and, within the PPN formalism, is characterized by the Whitehead parameter ξ . We present three different kinds of observation, from the Solar system and radio pulsars, to constrain it. The most stringent limit comes from recent results on the extremely stable pulse profiles of solitary millisecond pulsars, that gives $|\hat{\xi}| < 3.9 \times 10^{-9}$ (95% CL), where the hat denotes the strong-field generalization of ξ . This limit is six orders of magnitude more constraining than the current best limit from superconducting gravimeter experiments. It can be converted into an upper limit of $\sim 4 \times 10^{-16}$ on the spatial anisotropy of the gravitational constant.

Communicated by C M Will

PACS numbers: 04.80.Cc, 96.60.-j, 97.60.Gb

(Some figures may appear in colour only in the online journal)

1. Introduction

Since the 1960s, advances in technologies are continuously providing a series of formidable tests of gravity theories from on-ground laboratories, the Solar system, various pulsar systems, and also cosmology [42, 43]. Up to now, Einstein's general relativity (GR) passed all experimental tests with flying colors. However, questions related to the nature of dark matter and dark energy, and irreconcilable conflicts between GR and the standard model of particle physics, are strong motivations to study alternative theories of gravity. In addition, gravity as a fundamental interaction of nature deserves most stringent tests from various aspects.

For tests of gravity theories, one of the most popular frameworks is the *parameterized post-Newtonian (PPN) formalism*, proposed by Nordtvedt and Will [25, 40, 44, 42]. In the

standard PPN gauge, the framework contains ten dimensionless PPN parameters in the metric components as coefficients of various potential forms. These parameters take different values in different gravity theories. Hence, experimental constraints on these parameters can be directly used to test specific gravity theories [30, 42, 43].

In this paper, we concentrate on one of the ten PPN parameters which characterizes a possible Galaxy-induced anisotropy in the gravitational interaction of localized systems. Such an anisotropy is described by the Whitehead parameter ξ in the weak-field slow-motion limit [41]. We use $\hat{\xi}$ to explicitly denote its strong-field generalization. Besides Whitehead's gravity theory [39], ξ is relevant for a class of theories called 'quasilinear' theories of gravity [41]. In GR, the gravitational interaction is local position invariant with $\xi = 0$, while in Whitehead's gravity, local position invariance (LPI) is violated and $\xi = 1$ [41, 15].

An anisotropy of gravitational interaction, induced by the gravitational field of the Galaxy, would lead to anomalous Earth tides at specific frequencies with characteristic phase relations [41, 38]. The ξ -induced Earth tides are caused by a change in the local gravitational attraction on the Earth surface due to the rotation of the Earth with frequencies associated with the sidereal day. By using constraints on ξ from superconducting gravimeter, Will gave the first disproof of Whitehead's parameter-free gravity theory [41] (see [15] for multiple recent disproofs). Later Warburton and Goodkind presented an update on the limit of ξ by using new gravimeter data [38], where they were able to constrain $|\xi|$ to the order of 10^{-3} . The uncertainties concerning geophysical perturbations and the imperfect knowledge of the Earth structure limit the precision. Uncertainties include the elastic responses of the Earth, the effects of ocean tides, the effects of atmospheric tides from barometric pressure variation, and the resonances in the liquid core of the Earth [38] (see [16, 36] for recent reviews on superconducting gravimeters).

Limits from Earth tides are based on periodic terms proportional to ξ , while secular effects in other astrophysical laboratories can be more constraining. Nordtvedt used the close alignment of the Sun's spin with the invariable plane of the Solar system to constrain the PPN parameter α_2 , associated with the local Lorentz invariance of gravity, down to $\mathcal{O}(10^{-7})$ [28]. In the same publication Nordtvedt pointed out that such a limit is also possible for ξ , as the two terms in the Lagrangian have the same form. However, to our knowledge, no detailed calculations have been published yet. In section 3 we follow Nordtvedt's suggestion and achieve a limit of $\mathcal{O}(10^{-6})$.

A non-vanishing (strong-field) $\hat{\xi}$ would lead to characteristic secular effects in the dynamics of the rotation and orbital motion of radio pulsars. We have presented the methodologies in details to constrain the (strong-field) $\hat{\alpha}_2$ from binary pulsar timing [34] and solitary pulsar profile analysis [33] respectively. By the virtue of the similarity between $\hat{\alpha}_2$ - and $\hat{\xi}$ -related effects, in section 4 we extend the analysis in [34, 33] to the case of LPI of gravity. From timing results of PSRs J1012 + 5307 [19] and J1738 + 0333 [13], a limit of $|\hat{\xi}| < 3.1 \times 10^{-4}$ (95% CL) is achieved for neutron star (NS) white dwarf (WD) systems [35]. As shown in this paper, from the analysis on the pulse profile stability of PSRs B1937 + 21 and J1744 - 1134, a limit of $|\hat{\xi}| < 3.9 \times 10^{-9}$ (95% CL) is obtained, utilizing the rotational properties of solitary millisecond pulsars. This limit is six orders of magnitude better than the (weak-field) limit from gravimeter.

The paper is organized as follows. In the next section, the theoretical framework for tests of LPI of gravity is briefly summarized. In section 3, a limit on ξ from the Solar system is obtained. Then we give limits on $\hat{\xi}$ from binary pulsars and solitary pulsars in section 4. In the last section, we discuss issues related to strong-field modifications and conversions from our limits to limits on the anisotropy in the gravitational constant. Comparisons between our

tests with other achievable tests from gravimeter and lunar laser ranging (LLR) experiments are also given.

2. Theoretical framework

In the PPN formalism, PPN parameters are introduced as dimensionless coefficients in the metric in front of various potential forms [44, 42, 43]. In the standard post-Newtonian gauge, ξ appears in the metric components g_{00} and g_{0i} [44, 42, 43]. However, in most cases, it is relevant only in linear combinations with other PPN parameters like β , γ (see [42, 43] for formalism and details). Due to the limited precision in constraining these PPN parameters (see table 4 in [43] for current constraints on PPN parameters), it is not easy to get an independent stringent limit for ξ . For example, based on the Nordtvedt parameter (see (43) in [15]),

$$\eta = 4\beta - \gamma - 3 - \frac{10}{3}\xi - \alpha_1 + \frac{2}{3}\alpha_2 - \frac{2}{3}\zeta_1 - \frac{1}{3}\zeta_2, \quad (1)$$

one can only constrain ξ to the order of $\mathcal{O}(10^{-3})$ at most. Nevertheless, in the metric component g_{00} , -2ξ alone appears as the coefficient of the Whitehead potential [41],

$$\Phi_W(\mathbf{x}) \equiv \frac{G^2}{c^2} \iint \rho(\mathbf{x}')\rho(\mathbf{x}'') \left(\frac{\mathbf{x} - \mathbf{x}'}{|\mathbf{x} - \mathbf{x}'|^3} \right) \cdot \left(\frac{\mathbf{x}' - \mathbf{x}''}{|\mathbf{x} - \mathbf{x}''|} - \frac{\mathbf{x} - \mathbf{x}''}{|\mathbf{x}' - \mathbf{x}''|} \right) d^3\mathbf{x}' d^3\mathbf{x}'', \quad (2)$$

where $\rho(\mathbf{x})$ is the matter density, G and c are the gravitational constant and the speed of light respectively. This fact provides the possibility to constrain the PPN parameter ξ directly.

Correspondingly, in the PPN n -body Lagrangian, we have a ξ -related term for three-body interactions (see e.g. (6.80) in [42]),

$$L_\xi = -\frac{\xi}{2} \frac{G^2}{c^2} \sum_{i,j} \frac{m_i m_j}{r_{ij}^3} \mathbf{r}_{ij} \cdot \left[\sum_k m_k \left(\frac{\mathbf{r}_{jk}}{r_{ik}} - \frac{\mathbf{r}_{ik}}{r_{jk}} \right) \right], \quad (3)$$

where the summation excludes terms that make any denominators vanish. For our purposes below, we consider the third body being our Galaxy, and only consider a system \mathcal{S} (the Solar system or a pulsar binary system or a solitary pulsar) of typical size much less than its distance to the Galactic center R_G . Hence the Lagrangian (3) reduces to (dropping a constant factor that rescales G)

$$L_\xi = \frac{\xi}{2} \frac{U_G}{c^2} \sum_{i,j} \frac{G m_i m_j}{r_{ij}^3} (\mathbf{r}_{ij} \cdot \mathbf{n}_G)^2, \quad (4)$$

where U_G is the Galactic potential at the position of the system \mathcal{S} (associated with the mass inside R_G), and $\mathbf{n}_G \equiv \mathbf{R}_G/R_G$ is a unit vector pointing from \mathcal{S} to the Galactic center. In our calculations below we will use $U_G \sim v_G^2$, where v_G is the rotational velocity of the Galaxy at \mathcal{S} . Equation (4) is exact, only if the external mass is concentrated at the Galactic center, otherwise a correcting factor has to be applied, which depends on the model for the mass distribution in our Galaxy [21]. At the end of section 5, we show that this factor is close to two, as already estimated in [15].

From Lagrangian (4), a binary system of mass m_1 and m_2 gets an extra acceleration for the relative movement (see (8.73) in [42] with different sign conventions),

$$\mathbf{a}_\xi = \xi \frac{U_G}{c^2} \frac{G(m_1 + m_2)}{r^2} [2(\mathbf{n}_G \cdot \mathbf{n})\mathbf{n}_G - 3\mathbf{n}(\mathbf{n}_G \cdot \mathbf{n})^2], \quad (5)$$

where $\mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$ and $\mathbf{n} \equiv \mathbf{r}/r$. Because of the analogy between the extra acceleration caused by the PPN parameter α_2 (see (8.73) in [42]), the Lagrangian (4) results in similar equations of motion with replacements,

$$\mathbf{w} \rightarrow \mathbf{v}_G \quad \text{and} \quad \alpha_2 \rightarrow -2\xi, \quad (6)$$

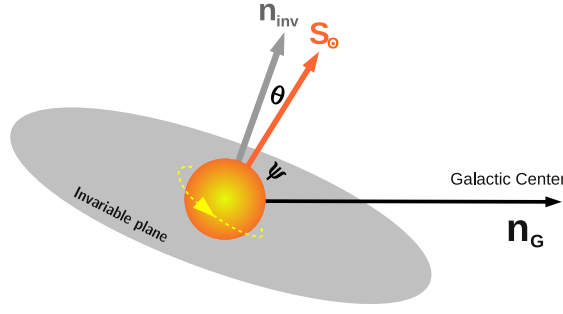


Figure 1. Local position invariance violation causes a precession of the Solar angular momentum \mathbf{S}_\odot around the direction of the local Galactic acceleration \mathbf{n}_G , which causes characteristic changes in the angle θ between \mathbf{S}_\odot and the norm of the invariable plane \mathbf{n}_{inv} . Due to the movement of the Solar system in the Galaxy, \mathbf{n}_G is changing periodically with a period of ~ 250 Myr.

where $\mathbf{v}_G \equiv v_G \mathbf{n}_G$ is an effective velocity [35]. With replacements (6), the influence of ξ for an eccentric orbit of a binary system can be read out readily from (17)–(19) in [34]. As for the α_2 test, in the limit of small eccentricity, ξ induces a precession of the orbital angular momentum around the direction \mathbf{n}_G with an angular frequency [34],

$$\Omega^{\text{prec}} = \xi \left(\frac{2\pi}{P_b} \right) \left(\frac{v_G}{c} \right)^2 \cos \psi, \quad (7)$$

where P_b is the orbital period, and ψ is the angle between \mathbf{n}_G and the orbital angular momentum. This precession would introduce observable effects in binary pulsar timing experiments (see section 4.1).

Similar to the case of a binary system, for an isolated, rotating massive body with internal equilibrium, Nordtvedt showed in [28] that ξ would induce a precession of the spin around \mathbf{n}_G with an angular frequency (note, in [28] $\xi^{\text{Nordtvedt}} = -\frac{1}{2}\xi$),

$$\Omega^{\text{prec}} = \xi \left(\frac{2\pi}{P} \right) \left(\frac{v_G}{c} \right)^2 \cos \psi, \quad (8)$$

where now ψ stands for the angle between the spin of the body and \mathbf{n}_G (see figure 1). This precession can be constrained by observables in the Solar system and solitary millisecond pulsars (see section 3 and section 4.2 respectively).

3. A weak-field limit from the Solar spin

At the birth of the Solar system ~ 4.6 billion years ago, the angle θ between the Sun's spin \mathbf{S}_\odot and the total angular momentum of the Solar system (its direction is represented by the norm of the invariable plane \mathbf{n}_{inv}) were very likely closely aligned (see figure 1 for notation), as suggested by our understanding of the formation of planetary systems. After the birth, the Newtonian torque on the Sun produced by the tidal fields of planets is negligibly weak (see (10)). Due to today's observation of $\theta \sim 6^\circ$, Nordtvedt suggested to constrain ξ to a high precision through constraining (8) [28]. Based on his α_2 test and an order-of-magnitude estimation, he already concluded $|\xi| \lesssim 10^{-7}$. Here we slightly improve his method and present detailed calculations to constrain ξ from the Solar spin.

For directions of \mathbf{S}_\odot and \mathbf{n}_{inv} , we take the International Celestial Reference Frame equatorial coordinates at epoch J2000.0 from recent reports of the IAU/IAG working group on cartographic coordinates and rotational elements [32, 1]. The direction of \mathbf{S}_\odot

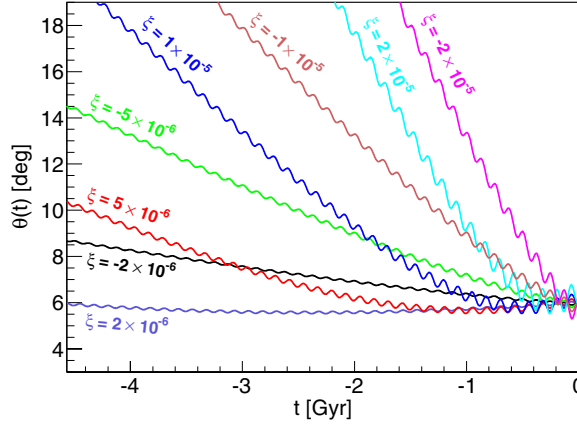


Figure 2. Evolutions of the misalignment angle $\theta(t)$ backward in time with different ξ vaules, which have taken both (8) and (10) into account.

is $(\alpha_0, \delta_0)_\odot = (286^\circ 13, 63^\circ 87)$ in the Celestial coordinates or $(l, b)_\odot = (94^\circ 45, 22^\circ 77)$ in the Galactic coordinates. The coordinates of \mathbf{n}_{inv} are $(\alpha_0, \delta_0)_{\text{inv}} = (273^\circ 85, 66^\circ 99)$ or $(l, b)_{\text{inv}} = (96^\circ 92, 28^\circ 31)$. The difference between these two directions is

$$\theta|_{t=0} = 5^\circ 97, \quad (9)$$

where $t = 0$ denotes the current epoch.

Assuming that the Sun's spin was closely aligned with \mathbf{n}_{inv} right after the formation of the Solar system, 4.6 Gyr in the past, one can convert (9) into a limit for ξ . For this, one has to account for the Solar movement around the Galactic center (~ 20 circles in 4.6 Gyr) when using (8) to properly integrate back in time for a given ξ . We show evolutions of the misalignment angle $\theta(t)$ in figure 2 for different ξ vaules. In calculations in figure 2, besides the contribution (8), we also include the precession produced by the Newtonian quadrupole coupling with an angular frequency,

$$\Omega_{J_2}^{\text{prec}} = \frac{3}{2} J_2 \frac{GM_\odot R_\odot^2}{|\mathbf{S}_\odot|} \sum_i \frac{m_i}{r_i^3}, \quad (10)$$

where M_\odot and R_\odot are the Solar mass and the Solar radius, m_i and r_i are the mass and the orbital size of body i in the Solar system, and $J_2 = (2.40 \pm 0.25) \times 10^{-7}$ [12]. The main contributions in (10) are coming from Jupiter, Venus and Earth. The coupling is very weak, and (10) has a precession period $\sim 9 \times 10^{11}$ yr, hence it precesses $\sim 2^\circ$ in 4.6 Gyr (notice a factor of two discrepancy with (15) in [28] mainly due to the use of a modern J_2 value). Such a precession hardly modifies the evolution of $\theta(t)$; besides, the precession (10) is around \mathbf{n}_{inv} which by itself does not change θ .

In figure 3 we plot the initial misalignment angle at the birth of the Solar system and the angle $\Delta\chi$ swept out by \mathbf{S}_\odot during the past 4.6 Gyr as functions of ξ . From figure 3 it is obvious that any ξ significantly outside the range

$$|\xi| \lesssim 5 \times 10^{-6} \quad (11)$$

would contradict the assumption that the Sun was formed spinning in a close alignment with the planetary orbits (say, $\theta_{\text{birth}} \gtrsim 10^\circ$). Limit (11) is three orders of magnitude better than that from superconducting gravimeter [38].

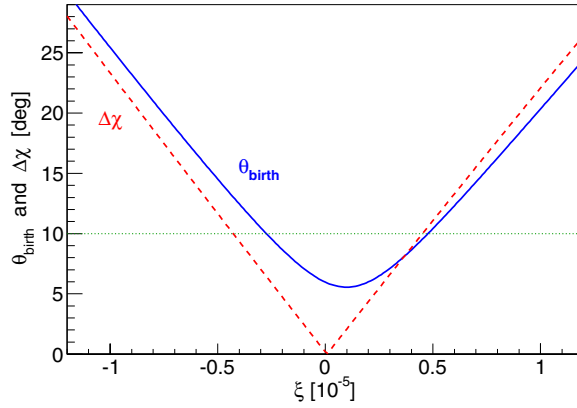


Figure 3. The initial misalignment angle θ_{birth} and the angle difference $\Delta\chi$ between current \mathbf{S}_{\odot} and \mathbf{S}_{\odot} at birth as functions of ξ . They are obtained from evolving \mathbf{S}_{\odot} according to (8) and (10) back in time to the epoch $t = -4.6$ Gyr.

4. Limits from radio millisecond pulsars

4.1. A limit from binary pulsars

According to (7), the orbital angular momentum of a binary system with a small eccentricity undergoes a ξ -induced precession around \mathbf{n}_{G} (here \mathbf{n}_{G} is the direction of the Galactic acceleration at the location of the binary). As mentioned in [35], this precession is analogous to the precession induced by the PPN parameter α_2 [34] with replacements (6). Hence the same analysis done for the $\hat{\alpha}_2$ test in [34] applies to the $\hat{\xi}$ test in binary pulsars.

Using the Galactic potential model in [31] with the distance of the Solar system to the Galactic center ~ 8 kpc, Shao *et al* [35] performed 10^7 Monte Carlo simulations to account for measurement uncertainties and the unknown longitude of ascending node (for details, see section 3 of [34]). From a combination of PSRs J1012 + 5307 and J1738 + 0333, they got a probabilistic limit (see figure 1 in [35] for probability densities from separated binary pulsars and their combination),

$$|\hat{\xi}| < 3.1 \times 10^{-4}, \quad (95\% \text{ CL}). \quad (12)$$

It is two orders of magnitude weaker than the limit (11) from the Solar spin, but it represents a constraint involving a strongly self-gravitating body, namely, NS-WD binary systems (see section 5).

4.2. A limit from solitary pulsars

Similar to the precession of the Solar spin, the spin of a solitary pulsar would undergo a $\hat{\xi}$ -induced precession around \mathbf{n}_{G} with an angular frequency (8). Such a precession would change our line-of-sight cut on the pulsar emission beam, hence change the pulse profile characteristics over time, see figure 1 in [33] for illustrations.

Recently, to test the local Lorentz invariance of gravity, Shao *et al* [33] analyzed a large number of pulse profiles from PSRs B1937 + 21 and J1744 – 1134, obtained at the 100 m Effelsberg radio telescope with the same backend, spanning about ~ 15 years. From various aspects, the pulse profiles are very stable, and no change in the profiles is found (see figures 2–7 in [33] for stabilities of pulse profiles). These results can equally well be used for a test of LPI of gravity.

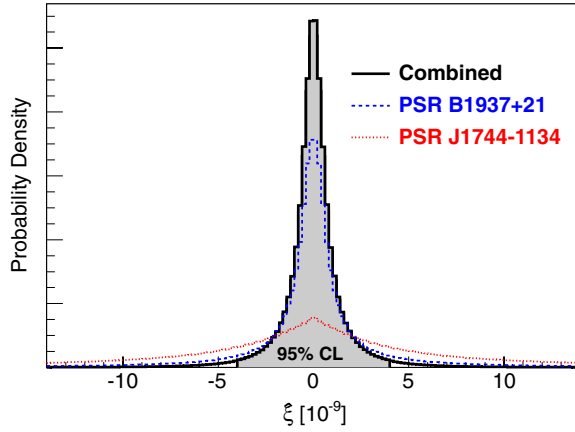


Figure 4. Probability density functions of the strong-field PPN parameter $\hat{\xi}$ from PSR B1937 + 21 (blue dashed histogram), PSR J1744 – 1134 (red dotted histogram), and their combination (black solid histogram). All probability density functions are normalized.

By using a simple cone emission model of pulsars [20], one can quantitatively relate a change in the orientation of the pulsar spin with that in the width of the pulse profile (see (10) in [33]). By using the limits on the change of pulse widths in table 1 of [33], we set up 10^7 Monte Carlo simulations to get probability densities of $\hat{\xi}$ from PSRs B1937 + 21 and J1744 – 1134. In simulations we use the Galactic potential model in [31] and all other parameters are the same as in [33] with replacements (6). The results are shown in figure 4 for PSRs B1937 + 21 and J1744 – 1134 and their combination. For the individual limits one finds

$$\text{PSR B1937 + 21: } |\hat{\xi}| < 2.2 \times 10^{-8}, \quad (95\% \text{ CL}), \quad (13)$$

$$\text{PSR J1744 – 1134: } |\hat{\xi}| < 1.2 \times 10^{-7}, \quad (95\% \text{ CL}). \quad (14)$$

They are already significantly better than the limit (11) obtained from the Solar spin. Like in [33], the analysis for PSR B1937 + 21 is based on the main-pulse. Also here, one could use the interpulse to constrain a precession of PSR B1937 + 21, which again leads to a similar, even slightly more constraining limit. As in [33], we will stay with the more conservative value derived from the main-pulse.

As explained in details in [34, 33], the combination of two pulsars leads to a significant suppression of the long tails in the probability density function. Assuming that $\hat{\xi}$ is only weakly dependent on the pulsar mass, PSRs B1937 + 21 and J1744 – 1134 give a combined limit for strongly self-gravitating bodies of

$$|\hat{\xi}| < 3.9 \times 10^{-9}, \quad (95\% \text{ CL}). \quad (15)$$

The limit (15) is the most constraining one of the three tests presented in this paper. It is more than three orders of magnitude better than the limit (11) from the Solar system and five orders of magnitude better than the limit (12) from binary pulsars. This is in accordance with the α_2 and $\hat{\alpha}_2$ results [28, 34, 33].

5. Discussions

Mach's principle states that the inertial mass of a body is determined by the total matter distribution in the Universe, so if the matter distribution is not isotropic, the gravity interaction

that a mass feels can depend on its direction of acceleration [7, 8]. The tests presented in this paper are Hughes–Drever-type experiments which originally were conducted to test a possible anisotropy in mass through magnetic resonance measurements in spectroscopy [14, 11]. We note that the constraint on LPI here is for the gravitational interaction, that is different from the LPI of Einstein’s equivalence principle related to special relativity, see e.g. [5, 2] and the review article [43].

Although we express our limits on the anisotropy of gravity in terms of the PPN parameter ξ (or its strong-field generalization $\hat{\xi}$), it is quite straightforward to convert them into limits on the anisotropy of the gravitational constant. From (6.75) in [42], one has

$$G_{\text{local}} = G_0 \left[1 + \xi \left(3 + \frac{I}{MR^2} \right) U_G + \xi (\mathbf{e} \cdot \mathbf{n}_G)^2 \left(1 - \frac{3I}{MR^2} \right) U_G \right], \quad (16)$$

where G_0 is the bare gravitational constant; I , M , and R are the moment of inertia, mass and radius of a system \mathcal{S} respectively; \mathbf{e} is a unit vector pointing from the center of mass of \mathcal{S} to the location where G is being measured (see [42]). The first correction only renormalizes the bare gravitational constant and is not relevant here. The second correction contains an anisotropic contribution. For solitary pulsars PSRs B1937 + 21 and J1744 – 1134, they both have $v_G^2 \sim 5 \times 10^{-7}$. Hence from (15), by using $I/MR^2 \simeq 0.4$ for a typical NS [18], one gets

$$\left| \frac{\Delta G}{G} \right|^{\text{anisotropy}} < 4 \times 10^{-16}, \quad (95\% \text{ CL}) \quad (17)$$

which is the most constraining limit on the anisotropy of G . It is four orders of magnitude better than that achievable with LLR in the foreseeable future [29].

For any ‘quasilinear’ theory of gravity, the PPN parameters satisfy $\beta = \xi$ [41]. Hence for such a theory, a limit on β of $\mathcal{O}(10^{-9})$ can be drawn, which is six orders of magnitude more constraining than the limit on β from the anomalous precession of Mercury [43]. Nordtvedt developed an *anisotropic PPN framework* [27] and suggested to use the binary pulsar PSR B1913 + 16 [26] and LLR [10, 29] to constrain its parameters. Our result shows that careful profile analysis of solitary pulsars can constrain some anisotropic PPN parameters more effectively. The standard model extension of gravity [4, 17] has 20 free parameters in the pure-gravity sector, of which a subset \bar{s}^{jk} appears in a Lagrangian term similar to (4) (see (54) in [4]), hence can be constrained tightly through our tests. We expect a combination of \bar{s}^{jk} (similar to (97) in [4]) can be constrained to $\mathcal{O}(10^{-15})$ ³.

At this point we would like to elaborate on the distinction between the weak-field PPN parameter ξ and its strong-field generalization $\hat{\xi}$. In GR, $\xi = \hat{\xi} = 0$, but a distinction is necessary for alternative gravity theories. Damour and Esposito-Farèse explicitly showed that in scalar–tensor theories, the strong gravitational fields of NSs can develop nonperturbative effects [9]. Although scalar–tensor theories have no LPI violation, one can imagine that similar nonperturbative strong-field modifications might exist in other theories with LPI violation. If the strong-field modification is perturbative, one may write an expansion like,

$$\hat{\xi} = \xi + \mathcal{K}_1 \mathcal{C} + \mathcal{K}_2 \mathcal{C}^2 + \dots, \quad (18)$$

where the compactness \mathcal{C} (roughly equals the fractional gravitational binding energy) of a NS ($\mathcal{C}_{\text{NS}} \sim 0.2$) is $\mathcal{O}(10^5)$ times larger than that of the Sun ($\mathcal{C}_{\odot} \sim 10^{-6}$). Hence NSs can probe the coefficients \mathcal{K}_i ’s much more efficiently than the Solar system.

Let us compare the prospects of different tests of LPI in the future. As mentioned before, the best limit on ξ from superconducting gravimeter [38] is of $\mathcal{O}(10^{-3})$. Modern superconducting gravimeters are more sensitive. They are distributed around the world, where

³ See relevant limits from LLR [3] and atom interferometry [22, 6] for comparison.

a total of 25 superconducting gravimeters form the global geodynamics project (GGP) network [36]. The sensitivity of a superconducting gravimeter, installed at a quiet site, is better than $1 \text{ nGal} \equiv 10^{-11} \text{ m s}^{-2}$ for a one-year measurement, which is less than the seismic noise level (a few nGal) at the signal frequencies of ξ [36]. However, the test is severely limited by the Earth model and unremovable Earth noises. Even under optimistic estimations for GGP, ξ is expected to be constrained to $\mathcal{O}(10^{-5})$ at best [36], which is four orders of magnitude away from (15). The analysis of LLR data usually does not include the ξ parameter explicitly, but with its analogy with α_2 , one can expect a limit of $\mathcal{O}(10^{-5})$ at best [23]. The Solar limit (11) is based on a long baseline in time (about 4.6 Gyr), hence it is not going to improve anymore. In contrast, the limits (12) and (15) will continuously improve with $T^{-3/2}$ solely based on current pulsars, where T is the observational time span [34, 33]. New telescopes like the Five-hundred-meter Aperture Spherical Telescope (FAST) [24] and the Square Kilometre Array (SKA) [37] will provide better sensitivities in obtaining pulse profiles, that will be very valuable for improving the limit of $\hat{\xi}$ (and also $\hat{\alpha}_2$ [33]), especially for the weaker pulsar PSR J1744 – 1134. In addition, discoveries of new fast rotating millisecond pulsars through FAST and SKA are expected in the future, which will enrich our set of testing systems and further improve the limits.

Let us elaborate on a possible correcting factor to our limits on ξ and $\hat{\xi}$, arising from a more rigorous treatment of the Galactic mass distribution. When estimating U_G , we have approximated it as $U_G \sim v_G^2$ which, e.g., at the location of the Sun gives $U_G/c^2 \simeq 5.4 \times 10^{-7}$. Mentock pointed out that the dark matter halo might invalidate such an approximation [21]. However, Gibbons and Will explicitly showed, by using a Galaxy model with spherically symmetric matter distribution, that such a correction is roughly a factor of two [15]. We use the Galaxy potential model in [31] that consists of three components, namely the bulge, the disk and the dark matter halo, and get a factor of 1.86.⁴ The results confirm the correcting factor in [15], and our limits on ξ and $\hat{\xi}$ should be weakened by this factor (as well as *all* previous limits on ξ in literature). Nevertheless, the limit (17) on the anisotropy of G will not change because only the product ξU_G enters in (16).

As a final remark, using the words of [15], also for pulsar astronomers Whitehead's gravity theory [39] ($\xi = 1$) is *truly dead*.

Acknowledgments

We thank Nicolas Caballero, David Champion, and Michael Kramer for valuable discussions. We are grateful to Aris Noutsos for reading the manuscript. LS is supported by China Scholarship Council (CSC). This research has made use of NASA's Astrophysics Data System.

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⁴ We also tested other models for the Galactic mass distribution. They all agree within 2%.

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