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Experimenting theory: The proofs of Kirchhoff's radiation law before and after Planck

> [My] remarks will show...that previous efforts at theoretical proof have not been on the right track at all, and also how little even in the simplest special cases they have been capable of making Kirchhoff's first law plausible.[1]

AMONG THE LAWS of physics established in the 19th century two were particularly outstanding and celebrated: Ohm's law of electrical conduction (1827) and Kirchhoff's law of the emission and absorption of radiation (1859). Both laws take the same mathematical form. The ratio of potential difference U and current I is a characteristic resistance R, or, $U/I=R$, as is the ratio of the emissive power e and the absorptive power a for fixed temperature and wavelength according to the expression $e/a=f$. [2] Both laws are notable for their simplicity and generality. Nonetheless, an essential difference separates them.

Ohm's law implies that each conductor has a distinctive resistance. Kirchhoff's law, on the other hand, has to do with universal quantities: "for rays of the same wavelength at the same temperature the ratio of emissive power and absorptive power is the same for all bodies." An equivalent and more common formulation states that the ratio of e to a "is a *universal* function of wavelength and temperature *only*."[3] In modern notation, $e/a=f(T, \lambda)$.

This universal function f acquired utmost importance at the end of the 19th century, as it described the unique emission spectrum for all "black bodies," bod-

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The following abbreviation is used: HP, Hilbert Papers, Manuscript Department, Niedersächsische Staats- und Universitätsbibliothek, Göttingen, Cod. Ms. D. Hilbert.

1. David Hilbert, "Vortrag Münster," HP, 586, 5f (manuscript for a lecture delivered at the 84th meeting of the Vereinigung Deutscher Naturforscher und Ärzte in Münster on 18 Sep 1912).

2. Georg Simon Ohm, *Die galvanische Kette, mathematisch bearbeitet* (Berlin 1827). In Ohm's notation $S=A/L$ where S denotes the electric current, L is a measure for the resistance ("reduced length"), and A the sum of all "tensions."

3. Gustav Kirchhoff, "Ueber den Zusammenhang von Emission und Absorption von Licht

HSPS, Volume 33, Part 2, pages 299-335. ISSN 0890-9997.

ies that absorb all incident radiation ($a=1$). The famous search for the correct formula for black-body radiation and Planck's solution to it amounted to the determination of Kirchhoff's law.

A second difference between the laws of Ohm and Kirchhoff lies in the histories of their justification and acceptance. Nobody in the 20th century would have demanded a new proof of Ohm's law, but, as the epigraph to his paper, taken from a talk by David Hilbert at the 1912 Naturforscherversammlung demonstrates, even more than half a century after the establishment of Kirchhoff's law and more than a decade after the final determination of its universal function in Planck's formula, no consensus about its validity had been attained.

Finally, Ohm stressed the mathematical nature of his proof of the law and emphasized that theory—Fourier's theory of heat conduction in fact—had been his constant guide.[4] Kirchhoff, on the other hand, not only arrived at his law in the course of experiments on Fraunhofer lines in the solar spectrum, he adopted much from experimentation with mirrors, prisms, and diaphragms in his proofs of this law—as many other physicists did in later attempts.

Hence, apart from a metaphorical use of the term "experimental" (when physicists test different and often incomplete sets of hypotheses and threads of different types of argument), experimental approaches played a direct role in the foundational discourse of Kirchhoff's law in theoretical reasoning.[5] I will call this phenomenon *experimenting theory*, as it concerns the role of experimental set-ups, the framing of the line of argument in experimental terms, the step-by-step procedures of varied situations, and, in particular, the postulating of idealized objects in thought-experiments.

This paper presents an example of experimenting theory from the field of classical radiation theory as a contribution to the discussion of the interplay of theory, experiment, and the use of instruments or tools (both material and conceptual) that has attracted much attention in recent years.[6]

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und Wärme," Akademie der Wissenschaften, *Monatsberichte* (Berlin, 1860), 783-787, 784, 786, reprinted in Gustav Kirchhoff, *Untersuchungen über das Sonnenspektrum und das Spektrum der chemischen Elemente und weitere ergänzende Arbeiten aus den Jahren 1859-1862* (Osnabrück, 1972), ed. Hans Kangro.

4. Christa Jungnickel and Russell McCormmach, *Intellectual mastery of nature. Theoretical physics from Ohm to Einstein.* Vol. 1. *The torch of mathematics, 1800-1870* (Chicago, 1986), 53.

5. Criticism of the physicists' lack of logic and rigor has been raised repeatedly; cf. Arthur Jaffe and Frank M. Quinn, "'Theoretical mathematics': Towards a cultural synthesis of mathematics and theoretical physics," *Bulletin (New Series)*, American Mathematical Society, *29* (1993), 1-13, which uses "experimental mathematics" to designate physicists' non-rigorous mathematics.

6. Cf. John Ackermann, "Review article: The new experimentalism," *British journal of philosophy of science, 40* (1989), 185-190; Michael Heidelberger and Friedrich Steinle, *Experimental essays - Versuche zum Experiment* (Baden-Baden, 1998); and Christoph Meinel, ed., *Instrument - Experiment: Historische Studien* (Bassum, 1999), esp. Klaus Hentschel,

I relate the history of Kirchhoff's law as far as it pertains to ideas about radiation and of Planck's work. This includes the evolution of the ascribed content of the law as well as its assumed foundational roots.[7] Planck's search for the correct justification of his radiation formula will be placed in the context of the debate over the derivation of Kirchhoff's law. Furthermore, I will ask what the analysis of the variety of approaches, arguments, and ontological claims that can be found in radiation theory may reveal about the conceptual framework available for Planck's researches. Finally, I will set forth the different forms of reasoning applied in proving a physical law. They range from procedures closely abstracted from experimental action like those found in Kirchhoff or Helmholtz, to a purely mathematical approach (such as Hilbert's) void of any experimental notion or object. I locate Planck's special method in a particularly powerful middle ground, while elucidating the great difficulties the establishment of a truly non-experimental theory in physics met before a new generation of quantum physicists appeared.

1. KIRCHHOFF'S LAW BEFORE PLANCK'S FORMULA

Fraunhofer's observation that specific absorption lines of the solar spectrum coincide with the main emission lines of sodium in a flame marked the beginning of many considerations of the relationship between the absorption and emission of luminous bodies. Kirchhoff, who characteristically preferred to complete a preexisting line of research than to begin a new one, attempted a full "drawing" of the spectrum of the sun in 1859. Simultaneously, he tried to condense qualitative knowledge about the coincidence of emission and absorption spectra into a quantitative law that would govern spectral analysis.[8] By making explicit the insight that information on the physical composition of the stars can be inferred from comparison of the spectra of the sun and the chemical elements, he (and Robert Bunsen) founded spectral analysis. While the qualitative rule that a body can emit all the wavelengths it absorbs applied generally to all radiating substances, the exact law—which Kirchhoff arrived at in December 1859—no longer applied to systems like salt in a flame. Rather, it referred to situations of equilibrium in which the mechanical theory of heat held. This limitation gave rise to much confusion about the

"Historiographische Anmerkungen zum Verhältnis von Experiment, Instrumentation und Theorie," ibid., 13-51.

7. This paper is a history of justifications rather than a history of discoveries. For the latter see Edmund T. Whittaker, *A history of the theories of aether and electricity*. Vol. 1. *The classical theories* (New York, 1951), chapt. 12; Hans Kangro, *Vorgeschichte des Planckschen Strahlungsgesetzes* (Wiesbaden, 1970); and "Kirchhoff und die spektralanalytische Forschung," in Kirchhoff, *Untersuchungen* (ref. 3), 1-54; Hans-Georg Schöpf, *Von Kirchhoff bis Planck. Theorie der Wärmestrahlung in historisch-kritischer Darstellung* (Berlin, 1978); Eisui Uematsu, "The role of Kirchhoff's 1859 work in the history of radiation theory" [in Japanese], *Kagakusi kenkyu: Journal of history of science of Japan, 25* (1986), 14-19.

8. Jungnickel and McCormmack (ref. 4), 299, 301.

validity and applicability of Kirchhoff's law. Specialists in the field like Aimé Cotton and Heinrich Kayser would later write extensive reviews to determine the range of application of Kirchhoff's law and examine its theoretical implications.[9]

Kirchhoff took it for granted in 1860 that the ratio of emissive power to absorptive power must be the same for all bodies. The open question was whether this relation applied to each wavelength separately.[10] Consideration of single wavelengths arose for him from his researches on emission lines of colored flames and absorption lines in the solar spectrum.

The necessity of a double proof

Kirchhoff's paper, "On the relation between emission and absorption of light and heat," was presented to the Berlin Academy of Sciences on December 15, 1859.[11] Originating from his observations of Fraunhofer lines, which he had presented seven weeks earlier to the same audience,[12] it reported a general law reached "by a very simple theoretical consideration." Kirchhoff's simple proof employed an idealization of the sodium colored flames he used in his experimental investigations: "it appears unobjectionable to postulate the existence of a body that, of all heat radiations, the luminous as well as the dark, emits only rays of *one* wavelength and absorbs only rays of the *same* wavelength."[13]

The proof involved two infinitely extended plates facing one another (figure 1) one of them, C, with the postulated properties for a wavelength Λ, the other, c, of arbitrary emissive and absorptive qualities. The back sides of the facing plates had perfect mirrors in order to retain all radiation between them (R and r). Light of the particular wavelength Λ therefore bounces between the plates, suffering partial absorption and partial reflection each time. Kirchhoff claimed that by summing the successively absorbed and reflected energies and using the condition of final equilibrium he could demonstrate the validity of his law. Then, imagining the second plate c replaced by an arbitrary plate c' at the same temperature, he concluded that the relation must be universal, the same for all bodies. Since according to the calculation the ratio e/a must equal for any plate c, c' etc. that of C (for

9. Aimé Cotton, "The present status of Kirchhoff's law," *Astrophysical journal, 9* (1899), 237-268; Heinrich Kayser, *Handbuch der Spectroscopie*. Vol. 2 (Leipzig 1902). Aimé Auguste Cotton (1869-1951) completed his thesis on "Recherches sur l'absoption et la dispersion de la lumiére par les milieux doués du pouvoir rotaire" in 1896 at the Ecole Normale in Paris. On Kayser's see Matthias Dörries, "Heinrich Kayser as philologist of physics," *HSPS, 26:1* (1995), 1-33, on 27-31.

10. Gustav Kirchhoff, "Ueber das Verhältnis zwischen dem Emissionsvermögen und dem Absorptionsvermögen der Körper für Wärme und Licht," *Annalen der Physik, 109* (1860) 275-301, on 276.

11. Kirchhoff, "Zusammenhang" (ref. 3).

12. Gustav Kirchhoff, "Ueber die Fraunhoferschen Linien," *Monatsberichte*, Akademie der Wissenschaften, Berlin, 1860, 662-665.

13. Kirchhoff, "Zusammenhang" (ref. 3), 784.

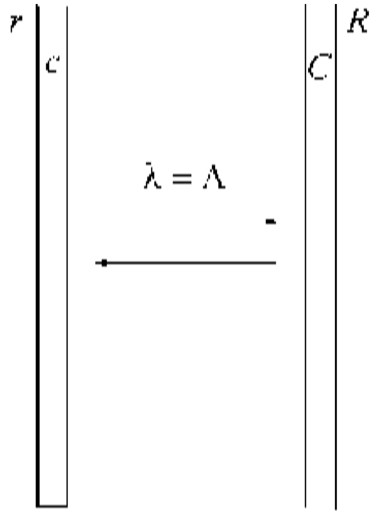

FIG. 1 Setup of Kirchhoff's first proof

$\lambda=\Lambda$). He added that his argument secured the theoretical foundation of the method he had proposed for the chemical analysis of the solar atmosphere.[14]

Only a few weeks later, Kirchhoff apparently changed his view that a general proof could be attained by the simple theoretical considerations he had invoked. In January 1860, he submitted a second much more involved proof without initially commenting on the fate of the first one. Two years later, he published a structurally improved version of this second derivation, which his editors chose for his collected works.[15] In this revision, he commented on the supposition of bodies that perform only at one specific wavelength:[16] "The necessary completion of the proof may easily be given when a plate is supposed to exist, having the property of

14. Ibid., 786f. Kirchhoff did not recognize that his law, which supposed thermal equilibrium, could hardly be used for non-equilibrium phenomena like burning gases. In 1903 Ernst Pringsheim spoke of the need to deprive Kirchhoff's law of the "nimbus" of being the theoretical foundation of spectral analysis; "Über die Strahlungsgesetze," *Zeitschrift für wissenschaftliche Photographie, 1* (1903), 391-417, on 394.
15. Gustav Kirchhoff, *Untersuchungen über das Sonnenspektrum und die Spektren der chemischen Elemente* (2nd edn., Berlin, 1962), appendix, "Über das Verhältnis zwischen dem Emissionsvermögen und dem Absorptionsvermögen der Körper für Wärme und Licht," 22-39; also in *Gesammelte Abhandlungen.* Vol 1 (Leipzig, 1882) 571-598, English trans. in D.B. Brace, ed., *The laws of radiation and absorption: Memoirs by Prévost, Stewart, Kirchhoff, and Kirchhoff and Bunsen* (New York, 1901), 75-97. The version of 1862 omits the final paragraph on the supposed validity of the law for fluorescent bodies.
16. Note that Kirchhoff's one-wavelength plate is now, by itself, a perfect mirror for all radiation with a wavelength different from the specified one.

transmitting undiminished rays whose wave length lies between λ and $\lambda+d\lambda$ and whose plane of polarization is parallel to the plane a; but which completely reflects rays of other wave lengths or of an opposite polarization." He now regarded his supposition as inadmissible. Instead, he relied on an even more intricate object: "[A] plate is possible which, of the rays striking it at the same angle, transmits and reflects them in different degrees according to their wave length and plane of polarization. A plate, which is so thin that the colors of thin films are visible and which is placed obliquely in the path, shows this."[17]

Was the case for this second theoretical object any better than that for the first? In 1860 Kirchhoff stressed that his second proof rested on the assumption of the existence of completely black bodies. The crucial assumption, however, was not the black body, but the diathermanous plate that was mentioned rather in passing. It "shows the colors of thin plates in visible radiation and, partly owing to its small thickness, partly owing to its material composition, neither emits nor absorbs a recognizable amount of radiation."[18]

The perfectly diathermanous plate was conceived to be fully transparent to heat waves and so thin—on the order of a wavelength—as to show the colors of the thin plates. Then it reflects radiation in accordance with the wavelength. But this object had no better justification than the original one-wavelength plate. Two years later, Kirchhoff introduced all his three assumptions—black bodies (the "essential aid" in the proof), completely diathermanous bodies, and perfect mirrors—right from the outset.[19]

Kirchhoff was soon subject to harsh criticism for both proofs.[20] In 1863, Frédéric de la Provostaye, and forty years later, Ernst Pringsheim (who held that "Kirchhoff's derivation is without any flaw"), doubted the admissibility of completely black, completely reflecting, and completely diathermanous substances.[21] In 1864 Wilhelm Wien demonstrated that one-wavelength plates would violate the second law of thermodynamics, the very starting point of Kirchhoff's considerations.[22] The French physicist Aimé Cotton, in his review of the status of Kirchhoff's

17. Kirchhoff (ref. 15), 26; cited in Brace (ref. 15), 79f.

18. Kirchhoff (ref. 10), 279.

19. Kirchhoff (ref. 15), 23 (76).

20. Gustav Kirchhoff, "Zur Geschichte der Spektral-Analyse und der Analyse der Sonnenatmosphäre," *Annalen der Physik, 194* (1862), 94-111, a justification against priority claims by Balfour Stewart and others; Daniel M. Siegel, "Balfour Stewart and Gustav Robert Kirchhoff: Two independent approaches to 'Kirchhoff's radiation law,'" *Isis, 67* (1976), 565-600; Kangro (ref. 3), 24-26. Stewart plays no role in what follows since his formulation did not lead to a universal function.

21. Kayser (ref. 9), 26; Ernst Pringsheim, "Herleitung des Kirchhoffschen Gesetzes," *Zeitschrift für wissenschaftliche Photographie, 1* (1903), 360-364.

22. Wilhelm Wien, "Temperatur und Entropie der Strahlung," *Annalen der Physik und Chemie, 52* (1894) 132-165, on 163. Owing to the Doppler effect, a plate that could completely reflect, absorb, and let penetrate certain wavelength ranges would lead to contradictory results when moved. Apparently referring to Kirchhoff's second proof, Wien concluded

law in 1899, concluded that the first proof, "which is too frequently reproduced in the classic works of the present day, does not establish the law rigorously," but he accepted the second proof since the "imaginary bodies [it assumed] may be realized with a higher and higher degree of approximation, and this renders their use legitimate."[23] The spectroscopist Heinrich Kayser shared this view for some of the "imaginary bodies" Kirchhoff had employed, but not for all. He could not accept the second proof.[24] Finally, in 1909, Wien, who himself had proposed a formula for the function Kirchhoff tried to establish, judged Kirchhoff's second proof to be "extremely artificial and onerous."[25] Only Woldemar Voigt called the proof "admirable."[26] Balfour Steward's dictum—"the proof of the Heidelberg Professor is so very elaborate that I fear it has found few readers"—turned out wrong in the long run.[27]

The second proof models itself even more on experimentation than the first, invoking an intricate setup of holes or diaphragms, completely black walls, ideal mirrors, and a perfectly diathermanous plate. Kirchhoff diagramed the argument of the proof in three drawings (figure 2).

The style of the proof may be illustrated by the following quotations:[28]

> Now suppose the surface 2 removed, and the opening closed by a portion of a perfectly reflecting spherical surface, placed directly behind it.
>
> In the arrangement described in Figure 2 imagine a plate of the kind described and designated as *P*, brought between the openings 1 and 2.
>
> Let opening 2 be closed by a black surface…and let opening 3 be closed in the first place by a similar surface, and next by a perfect concave mirror.
>
> If we now imagine the body *C* replaced by another black body of the same temperature.

We have a combination of imagined situations and actions with imaginary objects described as if a real experiment was being performed.

that one can only require that certain wavelengths be reflected, etc., to a high degree but not fully.

23. Cotton (ref. 9), 267.

24. Kayser (ref. 9), 27.

25. Wilhelm Wien, "Theorie der Strahlung," *Encyklopädie der mathematische Wissenschaften*. Vol. 5 (Leipzig, 1909), 282-357, on 285.

26. Woldemar Voigt, "Über die Proportionalität von Emissions- und Absorptionsvermögen," *Annalen der Physik, 67* (1899), 366-387, on 366.

27. Balfour Stewart, "Reply to some remarks by G. Kirchhoff in his paper 'On the history of spectrum analysis,'" *Philosophical magazine, 25* (1863), 354-360, on 359, cited after Siegel (ref. 20), 589.

28. Kirchhoff (ref. 15), 25-27 (English transl., 79-81).

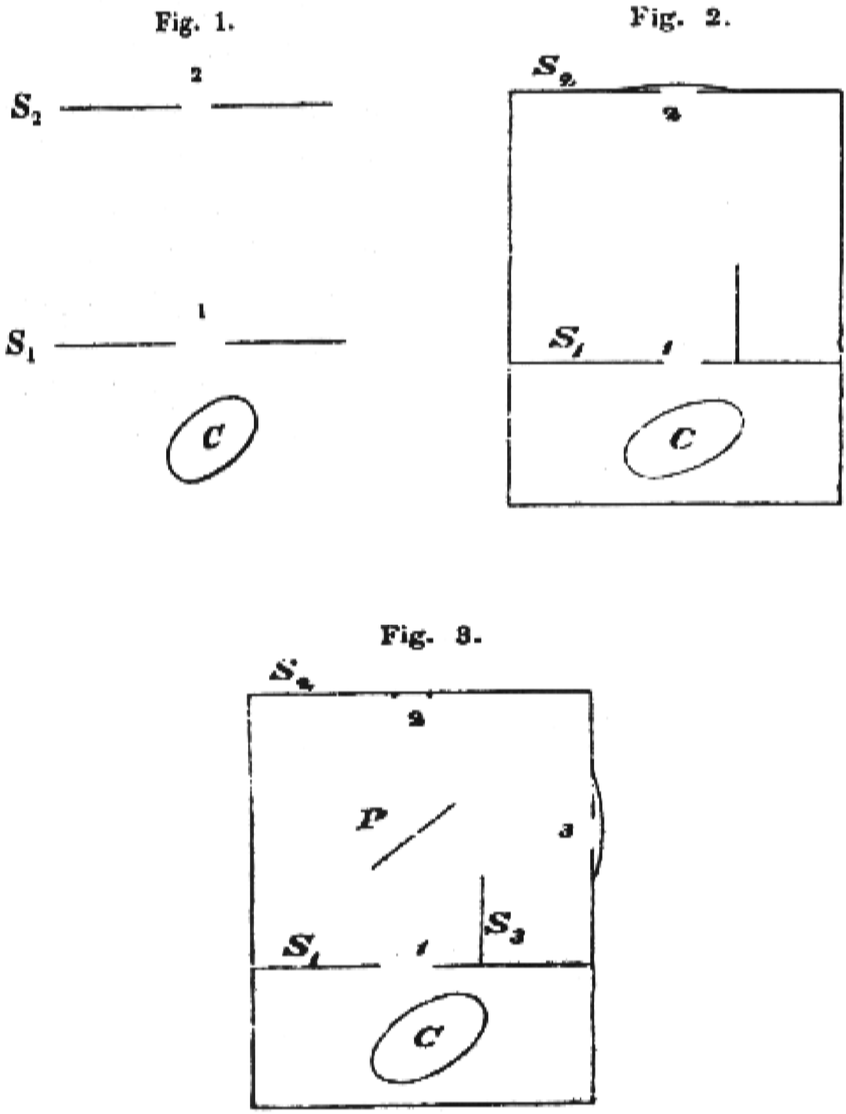

FIG. 2 Drawings used in Kirchhoff's second proof of his law. Kirchhoff (ref. 15), 24-26.

The central argument demonstrating the universality of Kirchhoff's function f follows by combining the relevant properties and laws of the objects employed, replacing the body C by a different C' , and varying thickness of the diathermanous plate P. The main step is to consider the emitted energy K of the body C that is reflected by the plate P

$$K = \int d\lambda \; e(\lambda) r^2 (\lambda),$$

where the function $r(\lambda)$ describes the reflection according to the theory of thin plates and $e(\lambda)$describes the emissive power of the black body C for wavelength λ. In the first step, C is taken to be black, that is, $a = 1$. In thermal equilibrium, K must not change when the body C is replaced by another C' with a distribution of the energy emission over the wavelength $e'(\lambda)$, i.e. $\int d\lambda (e - e')r^2 = 0$. The conclusion that this relation can only be satisfied when the functions for the emissive powers e and e' coincide, required a detailed Fourier analysis of the structure of the function $r(\lambda)$.

In distinction to the first proof, Kirchhoff now considered both polarized radiation and the case that the space in which the heat radiation propagates may be filled by a diffracting medium. With help of a law by Helmholtz, he further generalized the range of applicability of his law for cases with absorptive and reflecting media.[29] The much greater length of the second proof, hence, had two reasons: first, the more intricate experimental argument and, second, the extension of the general theory to a wider range of applicability for radiation within material media of various kinds.

Kirchhoff's second proof was criticized in print as early as 1863 by the French physicist Frédéric de la Provostaye, who would not admit perfect mirrors or fully diathermanous bodies.[30] Heinrich Kayser systematically discussed these objections, among others, in his very detailed study in 1902. He identified four questionable presuppositions: First, he pointed out that, "strictly speaking, a black body cannot exist. Under given circumstances, however, any body can play this role approximately." The assumption of the existence of completely diathermanous bodies presented much greater difficulties. A vacuum or possibly a dilute gas might pass the diathermal test, but since it would emit nothing it would not reach thermal equilibrium.[31] Perfect mirrors, which had been used in the literature since Fourier, were the least problematic abstraction. And last there was the implicit assumption that only the local physical condition would determine emission and absorption at a certain point.[32] Kayser's thorough analysis of Kirchhoff's second

29. Brace (ref. 15), 32; Hermann von Helmholtz, *Handbuch der physiologischen Optik.* Vol. 1 (Leipzig, 1856), 169.
30. Frederic de la Provostaye, "Considération théorique sur la chaleur rayonnante," *Annales de chimie et physique, 67* (1863), 5-51.
31. Kayser (ref. 9), 26-27.
32. Ibid., 30f. Kayser estimated that these dependencies were negligible.

proof arrived at the insight that although ideal mirrors and black bodies should be of no harm as idealizations in general (since they can be approximated), the completely diathermanous plate was as much a phantom as the one-wavelength plate.

Helmholtz and a prism

Kayser's analysis also mentioned a different but related treatment in Paul Drude's book on optics from 1900.[33] This new proof, however, was first given by Hermann von Helmholtz in his famous lectures of the early 1890s, which influenced many physicists, among them Max Planck. The relevant part of these lectures was not published until 1903.[34]

Helmholtz opened his proof (figure 3) with the requirement of thermal equilibrium and the consideration of black bodies.[35] Then all black bodies at the same temperature must emit the same radiation energy. However, the partial intensity for a certain color might exceed that for other black bodies provided that it was less for different colors. Hence, Helmholtz proceeded,[36]

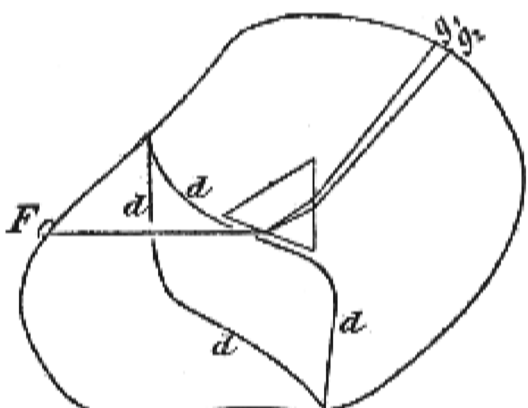

Fig. 28.

FIG. 3 Drawing used by Helmholtz in his proof (ref. 34), 165.

33. Paul Drude, *Lehrbuch der Optik* (Leipzig, 1900), 454-457.
34. Hermann von Helmholtz, *Vorlesungen über theoretische Physik.* Vol. 6. *Vorlesungen über Theorie der Wäerme* (Leipzig, 1903).
35. Helmholtz also presented two different proofs, the first of which, in *Vorlesungen* (ibid.), 162-164, resembles Stewart's argument. As it only applies to radiation absorbed and emitted perpendicular to the surface, the statement is of limited generality. Kayser (ref. 9), 8-12.
36. Helmholtz (ref. 34), 165f.

> Let us imagine a completely transparent prism in the interior of an absolutely black cover. Then appropriately arranged completely reflecting diaphragms *d* (Fig. 23) can insure that only radiation from one side of the cover that originates at a certain surface element *F* can enter a prism as a straight pencil and be refracted so that a point g_1 on the other side of the cover receives only radiation of the pencil with the color f_1, a different point g_2 only that of color f_2.

Following this scenario, Helmholtz demonstrated that when the emission for the first color f_1 at the point g_1 was greater than that for all other black bodies, and accordingly less for the second color, the temperature at the first point would decrease at the expense of the second. The central ingredient for this argument was the Helmholtz reciprocity theorem: for each light ray traveling a certain path, a light ray that traveled the same path but in reversed direction would undergo the same rate of absorption, reflection, diffusion, etc. as the original one. Helmholtz then generalized his result for arbitrary bodies in a rather cursory, if not misleading, manner in order to arrive at Kirchhoff's law.[37]

Helmholtz's line of argument, although very different from Kirchhoff's, employed a similar method of reasoning: an ingenious thought experiment. Like Kirchhoff, Helmholtz presupposed the existence of perfect mirrors and completely black bodies. In effect, he replaced the completely diathermanous plate by the completely transparent prism. Kayser made short work of the Helmholtz-Drude proof by noting that complete transparency *and* dispersion of light simply exclude one another. Hence, once again, the idealized object and the proposed thought experiment did not exist even in principle. The 19th-century proofs of Kirchhoff's law can be summarized as in table one.

Phenomenology at stake

Kirchhoff's theoretical work, which influenced a whole generation of German and European physicists, went back to Franz Neumann, Kirchhoff's teacher in Königsberg. Woldemar Voigt, another disciple of Neumann's, described the general character of the school's phenomenological approach as follows: theory "should describe the motions that occur in nature completely and in the simplest way," making possible "rigorous conclusions on the basis of a minimum of assumptions....Such a view is called phenomenological, which means that the foundations of the theoretical treatment are taken exclusively from direct observation." Voigt criticized the atomistic view of matter for its compulsion to obtain an *explanation* of effects at the cost of high ambiguity and arbitrariness.[38] The pheno-menologist,

37. He claims that the same ratio of emissive and absorptive power "that applies to the total radiation must also apply for each single kind of radiation separately," which would imply constancy with respect to the wavelength. Helmholtz (ref. 34), 166.
38. Woldemar Voigt, "Phänomenologische und atomistische Betrachtungsweise," Emil Warburg, ed., *Physik, Kultur der Gegenwart,* Ser. 3, *3:1,* ed. Emil Warburg (Leipzig, 1915),

Table 1: 19th-century proofs of Kirchhoff's law and its refutations

AUTHOR	*EXISTENCE CLAIM*	*AUTHOR*	*REFUTATION*
Kirchhoff Dec. 1859	bodies that emit and absorb only radiation of one specific wavelength (and fully reflect all others)	**Kirchhoff 1862**	justified by nothing
		Wien 1894	if in motion would violate 2nd law
Kirchhoff Jan. 1860	bodies showing colors of thin plates without emitting or absorbing any radiation themselves	**Provostaye 1863**	"hypothèses gratuites"
Kirchhoff 1862	completely diathermanous bodies	**Kayser 1902**	"The limit of such a body would be the vacuum."
Helmholtz c. 1890 Drude 1900	completely transparent prism	**Kayser 1902**	no dispersion with complete transparency
Richarz 1903	a diffraction grating would work instead of a prism	**Pringsheim 1903**	ray optics inappropriate for radiating ether

in contrast, does not conceive of possible molecular mechanisms to explain effects and laws in a reductionistic way, but rather describes effects quantitatively and accurately in straightforward equations. Considered "today's greatest physicist" by Voigt, and praised for his "caution and conscientiousness" by Helmholtz, Kirchhoff was widely seen as a model scientist. In Boltzmann's words he defined the "prototype of the German way to treat mathematical physical problems," as Euler, Gauss, and Neumann had done previously.[39] Radiation theory put phenomenology to the test since the basic phenomenological doctrine of banning the use of special hypothetical models naturally extended to the ban of conceptual objects with idealized properties never observed in reality. Was Kirchhoff's proof of his law, which met with so much criticism, the exception to this rule?

Boltzmann did not comment on this question. Rather, he praised the beauty of Kirchhoff's work on mathematical methods: highest precision in hypotheses, careful analysis ("feine Durchfeilung"), amplification of insights without concealing any difficulties, and specification of the slightest obscurities. He opposed Kirchhoff's approach with Maxwell's work on gas theory, where, by means of a miraculous substitution, "which to justify there is no time...the formulae spew out result after result, until, as a surprising final effect, the heat equilibrium of a heavy gas is obtained."[40] However, Kirchhoff's second proof does not appear so distinct from Maxwell's style as Boltzmann suggested. Kirchhoff apparently did not discuss his law in his lectures on mathematical physics suggesting that his theory—although with results of eminent importance—was no exemplar of mathematical physics.[41]

There was at least one physicist who saw the necessity to justify the phenomenologists' inclination to deal with conceptual bodies, Friedrich Pockels. In 1903, Pockels, Kirchhoff's former colleague at Heidelberg, commented on the form of Kirchhoff's argumentation: "Conceptual operations with bodies or processes that are, in reality, only approximately realizable, may appear strange at first sight; it is, however, perfectly admissible as a means of simplifying the argument, for the truth of the facts to be demonstrated cannot depend on the degree of perfection of our artificial instruments."[42]

Pockels observed that the Physical-Technical Imperial Institute had almost realized the production of these "artificial" black bodies. Whereas other commenta-

714-731, on 715f, Olivier Darrigol, *Electrodynamics from Ampére to Einstein* (Oxford, 2000), 43-49.

39. Quotations of Voigt, Helmholtz, and Boltzmann from Klaus Hentschel, "Gustav Robert Kirchhoff und Robert Wilhelm Bunsen," Karl von Meyenn, ed., *Die großen Physiker, 1* (Munich, 1997), 416-430, on 416.

40. Ludwig Boltzmann, "Gustav Robert Kirchhoff," in his *Populäre Schriften* (Leipzig, 1905), 53-75, on 73.

41. Gustav Kirchhoff, *Vorlesungen über mathematische Physik, Vierter Band.* Vol. 4. *Theorie der Wärme* (Leipzig, 1894).

42. Friedrich Pockels, "Gustav Robert Kirchhoff," Fritz Schöll, ed., *Heidelberger Professoren aus dem 19. Jahrhundert* (Heidelberg, 1903), *2,* 243-263, on 256f.

tors complained about the length and complexity of the proof, Pockels implied that without assuming these conceptual bodies, the proof still would be even more complicated. If Kirchhoff was a model theoretical physicist, and his approach exemplary, how does his proof of his law fit into the picture? In what sense did Kirchhoff's "subtle way" of deriving his law independently of any material properties later became "the methodological model for the investigations of Planck and Einstein"?[43]

Planck, who edited the publication of Kirchhoff's works on emission and absorption for *Ostwald's Klassiker* in 1898 did not mention any of the general objections discussed above.[44] Again, in 1906, when he presented his proof of the law, he did not indicate whether, if at all, his treatment was influenced by Kirchhoff. Three years later, Wien justified disregarding Kirchhoff's approach: "Today, it is hardly necessary to follow Kirchhoff's original proofs if the definitions and starting point are chosen slightly differently."[45]

Further analysis around 1900

Friedrich Richarz, who edited Helmholtz' lectures on the theory of heat, pointed out that he could save Helmholtz' proof by a simple modification, and thus would remain within the framework of the thought-experiment. Richarz reminded his readers that in his lectures, Helmholtz had mentioned the possibility of using a line grating instead of a prism in the critical steps. By using a grating, the contradiction in the properties assumed for the prism could be avoided. Richarz concluded that Helmholtz' "simplification" of Kirchhoff's proof was still "thoroughly flawless."[46]

Ernst Pringsheim provided the simple new proof in 1901. It required a retraction from Richarz: "Even in this way, the proof can still not be given in a manner free from objection, a fact to which my friend E. Pringsheim drew my attention; unless consideration is restricted to mutual irradiations of parts of the *surface of the body*, as Helmholtz did in his lectures." Since "an essential advance" in recent radiation theory was to treat "directly the *radiation present in the ether*, detached from the bodies that emitted it," Helmholtz' set-up, along with Kirchhoff's, no longer had any value.[47]

A professor in Breslau, originally from the Physical-Technical Imperial Institute at Berlin, Ernst Pringsheim concluded from his experimental work that Kirchhoff's law would "not be valid for all kinds of light but only for those phe-

43. Hentschel (ref. 39), 430.

44. Gustav Kirchhoff, *Abhandlungen über Emission und Absorption* (Ostwald's Klassiker der Exacten Wissenschaften, Nr. 100), ed. Max Planck (Leipzig, 1898), 37-41.

45. Wien (ref. 25), 285.

46. Franz Richarz, "Bemerkungen zur Theorie des Kirchhoffschen Gesetzes," *Zeitschrift für wissenschaftliche Photographie, 1* (1903), 5-8, on 8.

47. Franz Richarz, "Nochmalige Bemerkung zur Theorie des Kirchhoffschen Gesetzes," *Zeitschrift für wissenschaftliche Photographie, 1* (1903), 359-360.

nomena for which emission of light is a function of the temperature only." No gaseous light sources existed that obeyed the law.[48] Richarz had to concede that Helmholtz' proof could not be saved. Pringsheim had managed his flawless proof, however, by bringing in an argument that lay outside the theoretical framework in which the discussion had been conducted. Kirchhoff, Helmholtz, and Kayser supposed that radiation was emitted only at the surfaces of bodies and otherwise obeyed ray optics. Pringsheim worked with the new understanding of radiating ether or pure radiation. Light paths restricted by diaphragms so that that radiation would come from one well-defined surface area and travel to a distinct other one, could no longer be accepted.[49] Pringsheim did not need any of Krichhoff's questionable plates: "The derivation that is given in the following does not make assumptions of this kind, but starts with the empirical fact that arbitrarily many bodies exist or are producible, respectively, whose absorptive power varies in completely different ways from wavelength to wavelength."[50] Combined with Carnot's principle, which provided the indispensable basis of all arguments, this assumption sufficed.

Pringsheim took from prior derivations the realization that, after the implementation of thermal equilibrium, the next most important step lay not so much in the radiation geometry but in the replacement of one body by another with different physical properties. While Kirchhoff and Helmholtz treated the generalization of the law over all wavelengths and for all bodies in two quite separate steps, Pringsheim saw that both could be done together: The replacement changes emissive and absorptive properties generally (for example, being more or less black), and also the spectral variation of these properties. This fact can be exploited to deduce from the constancy of the ratio of total emission and absorption to that for each wavelength separately.

Pringsheim continued with two steps which Kirchhoff had taken as well (establishing the formula and exhibiting the unique radiation distribution for a black body), but in reverse order: First, he considered a body with absorptive power A_λ and the radiation within a cavity that contains this body, and established that, in equilibrium, a unique radiation distribution must be reached that is "quantitatively and qualitatively the same as a completely black-body would emit, if it existed."[51] In the demonstration, not the body but the cavities, or rather its walls, are replaced

48. Ernst Pringsheim, "Kirchhoff's Gesetz und die Strahlung der Gase," *Annalen der Physik, 45* (1892), 428-459, on 428f.

49. Woldemar Voigt, *Thermodynamik.* Vol. 2 (Leipzig, 1904), §117 (radiation "occurs" in a cavity), §125 (the thermodynamic treatment best fits unoriented radiation whereas Kirchhoff's law concerns oriented radiation).

50. Ernst Pringsheim, "Herleitung des Kirchhoffschen Gesetzes," *Zeitschrift für wissenschaftliche Photographie, 1* (1903), 360-364, on 361. Pringsheim presented his proof first in Berlin under the eyes of Planck as "Einfache Herleitung des Kirchhoff'schen Gesetzes," Deutschen Physikalischen Gesellschaf, *Verhandlungen, 3* (1901), 77, 81-84. In his first presentation, Prinsheim also introduced his assumptions in the course of the argument, while in 1903 he presented them at the outset.

51. Ibid., 363.

with others having different material properties, but maintained at the same temperature. Assuming that the distribution of the emitted radiation energy over the wavelength λ was different for different cavity materials $e_{1\lambda}, e_{2\lambda}, \ldots, e_{n\lambda}$, then, since the total absorbed energy of the body must not change,

$$\int_0^\infty A_\lambda e_{1\lambda} d\lambda = \int_0^\infty A_\lambda e_{2\lambda} d\lambda = \ \ldots \ = \int_0^\infty A_\lambda e_{n\lambda} d\lambda \, .$$

The absorptive power A_λ of the body within the cavities could be an arbitrary function of the wavelength unrelated to the emission of the different wall materials $e_{1\lambda}, e_{2\lambda}, \ldots, e_{n\lambda}$. These, in consequence, must be equal to a universal function for an ideal black body, since otherwise the integrals would not all coincide.

Kirchhoff's formula came as a second step, by considering the mutual irradiation of two surface elements—one of the body and one of the cavity—and assuming Helmholtz' (unproven) reciprocity theorem. Here Pringsheim set aside the new view of radiating ether. Indeed, Pringsheim never made explicit in his publications of his proof the new view on radiation he used to invalidate the Helmholtz-Richarz argument.

Pringsheim's proof agreed with Kirchhoff's to the extent that it employed an experimental way of thinking. Consider the style of argument:[52]

> Placing the same body K into an arbitrary number of cavities, one after the other, that all have the same temperature but are completely different in shape and composition of the bodies, the emission of the body K remains unchanged as does the absorptive power A_λ for each determinate kind of radiation.

In distinction to earlier thought-experiments, Pringsheim, for the first time, presents an experiment that can be performed in the laboratory, and his words can be read as an experimental observation. Only in the next step did it become clear that his statement arose from theoretical considerations. Pringsheim presented the variation of the absorptive power A_λ of the body K not as a replacement of the actual body, but as a consequence of the arbitrariness of the function A_λ.

The crucial step in the mathematical parts of the argument in the proofs of Pringsheim and Kirchhoff are similar: The integrals of certain emission functions for black bodies of different material properties are combined with a set of auxiliary functions to show that the emission functions are identical. Kirchhoff generated the set of auxiliary functions by changing the thickness of the (non-existent) diathermanous plate. Pringsheim generated them by supposing an infinite number of substances with different functions of absorptive power. Whereas Kirchhoff, using Fourier analysis, could claim mathematical conclusiveness but had prob-

52. Pringsheim, "Einfache" (ref. 50), 82.

lems with his ontology, Pringsheim had problems with both. He did not point to a mathematical theory that insured orthogonality of his function set, or that the functions of existing or producible materials actually formed a complete set. These subtle questions of functional analysis would surface later in a rigorous mathematical analysis of the proofs.

Just as the proofs of the theory differ, so did its scope. Pringsheim only considered radiation in otherwise empty space, whereas Kirchhoff, Helmholtz, and later Planck worked hard to extend the theory to radiation in transparent, diffusing, and absorbing media. The determination of the generality of the law must be seen as part of its foundation. That Pringsheim did not even mention this necessity indicates that the role and application of the law had undergone a transformation: The black body radiation that could be produced in cavities had become the more important object of research, the relation of emission and absorption of radiation in arbitrary media now received less attention.

2. PLANCK'S PROOF OF HIS PREREQUISITE

In the preface to the edition of his radiation theory in 1906, Planck observed that his treatment frequently deviated from the "customary methods" where "factual or didactic reasons" suggested this, "especially in deriving Kirchhoff's laws." He did not say where factual reasons compelled dismissal of the older account.[53] He took over twenty-five pages to derive Kirchhoff's law. From the outset, Planck considered the radiation within a medium and allowed absorption, reflection, refraction, and diffusion. He excluded diffraction "on account of its rather complicated nature." By requiring that surfaces should not have sharp edges, he used the most general case then available.[54]

At equilibrium, the absorbed and emitted energies of a volume element must be equal when summed over all wavelengths. Planck's first task was to prove equilibrium sets up for each separate wavelength as well. To do so, Planck considered an (approximately) infinitely extended, homogeneous, and isotropic medium and argued that: "The magnitudes e_v, a_v, and K_v [the intensity of radiation of frequency v] are independent of position. Hence, if for any single color the absorbed was not equal to the emitted energy, there would be, everywhere in the whole medium, a continuous increase or decrease of the energy radiation of that particular color at the expense of the other colors."[55] But this would be in clear contradiction to equilibrium. This new and very simple argument relied only on symmetry

53. Max Planck, *Vorlesungen über die Theorie der Wärmestrahlung* (Leipzig, 1906), v. "Hierbei bin ich öfters, wo es mir sachliche oder didaktische Gründe nahelegten, von der sonst üblichen Art der Betrachtung abgewichen." The English edition, *Theory of heat radiation* (New York, 1959, first published 1914), xi, translates "sachliche oder didaktische Gründe" as "the matter presented or considerations regarding the form of presentation."
54. Planck, *Vorlesungen* (ref. 53), 2 (English trans., 2).
55. Ibid., 27 (English trans., 25).

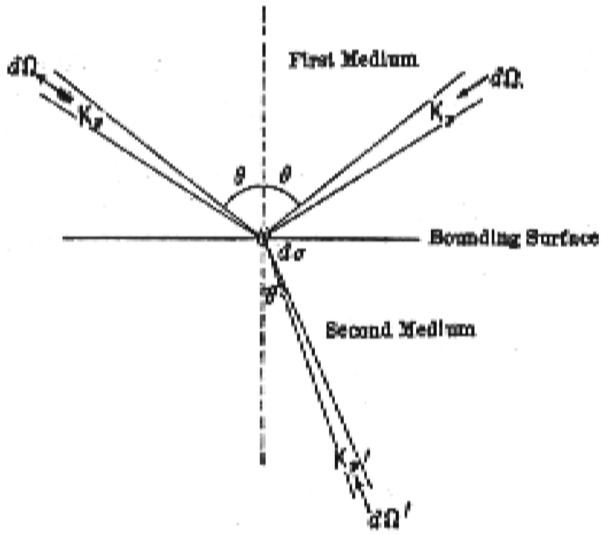

FIG. 4 Planck's diagram of radiation traversing a border between different media. Planck, *Theory* (ref. 53), 33.

principles. Planck exploited homogeneity and isotropy of space to establish Kirchhoff's relation for homogeneous media.

Next, Planck considered two infinite media of different refraction index bordering one another (figure 4). After some subtle discussion of the situation at the bordering surface, which again invoked Helmholtz' reciprocity theorem, Planck used this case to establish the independence of the ratio e_λ / a_λ from the material properties of the media (refraction indices). In Planck's words, the crucial insight was that, with respect to the second medium "*the ratio of emissive power to absorbing power of any body is independent of the nature of the body*. This ratio [in the second medium] is equal to the intensity of the pencil passing through the *first* medium which...does not depend on the second medium at all. The value of this ratio does, however, depend on the nature of the first medium."[56]

Again, the argument applied symmetry considerations to general principles. Finally, Planck argued that one could consider "*n* emitting and absorbing adjacent bodies of any size and shape whatever the state of thermodynamic equilibrium" and hence decompose the space containing the radiation in increasingly general ways, to approximate any physical situation.[57]

What remained from the experimental thinking of Kirchhoff, Helmholtz, and Pringsheim? The last step bore at least some relation to experimental strategies. Planck's lengthy discussion of Kirchhoff's law for a host of cases of absorbing,

56. Ibid., 43 (English trans., 40).
57. Ibid., 39 (English trans., 37).

diffusing, and diathermanous media and for various media with bounding surfaces of different properties corresponded to an experimental test series. Otherwise Planck completely did without diaphragms, lenses, mirrors, prisms, and other apparatus. Only the choice of bounding surfaces of regions of different material composition constituted the conceptualization of experiment-like set-ups. The kind of experimental thinking found in Kirchhoff, Helmholtz, and even Pringsheim gave way to an experimentally-motivated, stepwise treatment combined with an analysis of general principles.

Planck did not raise the point Pringsheim used against Richarz; the new understanding of radiating ether that invalidated all considerations that relied only on irradiations of surfaces. Although Planck avoided the use of surface elements, he still employed the language of ray optics in considering pencils passing through media; but since no devices were required to confine radiation of a certain kind (direction of propagation, wavelength, polarization, etc.) like diaphragms, mirrors and prisms, his considerations applied to every volume element and hence allowed each of them to emit or absorb radiation of arbitrary direction. Planck only noted that Kirchhoff's and Pringsheim's proofs had not considered the cases of absorbing and diffusing media.[58] By 1906, physicists had Planck's authoritative book and Pringsheim's generally accepted simple proof, which he had published in many versions. For most physicists the business may have been settled. Still, in the standard encyclopedia of physics, no clarity was obtained. In his article "Theory of radiation" in 1909, Wilheim Wien's argumentation fell beneath the level of the discussion set by Planck. Choosing elements of Pringsheim's reasoning, Wien first derived the law for the total radiation, which, however, is merely energy conservation. He then asserted that extension of the proof for each wavelength presented no difficulty. He deployed a thin plate like Kirchhoff's, to show that the radiation pressure (a favorite effect of Wien) would move the plate for different distributions of the energy density over the wavelength, which would give rise to an inadmissible *perpetuum mobile.* Wien thus revived a type of experimental thinking from the 19th century while subscribing to an odd ontological foundation:[59]

> That Kirchhoff's law is valid for each wavelength has its foundation in the fact that we possess instruments which can disperse radiation according to the various wavelengths present. For this reason, the radiation of each spectral region is independent of the existence of radiation from other spectral regions.

Wien's arrived peculiar view that our possession of instruments somehow accounts for the laws of nature can only be seen as a further indicator that radiation theory pointed to important deficiencies in the theoretical physics.

In 1899, Woldemar Voigt had already implicitly identified one important problem: The thermodynamic root of Kirchhoff's law clashed with Maxwell's electro-

58. Ibid., 43 (English trans., 40).
59. Wien (ref. 25), 282-357, on 285-287.

magnetic theory of light.[60] Twelve years later, Max Born and Rudolf Ladenburg claimed that they could make this contradiction explicit in a standard discussion of the foundation of Kirchhoff's law. Only with the additional condition of complete "disorder" of the radiation did the law become valid.[61] The two men who thus brought out the borderline problem of reconciling electrodynamics with thermodynamics were *Privatdozenten*, the former working under David Hilbert's guidance at Göttingen, the latter in Pringsheim's laboratory. Planck did not address this question explicitly or discuss the relation of his proof of Kirchhoff's law to previous ones before 1906. Only later, when his derivation was challenged, did he feel obliged to comment on Kirchhoff and Pringsheim. It was the Göttingen mathematician David Hilbert who forced Planck to end his silence about his predecessors.

3. ON MATHEMATICAL THINKING

By the autumn of 1912, David Hilbert was widely recognized as the world's leading mathematician. With the death of Henri Poincaré that year, he also became the most prominent mathematician concerned with the recent developments in physics.[62] Hilbert had proposed to lecture at the 1912 Münster meeting of the German Association of Natural Scientists and Physicians on the application of integral equations to the kinetic theory of gases but, at the last minute, changed his topic to radiation theory.[63] He had recently realized that the foundation of radiation theory required the mathematical tool of integral equations. He thought that radiation

60. Woldemar Voigt, "Über die Proportionalität von Emissions- und Absorptionsvermögen," *Annalen der Physik, 67* (1899), 366-387. Here he demonstrated that, for the emission and absorption of periodic and homogeneous oscillations, Kirchhoff's law does not hold (p. 373).
61. Max Born and Rudolf Ladenburg, "Über das Verhältnis von Emissions- und Absorptionsvermögen bei stark absorbierenden Körpern," *Physikalische Zeitschrift, 12* (1911), 198-202, on 198. "The possibility that in this case the combination of the thermodynamic and the electrodynamic point of view might give rise to contradictions exists and has only gotten a bit out of sight by the far greater difficulties that arise from the derivation of the complete radiation formula."
62. Leo Corry, "Hilbert and physics (1900-1915)," in Jeremy Gray, ed., *The symbolic universe. Geometry and physics 1890-1930* (Oxford, 1999), 145-188; Arne Schirrmacher: "Planting in his neighbor's garden: David Hilbert and early Göttingen quantum physics," *Physics in perspective, 5* (2003), 4-20.
63. On Hilbert's reputation for controversial presentations cf. the report on the 1903 meeting in *Naturwissenschaftliche Rundschau, 18* (1903), 553-556, on 554, with the following characterization of a Hilbert-Boltzmann dispute: "At the end [of the talk] the speaker and Mr. L. Boltzmann (Vienna) engaged in an extremely lively argument. We all know that nothing can raise one's own self-esteem as much as watching accepted authorities quarrel over a question and the fight of words was received by the audience with noisy amusement. It would be correct to say that the case of stability under discussion can be maintained readily in a physical-experimental sense, while the question of 'transcendental stability' in

theory would demonstrate "the fruitfulness and clarity of the method more simply and convincingly than kinetic theory."[64]

Hilbert had studied Planck's book of 1906 together with Hermann Minkowski.[65] Something of Minkowski's approach appears from the course on heat radiation he gave in the summer term of 1907 at Göttingen. He told the students:[66]

> In this course I address not only physicists, but also, and even more, pure mathematicians, who are usually more or less inclined to stay away from these fields. It is, in particular, my intention, and Professor Hilbert, too, is of similar opinion and pursues similar aims, to win over pure mathematicians to the inspirations that flow into mathematics from physics. It is not improbable that, during next year's seminars, we will consider mathematical-physical theories, especially of heat radiation.

This program was not executed as proposed. Minkowski turned increasingly to relativity theory, and Hilbert lectured on continuum mechanics while continuing and perfecting his research on integral equations that led to a book in 1912.[67] Hilbert realized early in 1912 that radiation theory might be a most telling application.[68] His lecture course on the "mathematical foundations of physics" became the detailed development of these ideas. Hilbert engaged Paul Ewald, a student of Sommerfeld's, in March 1912 to work through the literature on the proofs of Kirchhoff's law. He reported to Hilbert on April 11, a few days before the term started: "Concerning Kirchhoff's law, Planck's proof is the best known to me. Planck himself calls Pringsheim's proof full of gaps. Wien's hints in his encyclopedia article hardly can satisfy me."[69]

Ewald promised to find out about other proofs before he returned to Göttingen. With his physics assistant researching the literature, Hilbert developed his account of radiation theory and Kirchhoff's law in his lectures. These were, in turn, worked out formally by his mathematics assistant Erich Hecke.[70] Only a few days after the

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which not even one infinitesimal particle may acquire a finite velocity from an infinitesimal impulse, must remain undecided." For the physicists, Hilbert's problem was "transcendental," not something many of them would want to discuss.

64. Hilbert (ref. 1), 5. The manuscript deviates from the published paper only in marked passages, like those quoted. Hilbert provided reprints of his "Begründung der kinetischen Gastheorie," *Mathematische Annalen, 72* (1912), 562-577 at the meeting.

65. Hilbert remarked that this book gave rise to his work on radiation theory. David Hilbert, "Begründung der elementaren Strahlungstheorie," Deutschen Mathematiker Vereinigung, *Jahresberichte, 22* (1913), 1-20, on 18.

66. Hermann Minkowski, "Wärmestrahlung," notes to his lecture course, summer term 1907, in HP, 707, p. 2.

67. David Hilbert, *Integralgleichungen* (Leipzig, 1912).

68. Hilbert to Einstein, 30 Feb 1912, asking for Einstein's "theoretical works on gas and radiation theory," *Einstein collected papers.* Vol. 5, 439.

69. Ewald to Hilbert, 11 Apr 1912, HP, 98, item 1.

70. "Strahlungstheorie," summer 1912, notes taken by Erich Hecke, typescript at Mathematisches Institut Göttingen.

term had ended, Hilbert submitted his paper entitled "The foundation of elementary radiation theory" to the *Nachrichten* of the Göttingen Academy and the *Physikalische Zeitschrift*.[71]

Thus, four weeks later, on the morning of September 18, 1912, when Hilbert raised his voice to teach German physicists a lesson on the status of Kirchhoff's law and the proper way to establish it, he was well prepared. The joint session of the mathematics and physics sections of the association drew the largest audience of the entire meeting, some 140 persons.[72] Hilbert concluded that Kirchhoff's law on heat radiation, which had represented complex experimental results in a relation as simple and persuasive as Ohm's law, had not been made plausible, even in the most simple, special cases. For more than fifty years, physicists had failed to provide a proof for one of their most precious laws.

Hilbert made clear to physicists what in his eyes was the division of labor between physics and mathematics in the establishment of Kirchhoff's law: "This law appears here as a deep mathematical truth, whose content was found in physical experiment and predicted on the basis of physical deductions, whose proof, however, has become possible only by the theory of integral equations."[73]

The suggestive power or plausibility a physicist would find in the experiment-like structure of arguments like Kirchhoff's did not cast any spell over Hilbert. He tried to further his view on the emerging new relation between—or rather unification of—mathematics and physics by claiming: "If we did not have the theory of integral equations, the theories of gases and radiation would lead to it by necessity."[74] Given the asserted failure, the physicist remained surprisingly unstirred by Hilbert's talk. The chairman of the session, Arnold Sommerfeld, complimented the speaker for developing a framework in which everything would fit together beautifully. However, Sommerfeld added, the new approach could not produce Planck's theory of quanta, and thus the physicists could hope that at least the new field of quantum theory remained under their command. The physicist Merian von Smoluchowski recognized the "enormous progress" that had occurred through Hilbert's work and declared that "physicists will be grateful to him for it."[75] But not for long.

Hilbert considered radiation in an arbitrary, continuous medium with, in principle, variable values for emissive and absorptive powers, α and η, as well as for

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71. David Hilbert, "Begründung der elementaren Strahlungstheorie," in Akademie der Wissenschaften, Göttingen, Math.-Phys. Klasse, *Nachrichten* (1912), 773-789, *Physikalische Zeitschrift, 13* (1912), 1056-1064.

72. Gesellschaft deutscher Naturforscher und Ärzte, *Verhandlungen, 84* (1913), part II, 78. According to *Physikalische Zeitschrift, 13* (1912), 1009, ninety physicists attended the meeting and the main interest for them was the joint session with the mathematical section with talks by Hilbert, Nernst and von Smoluchowski.

73. Hilbert (ref. 71), 1062.

74. Hilbert (ref. 1), p. 15f.

75. Ibid; Hilbert (ref. 71), 1064.

the speed of light q (or refraction coefficient n) in each infinitesimal volume element. Moreover, these values could, in principle, depend on the neighborhood of the volume element considered; lack of this flexibility had been a criticism of Kirchhoff's proof.[76] To continue:[77]

> The most important question that now arises is that of the possibility of thermal equilibrium, or of the conditions that are necessary among the three coefficients q, η, and α so that equilibrium can be established. To settle this question, we first calculate the total energy density that arises in consequence of our assumptions about emission and absorption of matter at any arbitrary position *xyz*.

Hilbert considered in all generality the flow of radiation that arrives at a certain volume element and equated it, in equilibrium, with the emission from the same volume element. Hence, the emitted energy must be equal to the sum over all paths that bring radiation emitted somewhere else and partially absorbed by the medium in its way. The resulting integral equation incorporates a certain combination of the emissive and absorptive power functions and the velocity of light, with a certain symmetric kernel or propagator (e^{-A} / S).[78]

$$\eta - \frac{\alpha}{4\pi q^2} \iiint \frac{e^{-A}}{S} \eta(x_1 y_1 z_1)\, dx_1 dy_1 dz_1 = 0$$

The general theory of integral equations that Hilbert had recently put forward in his book transformed this equation into a relation for the three position-dependent functions q, α, and η. It thus immediately provided Kirchhoff's law (with position dependent on the velocity of light),

$$\frac{q^2 \eta}{\alpha} = \text{const.}$$

As this consideration would hold for all wavelengths and temperatures, Hilbert concluded that this combination must be a universal function of these quantities.

Only four lines after his conclusion that no earlier proof had been unobjectionable, Hilbert required that exchange of energy only take place by radiation "which we will suppose to be of the same constant frequency."[79] But, having said this,

76. Kayser (ref. 9), 30.
77. Hilbert (ref. 71), 1064.
78. Hilbert (ref. 71), 1058, 1059. *S* describes the evolution of a ray from *xyz* to x_1, y_1, z_1; *A*, the absorption along the path, i.e. $|\int \alpha\, ds|$.
79. Ibid., 1057.

Hilbert had presupposed the radiation to be in equilibrium for each frequency independently. Had he presupposed what he wanted to prove?

Two points particularly characterize Hilbert's approach. Planck had dismantled all the equipment of the experimenters' workshop (the diaphragms, mirrors, prisms, plates, etc.) but still stuck to the concepts of ray optics, where single pencils cross boundaries between one region and another of different material composition. With his view that each tiny volume element could have its own absorptive, emissive, and refractive properties, Hilbert departed fully from the classical view of mutual irradiations of surfaces and provided an appropriate model for radiating ether. Secondly, Hilbert's approach was completely free from experimental thinking. In contrast to the experimentally influenced ways of reasoning of earlier theorists, he set up a general manifold of possible situations and solutions and then imposed conditions (here of light propagation and equilibrium) that provided the solution. Mathematical necessity played the central role, not a mechanism, nor a conceived sequence of experimental actions.

4. ON NOT LEARNING A LESSON: HILBERT VERSUS PRINGSHEIM, 1912-1914

Pringsheim had published his proof several times in journals ranging from the proceedings of the German Physical Society through specialized periodicals for scientific photography and electrochemistry down to magazines directed at high-school teachers. Kayser had given Pringsheim's proof his blessing and Wien had made use of it.[80] Between Pringsheim and Hilbert, however, a notable controversy occurred.[81]

Apparently Pringsheim could not grasp Hilbert's argument as he found it in print since Hilbert's former student and colleague, Constantin Carathéodory, was asked to present it in the Breslau physics colloquium. He spoke in November 1912. Carathéodory did not succeed in conveying Hilbert's ideas convincingly, nor did he fully understand Pringsheim's objections. Only four weeks later, after "laborious discussions," was he able to grasp the main point, which he promptly communicated in a long letter to Hilbert.[82] The point Pringsheim raised may seem surprising, as it had nothing to do with Hilbert's questionable assumption of equilibrium for each wavelength separately, but rather attacked a completely new aspect, as he had done with Richarz. Pringsheim observed that the energy balance for a single volume element should include the energy exchanged via conduction as well as that exchanged by radiation. The latter can be decomposed by wavelength, the former cannot. Carathéodory concluded that there would still be an integral equation for this problem, but it would no longer yield Kirchhoff's law.[83]

80. Kayser (ref. 9), 27, 37-38; Wien (ref. 26), 285.
81. Max Born, "Hilbert und die Physik," *Die Naturwissenschaften, 10* (1922), 88-93, on 90f; Leo Corry, "Hilbert on kinetic theory and radiation theory (1912-1914)," *The mathematical intelligencer, 20* (1998), 52-58.
82. Carathéodory to Hilbert, 12 Dec 1912, HP, 55, item 4.
83. Ibid. This criticism was not one of Hilbert's arguments, but one of his assumptions,

Carathéodory, Rudolf Ladenburg, and Max Born discussed how Hilbert might reply during the Christmas holiday. They agreed that Born should try to mediate between Pringsheim and Hilbert.[84] Born knew both sides well. He studied mathematics with Hilbert and acted as his assistant in Göttingen, and had learned to do experiments with black bodies under Pringsheim.[85] Born returned to Göttingen a few days after his meeting with Pringsheim. Now, Hilbert decided to publish an extended version of his paper in the journal of the Association of German Mathematicians.[86]

The extended paper contained a new section marked by a footnote referring to Pringsheim's proof of 1903. Carathéodory had called Hilbert's attention to this paper and had suggested that, although much could be said against Pringsheim's proof, the guiding idea might be useful. Ewald, who had analyzed Pringsheim's proof for Hilbert, might have overlooked the suggestive presentation of 1903. Although Hilbert was motivated to append some pages comparing different approaches to the proof, he insisted that his specific integral equation crucially depended on his assumptions, and he did not refer to Pringsheim's objection about the neglect of heat conduction. Moreover, he revealed that the correct integral equation would be at stake in the same way whether equilibrium held for each wavelength separately or only for the total energy. Carathéodory judged that the amended paper fully clarified the situation and "must satisfy every physicist." It did not satisfy Pringsheim.[87]

Pringsheim's rebuttal appeared in *Physikalische Zeitschrift* in April 1913.[88] Hilbert's claim that his was the only reasonable proof of Kirchhoff's law had irked Pringsheim. Only in the amended version published in the journal of the German Association of Mathematicians, did Hilbert point out that his discussion was meant to be an axiomatic treatment. Later, Hilbert claimed that the presentation in his Münster talk foreshadowed a proof that "satisfies the modern [*neueren*] requirements of axiomatic treatment after the model of geometry."[89] In this axiomatic way, Hilbert now made clear the difference between the approaches of Pringsheim, Planck, and himself. According to him, his main axiom was the requirement of separate equilibrium for each color, Planck's was the local determination of the

since Hilbert had explicitly required that there was no heat conduction. Hilbert (ref. 65), 2.

84. Born to Hilbert, 7 Jan 1913, HP, 40A, item 4.

85. Born characterized Pringsheim as a "quiet thinker, elegant in manners and attire, cautious and reserved in his statements, modest and unobtrusive." Max Born, *My life* (New York, 1978), 123.

86. Hilbert (ref. 65).

87. Carathéodory to Hilbert, 4 Apr, 1913, HP, 55, item 5.

88. Ernst Pringsheim, "Bemerkungen zu der Abhandlung des Herrn D. Hilbert 'Begründung der elementaren Strahlungstheorie'," *Physikalische Zeitschrift, 14* (1913), 589-591, dated 15 Apr.

89. David Hilbert, "Bemerkungen zur Begründung der elementaren Strahlungstheorie," Akademie der Wissenschaften, Göttingen, Math-Phys. Klasse, *Nachrichten* (1913), 409-416 and *Physikalische Zeitschrift, 14* (1913), 592-595, on 592.

coefficients, and Pringsheim's was the postulate of the existence of matter for each given absorption function. Hilbert demonstrated that his axiom I would suffice to derive the law, in its widest sense, applying to radiation in arbitrary media, whereas both Planck's and Pringsheim's, which he called axiom II and axiom III, respectively, would fail. Only through the combination of their fundamental axioms II and III could the job be accomplished as well.[90]

In his first article criticizing Hilbert, Pringsheim raised three points. First, he deprived Hilbert's derivation of its generality:[91]

> [It] derives an equation that coincides formally with Kirchhoff's law. However, it treats only an ideal and experimentally unrealizable limiting case, in which the whole radiation present is monochromatic. The meaning of Kirchhoff's law, however, lies in the fact that in the only physically interesting case, which is that of mixed radiation, and which consists of oscillations of infinitely many different frequencies, Kirchhoff's law is satisfied for each single frequency.

As a consequence, Pringsheim rejected Hilbert's claim that the law was a "deep mathematical truth" only revealed by the tool of integral equations. The ground of the rejection was experimental unfeasibility. Pringsheim next observed that Hilbert's axioms, being far too fundamental for a physicist, could not be "tacitly assumed." Anyway they involved a vicious circle: "Therefore the content of axiom I, which according to Mister Hilbert provides the basis of his derivation of Kirchhoff's law, is physically equivalent to Kirchhoff's law."[92] Here Pringsheim placed no value on the work needed to relate the two equivalent statements *mathematically*.

Finally, Pringsheim objected that Hilbert's axiom III, which described Pringsheim's assumptions, did not correspond to the situation he had had in mind—the radiation of an extended body surrounded by empty space, not embedded within a medium.[93] Hilbert replied to all of this that Pringsheim's objections were "in no way justified."[94] How could they be justified? In an axiomatic framework, all conclusions are already in the axioms, otherwise they could not be deduced logically from them. Pringsheim's denouncement of equivalence did not count from the mathematical point of view. Second, in a mathematical calculation of a physics problem, each step need not be checked for a corresponding, physically possible situation. Such a requirement obviously corresponds to an experimental approach, whereas the mathematical validity of an argument cannot depend on an intermediate ontology as Pringsheim required.

On the other hand, the main point in Kirchhoff's law to Hilbert cannot have been the problem of spectral composition as it was initially for Kirchhoff and later

90. Hilbert (ref. 65), 19.
91. Pringsheim (ref. 88), 589.
92. Ibid., 590.
93. Ibid., 591.
94. Hilbert (ref. 89), 594. Hilbert's paper was received by the journal three weeks after Pringsheim's critique.

for Pringsheim. Rather, the point was the validity of the relation for arbitrary media with varying physical properties, as it had been for Kirchhoff to some extent and definitely for Planck, who had solved the decomposition question in a few lines but had dedicated many pages to various arrangements of media. To an unprejudiced reader of Planck's book, the relation for arbitrary media appeared to be the central problem at stake. Pringsheim realized that he stood closer to Kirchhoff than to Planck, who already shared some common ground with Hilbert:[95]

> Following Planck's lead, Mister Hilbert considers the radiation within an absorbing substance and talks about the absorption coefficient as a function of the space coordinates. I, however, following Kirchhoff, treat the radiation in empty space and consider the absorbing power of an extended body.

Hilbert's brief footnote with its wholesale repudiation of his criticism prompted Pringsheim to reply with polemics and half-truths. He complained that Hilbert turned to the axiomatic point of view only after Carathéodory had communicated to him that "his alleged derivation of Kirchhoff's law tacitly assumed its essential physical content."[96] The complaint would scarcely have impressed anyone familiar with Hilbert's work, however, since he had long advocated the axiomization of physics.[97] Pringsheim turned to the applicability of the axiomatic method to physics. Reiterating that Hilbert's derivation rested on a very special condition (single wavelength), he concluded that "strictly speaking, even all five of Hilbert's axioms together are not sufficient for deriving Kirchhoff's law generally." Furthermore, if Hilbert put everything he needed into axioms, one would have far too many of them. That brought Pringsheim to an axiom of his own: "We always will arrive at this difficulty when we try to found a physical discipline axiomatically." And to an obvious inference: "Physics is no appropriate field for the axiomatic method." Pringsheim did not find criticizing Hilbert pleasant. But it had to be done to protect physicists from accepting Hilbert's errors as truth on the basis of his high standing in "the mathematical world."[98]

5. ON LEARNING A LESSON: HILBERT AND PLANCK, 1912-13

The shortcoming of Hilbert's treatment could have been cured easily by Planck's concise argument that in an isotropic medium, including vacuum, one wavelength must not take precedence at the expense of another. (For Pringsheim's understanding of Kirchhoff's law this was already a proof.) But it was exactly this

95. Pringsheim (ref. 88), 590.

96. Ernst Pringsheim "Über Herrn Hilberts axiomatische Darstellung der elementaren Strahlungstheorie," *Physikalische Zeitschrift, 14* (1913), 847-850, on 847.

97. Leo Corry, "David Hilbert and the axiomatization of physics (1894-1905)," *Archive for history of exact sciences, 51* (1997), 83-198.

98. Pringsheim (ref. 96), 848f.

argument that Hilbert criticized in the extended version of his Münster talk.[99] The resultant exchange between Hilbert and Planck developed completely differently from that with Pringsheim. It started a few days after the meeting of 1912 when Hilbert sent Planck a reprint of the paper underlying his talk. Planck answered in October that because the production of the second edition of his book on the theory of heat radiation was too far advanced, he could not take this "interesting method" into consideration.[100] Also in October 1912 and again in January 1913, Planck touched on radiation problems. Hilbert had criticized Max Abraham's treatment of black body radiation in his textbook on *Electromagnetic theory of radiation* (1905, 1908). Planck defended Abraham, who had been his student. But the point raised was not a central one in the proof of Kirchhoff's law.[101]

Hilbert's criticism rested on an argument typical of his mathematical style: he constructed a solution for α, η, and q that satisfied Planck's axiom but did not obey Kirchhoff's law.[102] This approach, however, did not determine wherein lay the gap in Planck's reasoning; Hilbert could only conjecture that Planck's argument might not apply in the general case of inhomogeneous medium or bordering homogeneous media.[103] In turn, Planck maintained that it was possible "to proceed step-by-step to the general case of arbitrarily connected homogeneous media" without recourse to Pringsheim's assumptions, i.e., axiom III. The laws of reflection and refraction would suffice. Turning to Hilbert he wrote:[104]

> I see the sole physical significance of your method of proof in its application to inhomogeneous media. But, on the other hand, the propagation of energy in such media is not determined by the principle of fastest arrival that you use, since determinate light paths do not exist, but rather "diffusion" of light occurs, which neither you nor Pringsheim take into account.
>
> I would be very pleased if you could tell me your thoughts on these points. For

99. Hilbert (ref. 65), 18.
100. Planck to Hilbert, 4 Oct 1912, HP, 308A, item 1.
101. Max Abraham, *Theorie der Elektrizität*. Vol. 2. *Elektromagnetische Theorie der Strahlung* (Leipzig, 1905, 1908). Planck to Hilbert, 20 Oct 1912, and 24 Jan 1913, HP, 308A, items 2 and 3. Planck explained to Hilbert that a certain equation in Abraham's book (no. 227, p. 340, 2nd edn.) is a simple consequence of the second law of thermodynamics.
102. Constructing counter-examples was a typical ingredient of Hilbert's axiomatic method. Cf. his demonstration of the independence of the parallel axiom in Euclidean geometry in David Hilbert, "Grundlagen der Geometrie," *Festschrift zur Feier der Enthüllung des Gauss-Weber Denkmals in Göttingen* (Leipzig, 1899), chapt. 2 §10.
103. Hilbert (ref. 65), 18.
104. Planck to Hilbert, 4 Apr 1913, HP, 308A, item 4. Hilbert acknowledged the criticism regarding the velocity of propagation in David Hilbert, "Zur Begründung der elementaren Strahlungstheorie (Dritte Mitteilung)," Akademie der Wissenschaften, Göttingen, Math.-Phys. Klasse, *Nachrichten* (1914), 275-298, on 277; also in *Physikalische Zeitschrift, 15* (1914) 878-889, and Hilbert, *Gesammelte Abhandlungen*. Vol. 3 (New York, 1965), 238-257. In the text of the *Gesammelte Abhandlungen* the phase velocity that Hilbert initially took as propagation velocity was tacitly corrected to the more appropriate group velocity.

> I would rather not give the impression to the "outside" that I agreed with your published view.

A few days later Planck repeated his criticism in much detail. First he turned the tables on Hilbert:[105]

> In your "proof of impossibility" I see a gap in the fact that your equation (26) does not begin to contain the content of my axioms. The essential ones are the following:
> 1. In an arbitrarily limited body with finite absorptive and emissive powers for each temperature *a single* state of thermal equilibrium is possible (maximum of entropy and minimum of free energy, respectively).
> 2. η, α, and q depend only on the nature of the matter (your axiom II.)

The main point Hilbert missed was the first axiom, which determined the radiation for each wavelength: in equilibrium, every characteristic quantity, and notably the radiation density, is determined by the temperature. Hilbert accepted the validity of Planck's treatment, which now succeeded through temporarily agreeing on the use of Hilbert's style of reasoning.[106]

Hilbert later evaluated the contribution he believed he had made to radiation theory: "One of the most noteworthy results of my first communication lies in the fact that the statement, the ratio $q^2\eta/\alpha$ has the same value for each point of a system in thermal equilibrium, can be inferred from an integral equation, without performing any *translocation* of matter or *change* of its physical nature in the proof, which is otherwise always done proving Kirchhoff's law."[107] Hilbert understood that his style of reasoning was completely free from the physicists' experimental thinking, which, for him, was a major advance. The prospect of a new mathematical physics shimmered on the horizon.

The debate over the proof of Kirchhoff's law provides a good example of how the mathematization of physics developed and how it came to a temporary halt. Hilbert may have succeeded methodologically by making Planck argue axiomatically, because only in that way could discrepancies in the various proofs be made obvious. However, physicists still did not embrace axiomatics; even Planck rejected the approach as "inappropriate for the foundation of a proof of Kirchhoff's law." Hilbert's set of axioms was "completely arbitrary." Pringsheim's assumption of the existence of a continuous sequence of materials with respect to certain physical properties was "*strictly speaking* clearly wrong." Nonetheless, and to Hilbert's astonishment, Planck added that Pringsheim's proof was still the "simplest and most transparent." Furthermore, it was also "factually the most profound," since it derived the law from its real root—the second law of thermodynamics. And Planck's statement that he followed the lead of Kirchhoff—who

105. Planck to Hilbert, 15 Apr 1913, HP, 308A, item 5.
106. Hilbert (ref. 89), 593.
107. Hilbert (ref. 104), 276, Hilbert's emphasis.

made his proof "so complicated" only to free it from its shaky assumption based on our imperfect material world showed Hilbert that Planck was no ally in axiomatizing physics.

In his third communication on the foundation of radiation theory, Hilbert came to grips with Pringsheim's criticism concerning intrinsic reflection. Hilbert had asked Wilhelm Behrens to tackle this problem as his habilitation thesis.[108] Hilbert could now present "under rigorous consideration of reflection" new and elementary proofs of Kirchhoff's law, while also solving the question ("of equal importance") of the freedom from contradictions of the axioms themselves and in combination with the laws of optics.[109] While the axioms remained basically the same (characterizing his, Planck's and Pringsheim's approaches), the proof now employed a different strategy.[110] The central integral equation no longer played a major role.

While the proof of Kirchhoff's law in Hilbert's third communication followed his characteristically purely mathematical reasoning, the demonstration of the axioms' freedom from contradictions required something new. For example, in discussing the compatibility of the axioms with the elementary laws of optics, Hilbert now invoked thought experiments (figure 5):[111]

> Now we imagine that the plane e and the points A, A' are fixed...and rotate the system in such a way that....We now imagine that the space around the plane e is filled on one side with a substance with optical coefficients q, α, η, and on the other side with a substance with optical coefficients q_*, α_*, η_*; and furthermore that a ray from O meets the plane e at A and is then refracted to B and reflected to C.

Although Hilbert tried to keep the door of the experimenter's workshop shut, with this exceptional case he obviously adopted something of Planck's style. In one almost ironic instance, Hilbert as much as revived Kirchhoff's initial postulate of a one-wavelength plate: He considered as an axiom a variant of Pringsheim's postulated existence of substances that reflect all radiation except for a single wavelength.[112]

What had happened to Hilbert? He had temporarily become a theoretical physicist. He no longer gave his courses titles like "Mathematical Foundations of Physics" (summer 1912 and winter 1912/13) or "Seminar on the Axioms of Physics"

108. Wilhelm Behrens, "Lichtfortpflanzung in parallel geschichteten Medien," *Mathematische Annalen, 76* (1915), 380-430. Behrens acknowledged (p. 382f) that this work, which showed how to derive the laws of radiation theory from Maxwell's theory in approximation, was motivated by Hilbert's publications, 382f.
109. Hilbert (ref. 104), 276f.
110. The coefficients α, η, q are taken as functions of parameters p that describe the material; differentiation with respect to these parameters vanishes for the combination $q^2\eta/a$.
111. Hilbert (ref. 104), 291.
112. Ibid., 297. See also Wien (ref. 22).

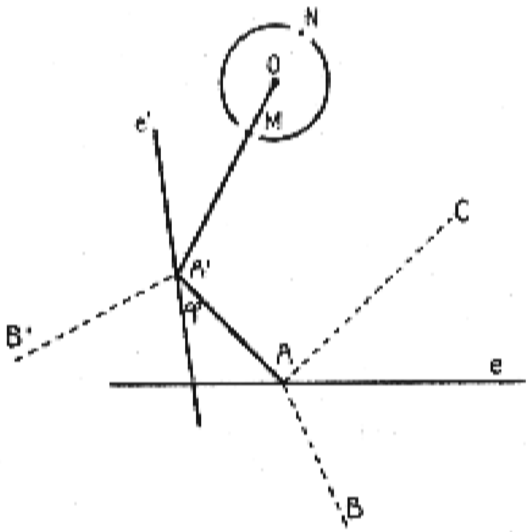

FIG. 5 Hilbert's drawing supporting his argument for the independence of axioms. Hilbert (ref. 104), 290.

(1912/13), but simply "Theory of Electron Motion" (1913), "Electromagnetic Oscillations" (1913/14), "Selected Topics of Statistical Mechanics" (1914), etc. This episode lasted until 1916, when by "Principles of Physics" Hilbert meant the foundations of Einstein's theory of general relativity. Hilbert had learned that the "inspiration that flows into mathematics from physics" can only be gained by doing physics the physicists' way. He came to acknowledge the legitimacy of Planck's approach and realized the practical problems with axiomatization.

Shortly after his third paper on radiation theory, Hilbert managed to pick a fight with Einstein. He now presented "a new axiomatic system of fundamental equations of physics that are of ideal beauty and that contain...the solution of the problems of Einstein and Mie at the same time."[113]

He did not, however, claim priority. As in the discussions on radiation theory, there were more substantial issues at stake than determining priority. Hilbert and Einstein had different aims and followed different paths in their researches. At one point they met, however. For Einstein this meant a major breakthrough for general relativity, for Hilbert, only a step in a program for a unified theory of matter.[114] Incongruence of the objects of justification (the content of the law) and the ways of reasoning employed does not require fifty years to develop as it did in radiation theory.

113. David Hilbert, "Die Grundlagen der Physik (Erste Mitteilung)," Akademie der Wissenschaffen, Göttingen, Math-Phys. Klasse (1915), 395-407, on 395.
114. Cf. David Rowe, "Einstein meets Hilbert: At the crossroads of physics and mathematics," *Physics in perspective, 3* (2001), 379-424.

It was owing to Einstein that disputes over Kirchhoff's law evaporated. On the basis of Planck's oscillator model, Einstein defined emission and absorption coefficients A_m^n and B_m^n as a measure of the probabilities of quantum transitions between energy levels E_n and E_m that had no direct equivalent in the theory of Kirchhoff's law.[115] A universal function cannot be found, since the theory of thermal equilibrium cannot fully be recovered in the quantum description. Thus, reflecting on the emergence of quantum physics from a proto quantum problem, we may conclude—adopting a figure from Wittgenstein—that Einstein threw away the ladder Planck had climbed.[116] But again, Einstein's treatment of radiation phenomena employed Planck's makeshift concept of virtual oscillators and, hence, revived a way of experimental thinking that left its mark on his theory.[117]

6. SUMMARY AND CONCLUSION

One does not have to search far to find fundamental physical laws that, like Kirchhoff's, lacked full justification over comparatively long periods. Consider the one prerequisite every proof of Kirchhoff's law employed: the reciprocity law, usually ascribed to Helmholtz.[118] But he did not prove it, and neither did Kirchhoff, Pringsheim, or Planck. Planck even presented a generalized reciprocity theorem in 1906, also without proof,[119] though one was available. Its author, Richard Straubel, found it "strange" that both Kirchhoff and Clausius had missed this "general law of theoretical optics" or had "refrained from formulating it."[120] Hilbert took up the problem in two ways, making no appeal to physical intuition. First, he pointed out that the theorem could be transformed easily into a problem of the theory of surfaces that had been solved by Pierre-Ossian Bonnet and Gaston Darboux. Second, he gave a detailed, direct proof in his lectures—explicitly addressing physicists.[121] No doubt a closer look would reveal many more examples of this kind, for example, in elasticity theory or quantum theory.

Thus it appears that physics does not generally progress from a solid homeland to vague frontiers, but rather that it reserves, or resolves in different ways at different times, questions about its general laws and principles. What these laws and principles meant and to what phenomena they actually applied, varied from time to time.

115. Albert Einstein, "Strahlungs-Emission und Absorption nach der Quantentheorie, Deutschen Physikalischen Gesellschaft, *Verhandlungen, 18* (1916), 318-323, on 321.
116. Ludwig Wittgenstein, *Tractatus logico philosophicus* (London 1922), sentence no. 6.54.
117. J.L. Heilbron, "The virtual oscillator as a guide to physics students lost in Plato's cave," *Science and education, 3* (1994), 177-188.
118. Helmholtz (ref. 29), 169, and (ref. 34), §42.
119. Planck (ref. 53), §46.
120. Rudolf Straubel, "Über einen allgemeinen Satz der geometrischen Optik und einige Anwendungen," *Physikalische Zeitschrift, 4* (1903) 114-117, on 115.
121. Hilbert (ref. 70), 59; Manuscript of this part of his 1912 lectures in HP, 728.

The history of Kirchhoff's radiation law followed a typical "life cycle." It had its origin in the study of colored flames and stellar spectra—phenomena to which the abstracted law no longer applied. It vanished with the application of quantum theory to emission and absorption phenomena—a theory that resulted from the search for Kirchhoff's universal function. The later quantum theory of radiation had virtually no overlap with classical radiation theory.[122] During its life cycle neither its content nor its foundational roots remained constant. The various forms are summed up in table two.

The metamorphosis of the question can be seen, first, in the fact that the wavelength problem and Hilbert's arbitrary medium problem had no overlap in content though both were called Kirchhoff's law. Secondly, the instruments used—both the real or conceived objects and the mathematical tools—also changed so much as to cut out the common ground even among Kirchhoff and Planck. At the same time, with his renunciation of special objects and his search for a more general scope of application of the law, Planck occupies an intermediary position in the table. Characterization of the objects, tools, and scope identifies Planck's approach to black body radiation.

The scientists involved in the story later obscured this development. For example, Max Born, who in 1913 took over Hilbert's criticism of Planck's proof and who still told his students ten years later that Hilbert had given the rigorous mathematical proof of Kirchhoff's law (although using Pringsheim's idea), fully rehabilitated Kirchhoff, in 1929, as the scientist who had proved "on the basis of indisputable thermodynamic arguments that radiation released through a small hole in a glowing oven must have a spectrum of a universal kind." He did not comment on the matter in his autobiography, nor did Hilbert's biographer mention it.[123] Planck had already established his view in his letters to Hilbert in 1913. In the textbooks that condensed his legendary teaching Sommerfeld essentially went back to 1859, invoking a filter transparent for one wavelength only.[124]

Historical research on early quantum theory tends to ignore the role of Kirchhoff's law.[125] A few biographical studies allow that Kirchhoff's work was crucial to the development of quantum theory either as "the key to the whole thermodynamics of radiation...the key to the new world of the quanta," or as the "methodological example for Planck's and Einstein's investigations."[126]

122. E.g., Walter Heitler, *The quantum theory of radiation* (Oxford, 1936).
123. Max Born, "Die Theorie der Wärmestrahlung und die Quantenhypothese," *Naturwissenschaften, 1* (1913) 499-504, on 501; lectures on "Kinetische Theorie der Materie" (Winter term 1922) worked out by Luise Spieker, Göttingen Mathematical Institute, on 170; "Über den Sinn der physikalischen Theorien," *Naturwissenschaften, 17* (1929), 109-118, on 114; and Born (ref. 85).
124. Arnold Sommerfeld, *Thermodynamik und Statistik* (Wiesbaden, 1952), 131f.
125. Hans Kangro, *Vorgeschichte des Planckschen Strahlungsgesetzes* (Wiesbaden, 1970), 3, Armin Hermann, *Frühgeschichte der Quantentheorie (1899-1913)* (Moosbach, 1969).
126. Leon Rosenfeld, "Gustav Robert Kirchhoff," in *Dictionary of scientific biography*, ed. Charles Coulston Gillispie. Vol. 7 (New York, 1973), 379-383, on 382. Hentschel (ref. 39), 430.

Table 2: The content of Kirchhoff's law in relation to objects and tools used in proof

PROOF BY	*CONCERNING*	*THE λ-PROBLEM*		*THE MEDIUM-PROBLEM*		*OTHER ISSUES*
		objects	*tools*	*generality*	*tools*	
Kirchhoff 1859	e/a	one-λ-plate*				
Kirchhoff 1860/62	e/a	diatherm. plate*	Fourier theory	homogeneous		
Helmholtz	e/a	prism*				
Pringsheim	$u/(\lambda)$	cavities	$\int$-argument			
Planck	e/a		expense argument	piecewise homogeneous		
Hilbert 1912	e/a			general inhomogeneous	$\int$-equation	
Hilbert 1913	e/a			general inhomogeneous	$\int$-equation	axiomatics
Hilbert 1914	e/a			general inhomogeneous	differential argument	axiomatics
(Einstein)	$E_n \rightarrow E_m$	oscillator...				
Sommerfeld	$u / (\lambda)$	λ-filter*				

* not tenable assumptions or arguments

The birth of quantum theory occurred with Einstein's and Ehrenfest's interpretation of the role of energy elements and the establishment of a quantum discontinuity, rather than through Planck's formula alone.[127] Still, the instrumentation used to confirm Kirchhoff's law, and, in particular, his introduction and proposals for the construction of black bodies provided the material basis for the experimental history of early quantum theory.[128] Hence, concerning the relation between theoretical proof and experimental confirmation, the new physics came mostly from the presentation and interpretation of the derivations or proofs of the radiation formula. In the first years after Planck's derivation, textbooks still claimed other formulas would account equally well for the measurements.[129] When Max Born first learned about Planck's formula in a lecture on astrophysics by Karl Schwarzschild in 1905, he was told that Wien's formula fit more accurately than Planck's.[130] The same situation applied to Kirchhoff's law: The fact that high accuracy measurements suggested the independence of the energy distribution of radiation from the material properties of the experimentally approximated black bodies in the laboratory did not make a proof unnecessary. But no valid proof of Kirchhoff's law, which was an essential prerequisite for Planck's theory, existed when he set up his formula. Perhaps as a consequence of this situation, the ways of deriving the existence of the universal radiation function diverged from ways of deriving its actual expression.

The process of establishing two general physical statements, one of which was needed to establish the other, was not consecutive, but had considerable temporal overlap. The life span of Kirchhoff's law roughly coincided with the period in which theoretical physics in Germany developed into a new discipline. Can we relate this development directly to a turn in the theoretical justification of physical laws from older types of experimental thinking to novel ways of a distinctly mathematical kind? The turn can be seen in Planck's middle way between Kirchhoff's purely experimental style of thinking and Hilbert's purely mathematical way of reasoning. Or, rather, it superposed elements of both, in Kirchhoff's radiation law and in his own radiation formula. This conciliatory approach agreed well with the physics of this period. Questions about the foundations of quantum mechanics would shift matters in Hilbert's direction.

Planck was not a follower of the phenomenological school, which had roots in experimental manipulation to which he had no intimate relation. Still, he subscribed

127. Thomas S. Kuhn, *Black-body theory and the quantum discontinuity* (New York, 1978).
128. Dieter Hoffmann, "Schwarze Körper im Labor, Experimentelle Vorleistungen für Plancks Quantenhypothese," *Physikalische Blätter, 56* (2000), 43-47, "On the experimental context of Planck's foundation of quantum theory," *Centaurus, 43* (2001), 240-259.
129. Orest D. Chwolson, *Lehrbuch der Physik.* Vol. 2. *Lehre vom Schall (Akustik) - Lehre von der strahlenden Energie* (Braunschweig, 1904), 230.
130. Karl Schwarzschild, "Astrophysik," lecture notes taken by Max Born for the winter term, 1904, Manuscript Department, Niedersächsische Staats- und Universitätsbibliothek, Göttingen, Cod. Ms. K. Schwarzschild 13, item 1, 115-119.

to such aims of the phenomenological school as the economic description of observable phenomena without reference to special, microscopic, pictures. He did not follow Kirchhoff and Helmholtz, whom he acknowledged as authorities, in their (thought-) experimental thinking with special conceptual objects. His main pillars for the building of physics were general principles and universal constants. Physical knowledge should be independent not only of human actions, but also of experimental objects. Hence Pringsheim's arguments in terms of experimental realizability, had no appeal to Planck. But he was physicist enough to work by stepwise attack of complex and complicated physical phenomena: first solve a simple paradigmatic case employing the main principles, then generalize to more complex and hence more natural or real cases.

Despite his program for eliminating anthropomorphic elements from physics, Planck did not share Hilbert's view that the task of mathematical physics was to remove all unnecessary physical ornament from discussion, to identify physical assumptions (axioms), and to transform the deduction of the law into a purely mathematical problem. This axiomatic approach, which began with the most general case and its corresponding space of possible solutions and then imposed constraining relations, ran counter to Planck's use of paradigmatic simple situations as starting points (the infinitely extended homogeneous medium for Kirchhoff's law, the oscillator for his radiation function). Hilbert's main concern was whether a set of assumptions sufficed for a logical derivation. But this ideal hardly met physicists' needs for explanation and understanding.

It is hard to say what the concept of proof outside of mathematics should mean. There is no theory of proof in physics. Nonetheless, physicists were and are much concerned with the proofs of their theories. Hence, a history of proof in physics must start with the changing divide between those insights that physicists considered in need of a derivation or proof, and those that they apparently considered evident beyond question. Such a history would exhibit the developing rationality of the science. A historical concept of the proof in physics would incorporate at the least, the following four elements.

- *Methodological assumptions* that determine the conclusiveness of a proof. This conclusiveness is different from logical implication: for example, the phenomenological ideal of simple relations between experimentally observable quantities, Planck's admonition to Hilbert that hypotheses and axioms must not be "completely arbitrary," or Aimé Cotton's insight that reasoning with imaginary bodies has a particular "suggestive value" that can lead to the discovery of new facts and laws. The acceptance of certain idealizations and simplifications belongs here, too.[131]

- Types of objects, or the *ontology*, of the proof. Here one might refer to the range of objects applied in derivations of Kirchhoff's law: typical real

131. Ref. 105; Cotton (ref. 9), 267.

ones like diaphragms and radiating bodies; special real ones like cavities and line gratings; also generalized and idealized ones like pure radiation, ideal mirrors, and completely black bodies; and fictitious ones that contradicted assumed theory like the one-wavelength plate or the completely transparent prism.

• Mathematical and logical *tools* that characterize the proofs. Examples are Fourier theory, the theory of integral equations, symmetry relations and reciprocity statements.

• Types of *actions* described in the proofs. These can be both conceived experimental actions like replacing objects, varying parameters, or comparing temperatures at two points, as well as actions exhibited in stepwise procedures, the construction of counter-examples, or reference to mathematical necessity.

The history of proof in physics is, therefore, particularly a history of experimenting theory. A proper history of physics must necessarily combine experiment and theory. There is always theory in experiment and experiment in theory.

ARNE SCHIRRMACHER

Experimenting theory: The proofs of Kirchhoff's radiation law before and after Planck

ABSTRACT:
The role of experimental thinking and action in theorizing is investigated using an example from classical radiation theory. The history of Kirchhoff's law exhibits both the development of the views on radiation and the evolution of the content as well as the assumed foundational roots of this law. Planck's search for the correct justification of his radiation formula is placed into the context of the contemporary debate over his prerequisite. It is then asked what the analysis of the variety of approaches, arguments, and ontological claims that can be found in radiation theory can reveal to us concerning the conceptual framework that was available in Planck's researches. Next, the different forms of reasoning applied in proving a physical law will be exemplified, which range from procedures that are closely abstracted from experimental action like those found with Kirchhoff or Helmholtz, to a purely mathematical approach—as that of Hilbert—which is void of any experimental notion or object. This discussion shall finally both locate Planck's specific method and elucidate the great difficulties the establishment of a truly non-experimental, i.e., mathematical, theory in physics met before a new generation of quantum physicists appeared.
