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Mach's Thesis: Thermodynamics as the Basic Theory for Physics Teaching

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ABSTRACT: Recently, some teachers have launched proposals for teaching physics by starting from thermodynamics rather than from mechanics. Such proposals are included in Mach's thesis, according to which thermodynamics is more basic for theoretical physics than mechanics. The past history of such a thesis is quickly sketched. Then, by recent results in theoretical physics, it is qualified in a new, modern version. As such it appears as an instance of a general conflict between two radically different ways to produce theoretical physics. The conflict results from two different options, about both the kind of mathematics and the kind of the organization of the theory. After such a general illustration of the great relevance of history of physics for physics teaching, the cultural bounds of such proposals are recognized and discussed.

1. A NEW CHARACTERIZATION OF PHYSICS TEACHING

By an historical analysis I suggest a general scheme for interpreting the history of classical physics;[1] such scheme is sketched in Table I. Two basic choices characterize any physical theory:[2] among the possible four models of scientific theory the Newtonian one dominated the history of classical physics; correctly it may be called the paradigm of classical physics. However, a different model, the Carnotian one, is highly relevant, being followed by many physical theories.

Such choices stood unrecognized for the past three centuries. Rather, scientists surrogated them by means of basic physical notions; for example, they surrogated AO by means of the notion of a priori space. In this way, any basic notion acquired, beyond the experimental meaning, a philosophical meaning. Such an odd way of conceiving physical notions actually constituted a particular culture which is a specific feature of the physical community. Past history of physics itself has been determined by such a subjective way of looking on the reality.

A different history is presented to students by textbooks and teachers. This history is a mere addition to the illustration of a long list of objective techniques by which one may re-construct a sketch of the theory as well as one may study some applications. No basic question about foundations of physics is mentioned. Moreover, no inquiry about physical notions is presented, even for introducing their definitions. Although confined to techniques only, such didactics does not present the whole objective realm. At the undergraduate level it presents only some representative, mathematical tools pertaining to theoretical physics. Furthermore, it is shown by Table I that present-day didactics ignores the alternative value of the

TABLE I

Effective history (as scientific geniuses determined it through two fundamental options)	**Subjective history** (as scientists conceived it through surrogatory notions)	**Objective history** (as teachers formalized it through tools of reasoning)
Newtonian MST (AO + AI)	'Dissolution of the finite cosmos and geometrization of space'	Classic logic *Analytic method* Infinitesimal analysis (main example: differential equations of the 2° order)
Carnotian MTS (PO + PI)	'Evanescence of the force – cause and discretization of matter'	Non-classical logic *Synthetic method* (Reasoning by a cycle (main example: S. Carnot's cycle in thermodynamic)

Legenda: MST = Model of Scientific Theory; AO = Aristotelian Organization: PO = Problematic Organization; AI = Actual Infinity; PI = Potential Infinity.

formal tools of the Carnotian model; it relegates the cyclic way of reasoning to be a mere engineer's tool. The synthetic way of reasoning is ignored as well as non-classical logic (although the latter one is well-recognized by modern theoretical physics as the logic of quantum mechanics).

By following the main ideas of the previous scheme, some years ago I started to conceive physics teaching in a different way from the dominant one. In a previous paper[2] I suggested a feasible way for radically reverting the attitude of present-day physics teaching, at least with regard to thermodynamics. By making a few alterations of the original text one may use the book written by S. Carnot as a textbook of modern theory; such a proposal joins history of physics with physics teaching. Furthermore by such a book one may illustrate the basic choices of the Carnotian model of scientific theory, in opposition to the basic choices of the Newtonian model, as they appear in Newton's principles; hence, one may introduce pupils even to the foundations of physics.

In the following, I want to suggest further innovations which concern the subjective realm only. I will present a new teaching of the basic concepts of both thermodynamics and mechanics.

2. A NEW PROGRAM WITH RESPECT TO TRADITIONAL AND MODERN PHYSICS TEACHING

Let us focus our attention upon traditional didactics of subjective physics and list its main features. According to such a didactics, mechanics is the first theory to be taught. Not only because it has been conceived as the first physical theory – although it was geometrical optics that was the first mathematized physical theory – but because both its main concepts and

TABLE II

Continuous functions	Mechanics	Thermodynamics
1. Axioms of vector spaces	Axioms of forces	Axioms of entropy
2. Definition of a continuous functions	Principles of mechanics	Principles of entropy production
3. Theorems about all continuous function	Theorems about all mechanical systems	Theorems about all thermodynamical systems
4. Definitions of special functions, and their properties	Definitions of special systems and materials, and their properties	Definitions of special systems and materials, and their properties

its principles are conceived in a so universal way that they would play a basic role in illustrating the whole of theoretical physics. However, when traditional physics teaching illustrates thermodynamics after mechanics, it confesses to the pupils the incompleteness of such a program; mechanical notions do not apply to such a new theory. In other words, traditional didactics over-estimates mechanics; however, it presents thermodynamics in a respectful way.

After the Sputnik's launch, physics teaching has been forced to be more efficient. It changed in order to support what nuclear physicists think to be the relevant classical theories; surely not thermodynamics, which is considered as already included in statistical mechanics. They think instead that both classical mechanics and electromagnetism have to be taught extensively. In this way, present physics teaching – e.g. PSSC – no more follows a pluralistic attitude with respect to the theories of classical physics; it emphasizes just what a pupil needs for present-day curriculum in higher physical studies and it disregards the theoretical relevance of thermodynamics. Such an attitude is re-enforced by current research in history of physics; for example in the 1960s C. Truesdell launched a program for re-discovering the 'true' history of thermodynamics;[3] it is based upon so intensive a use of modern analytical tools that thermodynamics is reduced to a section of continuum mechanics.

Here the basic notions of traditional thermodynamics suffer radical variations in meaning.

My program is characterized by a different direction. I will introduce the two following innovations with respect to traditional physics teaching: (i) to restore the traditional pluralistic attitude; in the following, by appealing to Table I, I will qualify such a pluralism as originating from the two antagonistic, models of scientific theory – i.e. the Carnotian one and the Newtonian one; (ii) in opposition to the great emphasis traditional physics teaching gives to classical mechanics, give emphasis to thermodynamics; in the following I will qualify such an emphasis by appealing to Mach's thesis.

3. THE DIFFICULT DISCUSSION ABOUT THE BASIC PHYSICAL NOTIONS

In the above I characterized common physics teaching as an illustration of essentially a list of relevant techniques. However, when teachers analyze their ways of teaching physics, they correctly enlarge their views to include the subjective realm, i.e. they refer to the main features of the physical *notions*. Only in this way cultural variables may matter at least as much as mathematical techniques do.

Unfortunately, their analyses are often ineffective. Indeed, in the history of the intellectual life, our basic concepts suffered an analysis long before the rise of modern science. Ancient Greek philosophers scrutinized the basic notions of our thinking. Then such a kind of analysis became a subordinate activity of some particular philosophers, all of them following divergent directions. The present crisis of the philosophy of knowledge denies a general agreement about a particular kind of philosophical analysis of the basic notions belonging to physical theories.

In our century a new philosophical attitude gained relevance, i.e. cognitive theory.[4] It suggests both a general method of inquiring about the pupils' acquisitions and some remarkable problems. By following such a theory physics teachers achieved for a first time a common language for discussing their problems.

However, some decades ago, a scholar of history of science sharply characterized the basic concepts of great part of classical physics. A. Koyré offered an authoritative analysis of the rise of modern science, including Newtonian theory, by means of the following concepts: 'dissolution of the finite cosmos and geometrization of space'.[5] In particular, Koyré stressed – like the previous scheme does – the notion of infinity as a crucial one for the rise of modern science. By comparison, cognitive science is not able to deal with such a notion.

Moreover, the previous scheme suggests one more crucial notion for a scientific theory, i.e. the organization of a theory. Such a category, together with that of infinity suggests that physical theories may follow at least two distinct ways of conceptualizing the basic notions, i.e. the Newtonian one and the Carnotian one. Unfortunately, such suggestions too are ignored by cognitive science, which results in inadequacies with respect to Koyré's historical analysis and even more with respect to the scheme of Table I. I conclude that cognitive science may represent a mere first approximation to the analysis a teacher needs for achieving cultural control of his own scientific didactics.

In the following, by means of Mach's viewpoint we will throw new light on the basic notions of both thermodynamics and mechanics; finally, we will make clear why a physics teacher meets great difficulties when they naively attempt to inquire into the basic notions of physics.

4. THERMODYBNAMICS AS THE BASIC THEORY FOR THEORETICAL PHYSICS. THE HISTORY OF MACH'S THESIS

One has to remember that in the last century E. Mach supported the attitude of emphasizing thermodynamics in both theoretical physics and philosophy of science. In particular, he stated his celebrated thesis that in the historical development of physics, classical mechanics came first only by fortunate accident, but from a foundational viewpoint thermodynamics has to be considered as the basic theory for the whole of theoretical physics.[6]

In the following I will argue for a re-evaluation of Mach's thesis also for physics teaching – although Mach never suggested a new physics teaching.

It is well-known that in the fourth quarter of last century theoretical physicists debated about the relevance of thermodynamics for mechanics. Rankine, Helm, Voigt, Duhem, Ostwald, Planck, among many others, were the supporters of thermodynamics. They have been called 'energetists' inasmuch as they stressed overall the relevance of the energy concept with respect to the notion of force. However, Mach's thesis represents the true core of the theoretical claims of such a group of supporters, although everyone shared it in his own particular way.

The main arguments supporting Mach's thesis were the following ones. Mechanics is an idealistic theory – or at least a very abstract theory – on the real world inasmuch as it disregards friction, which instead is an universal phenomenon. Thermodynamics is an empirical and operative theory, whereas mechanics appeals to abstract and even idealistic notions (like absolute space and absolute time). In mechanics time is a reversible parameter whereas thermodynamics – in according to everyday life – suggests that time is irreversible in nature. Moreover, thermodynamics is a universal science even more that mechanics. For supporting such a claim Mach suggested an original use of the Carnot's cycle for describing in an analogical way many situations belonging to mechanics, to electrostatics, etc.[7]

However, Mach's thesis – as the program of re-founding theoretical physics upon thermodynamical concepts – met very hard obstacles: (a) In thermodynamics mathematics works at an elementary level, whereas in mechanics the higher mathematics of differential equations occurs as an essential component. It was an unsolved question in what way the a-typical mathematical notions belonging to thermodynamics may constitute a basis for originating the differential equations which mechanics make use of. (b) The basic principle for passing from thermodynamics to mechanics is the conservation of energy. But a mathematical version of it in mechanics requires the existence of a primitive function of dW, i.e. the differential of the work. Some authors showed that the problem is solvable in some particular cases only;[8] hence any mathematical version of conservation of energy cannot represent it as a universal principle. (c) Energy conservation as a scalar law is unable to give the description of a three dimensional motion. (d) In an ergodic motion the trajectories are not severed by the constants of motion.[9]

By summarizing, the program of founding theoretical physics upon thermodynamics was unable to develop a new suitable mathematical formalism. Furthermore, in 1909 the mechanicist-oriented Carathéodory introduced in thermodynamics differential equations.[10] Such a result appeared to most people a decisive step – it suggested at the same time the first axiomatics for a physical theory – to consider the traditional formulation of thermodynamics as a naive attempt performed by empirical engineers who were unaware of sophisticated mathematics.[11] As a consequence, Mach's thesis was considered a mere cultural suggestion without consequences for physical formalism.

5. MACH'S THESIS RE-EVALUATED IN MODERN TERMS

However, since the 1930s thermodynamics has received increasing attention because the new theory of irreversible processes proved that the old thermodynamics is still a fertile theory, even suggesting new ideas for biology. Among others, Prigogine is the most famous supporter of the cultural relevance of thermodynamics. He suggests thermodynamics as the basis for a 'new alliance' of mankind with the world of life with respect to the past alliance that mechanics supported unsuccessfully.[12]

Moreover, Mach's thesis has been suggested again by some prominent theorists. From a methodological viewpoint Giles[13] characterized thermodynamics as a true alternative to mechanics with respect to both experimental physics and theoretical physics; explicitly he supports the idea of emphasizing thermodynamics versus mechanics.

In recent years two authors stressed in a formal way the basic role of thermodynamics for the whole of theoretical physics. In 1974 Cällen remarked that thermodynamics presents a very attractive feature, namely every variable represents a conserved quantity by a specific symmetry – either a classical symmetry or a non-classical one. Hence, thermodynamics is actually a science of symmetry.[14]

Furthermore, in a wide review of theoretical physics Barut[15] suggested that since Kepler's works, theoretical physics has presented two basic attitudes, i.e. the dynamic's one and the symmetry's one. In some cases they are complementary, in some cases they are opposite one to another. We easily recognize in the dynamic approach the theoretical tradition of mechanics, and in the symmetry approach – via Cällen's result – the theoretical tradition of thermodynamics.

The last result leads us to qualify Mach's thesis in modern terms: '*The traditional claim of theoretical mechanics for a monopoly of – or at least for a higher relevance in – theoretical physics is unjustified; thermodynamic theory enjoys an equal theoretical dignity with mechanics although its tech-*

nique of reasoning, i.e. symmetry, has been formalized in mathematical terms much later than infinitesimal analysis. It is a matter of personal opinion to choose the former theory, or the latter one, as the basic theory in physics'.

6. MACH'S THESIS IN PHYSICS TEACHING

From the above re-evaluation of Mach's thesis we conclude that there is no objective criterion for preferring mechanics to thermodynamics. Thus it is legitimate to start physics teaching either from mechanics as the basic theory or from thermodynamics.

Actually, in recent years some authors[16] advanced interesting proposals for emphasizing the role of thermodynamics in physics teaching. In the 1970s the energy debate in society and then in the 1980s peace education motivated many teachers to teach physics by starting from social problems. The notions of work and energy may be considered as 'social' notions when compared with the anthropomorphic, individual notion of 'force'. A physics teaching based upon the energy notion leads directly stressing the relevance of thermodynamics. Some other teachers, reached the same kind of teaching by following an internalist viewpoint of science; they echoed the great relevance thermodynamics acquired in the philosophical debate after the proposal of a 'new alliance'.

Like in Mach's thesis, in such proposals it is a hard problem to link thermodynamic notions to mechanical notions. A radical thermodynamicist may retort that the traditional teaching also meets the same problem. Also the teaching which starts from mechanics does not offer a clear way of linking mechanical notions to thermodynamical notions. However – he may stress – thermodynamics offers one basic notion as a common notion, i.e., energy. Moreover the notion of pressure is more adequate to pupils' minds for suggesting the notion of mechanical force. But he cannot go further. The remaining notions are very different in nature. Hence the theoretical systems of the two theories may be correlated by means of some reductive analogy, for example by means of the well-known analogy of the hydraulic wheel – really, a misleading one with respect to the nature of heat.

7. IMPROVING THE PROPOSALS BY CHOOSING THE SUITABLE FORMULATION FOR A THEORY

Moreover, the greatest difference between thermodynamics and mechanics appears to be the kind of mathematics – ignored by Mach's thesis in the last century. However, an analysis of the history of thermodynamics shows that its elementary mathematics is not a mere first attempt for introducing the same advanced mathematics pertaining to mechanics; in particular the Carathéodory's formulation was weaker than the traditional one.[17] As a consequence, one has to inquire whether mechanics also may be formulated without differential equations. Really, in the history of mechanics, since 1783 L. Carnot offered a mechanics whose mathematics includes algebraic equations only. That is, the equations of the conservations of energy, momentum and momentum-of-momentum, all to be solved in velocity only, not in the space. Actually such a formulation obtains the conservation laws by means of a symmetry technique.[18] Moreover, L. Carnot inspired both directly and indirectly the S. Carnot's invention of thermodynamics.[19] Thus, we discover that our looking for a mathematical linkage between thermodynamics and mechanics receives a solution by an effective historical linkage, but considered in the reverse direction. In this linkage between L. Carnot and S. Carnot, more notions are produced that pertain to both theories. An example is the notion of irreversibility – because in L. Carnot's mechanics the basic phenomenon is the shock, possibly the inelastic shock, and the energy balance always includes the quantity 'lost energy'. Moreover both theories deal with systems, instead of with elementary components. Finally both theories come from reflections about machines.

In passing, it is remarkable that by L. Carnot's formulation one can teach mechanics at a high-school level by means of symmetries; indeed no technical difficulties arise, because L. Carnot's mathematics is algebraic. The cost of such teaching is the introduction – eventually in the place of Newton's second law – of a general equation $\Sigma\, m_i U_i u_i = 0$ which is a generalization of the principle of virtual work.

An improved teaching approach is gained when thermodynamics itself is formulated in a more suitable way. In 1942 Brønsted[20] suggested that any kind of work may be written as $X \Delta Y$, i.e., as the transfer of a quantity X from the two levels of a potential Y. In thermodynamics the second principle merely introduces a thermal work as expressed by two similar functions, the potential T and the quantity S, all defined by suitable instruments. (By such a formulation one generalizes what L. Carnot did in mechanics two centuries ago, that is to identify any mechanical work with the rising of a weight, that is, $p \Delta h$). In this way the energy notion is sharply defined, and the entropy notion becomes a natural one. Brønsted's formulation stresses that heat is an anthropomorphic notion which may be used in the irreversible processes only.

Such ideas suggest an analogous thesis in mechanics: the true notion is that of energy potential which actually is well-defined in mathematical terms. Instead force – just the thesis supported in the past by D'Alembert, Carnot and Mach – is an anthropomorphic notion which is introduced for surrogating our intuition when we are unable to obtain in a mathematical way a potential. This is an example of the new interpretations of the basic mechanical notions such a linkage may suggest to us.

8. THE BOUND OF INCOMMENSURABILITY

Why do thermodynamic notions approach more or less to the mechanical notions according to the particular formulation which we are choosing for anyone of them? We have to remember the suggestion by Feyerabend and Kuhn, that is, two physical theories may be *incommensurable*. In such a case almost any notion belonging to a theory is radically different from the corresponding notion, if any, belonging to the other theory. Really, the traditional teaching, which starts from mechanics as first theory, manifests an incommensurability when it is not able to introduce thermodynamics by means of its basic notions (space, time, etc.). As a further example, let us remember that force in thermodynamics is rather a pressure. Almost the same occurs when by starting from thermodynamics we try to suggest a continuity between the notions of thermodynamics and the notions of Newtonian mechanics.

Elsewhere, I offered a new, sharp definition of incommensurability:[21] two theories are incommensurable when they differ in at least one basic choice. As a consequence, Newtonian mechanics and thermodynamics are clearly incommensurable theories because they differ in the kind of mathematics; it is a natural consequence that their notions suffer radical variations. By an historical analysis I found out a list of the radical variations between the basic notions of Newtonian mechanics on one side, and on the other side the basic notions of L. Carnot's mechanics and S. Carnot's thermodynamics. They are listed on the same paper.[21]

In contrast, the difference between the two sets of notions is not great when the two formulations are commensurable ones. This is the case for L. Carnot's mechanics and either S. Carnot's or Brøsted's formulation of thermodynamics.

As a natural consequence one may cumulate the greatest number of notions belonging at the same time to two theories only provided that he presents such theories by means of commensurable formulations. On the contrary, one finds very few notions belonging at the same time to two incommensurable formulations.

9. CONCLUSIONS

As a general conclusion, the present paper suggests that physics teaching should not monopolize theoretical physics under only one theory, be it mechanics of thermodynamics. When pluralism is followed, two strategies are possible. Without trying to connect their notions, the different theories are presented as separate theories, in order to stress the richness of theoretical physics. Otherwise, a unitarian coherent theoretical approach is presented by linking mechanics and thermodynamics by means of their common notions. But in such a case one has to be careful to choose commensurable formulations in order to present not a loose analogical view, but a unitarian way to look at foundational notions of physics. In particular, when one wants to stress the theoretical relevance of thermodynamics, one may present mechanics by means of L. Carnot's formulation, eventually by the symmetry technique. In the last case it would be highly desirable to find a suitable way for teaching non-classical symmetries in order to present thermodynamcis according to Cällen.

NOTES

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