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Binary Stars and the Velocity of Light*

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De Sitter showed that if the velocity of light is constant with respect to the source, distant binaries will exhibit certain peculiarities that have not been observed. This is usually regarded as proof of the validity of Einstein's second postulate of special relativity. But there are several ways of invalidating the proof. One of the most promising ways—that of employing Riemannian space for light—is considered in this paper.

Data on visual binaries, spectroscopic binaries, and cepheids are calculated for Euclidean space and for Riemannian space of constant positive curvature ($R=5$ light years). The acceptance of Riemannian space allows us to reject Einstein's relativity and to keep all the ordinary ideas of time and all the ideas of Euclidean space out to a distance of a few light years. Astronomical space remains Euclidean for material bodies, but light is considered to travel in Riemannian space. In this way the time required for light to reach us from the most distant stars is only 15 years.

1. INTRODUCTION

THE principal hypothesis[1] of special relativity is that in free space, the velocity of light is constant *with respect to the observer*, independent of motion of source or observer. This assumption is contrary to all human experience, and it can be included in the theory only by abolishing ordinary ideas of space and time.

The alternative assumption is that the velocity of light is constant *with respect to the source*, as advocated in the "emission theory" of Ritz.[2] Apparently the only evidence in favor of the Einstein hypothesis is given by the behavior of binary stars.[3] If the velocity of light is independent of the velocity of the stellar source, then the observed motion of the star in its orbit will be the true motion, except for the constant time interval required for light to travel from star to earth. On the other hand, if the velocity of the star and the velocity of light are additive, the apparent orbit will be distorted and the apparent stellar magnitude will vary.

Lively discussion on this subject took place in 1913 among de Sitter,[3] Freundlich,[4] Gutnick,[5] and Zurhellen.[6] Again in 1924, arguments were advanced by de Sitter,[7] La Rosa,[8] Bernheimer,[9] and Thirring.[10] The present paper reopens the subject to investigate possibilities that were ignored in the earlier discussions.

2. CRITERIA

Consider a binary star (Fig. 1), and an observer who is in the plane of the orbit and at distance r ($r \gg a$) from the center of the orbit. The outer star is luminous and has a uniform velocity v. Its companion is assumed to radiate negligible light. The center of mass of the binary moves at velocity w with respect to the observer.

Assume two clocks, one at the star and one at the observer. The reading of the former will be called t_s and that of the latter will be called t. Einstein's relativity is not used, so the clocks can be synchronized.

Assuming composition of velocities, we find that the

* Presented at the meeting of the Optical Society of America, New York, March 21, 1953.

[1] A. Einstein, Ann. Physik 17, 891 (1905).
[2] W. Ritz, Ann. chim. et phys. 13, 145 (1908).
[3] W. de Sitter, Physik. Z. 14, 429, 1267 (1913).
[4] E. Freundlich, Physik. Z. 14, 835 (1913).
[5] P. Gutnick, Astron. Nachr. 195, 265 (1913).
[6] W. Zurhellen, Astron. Nachr. 198, 1 (1914).
[7] W. de Sitter, Bull. Astron. Inst. Netherlands 2, 121 (1924); *ibid.*, p. 163.
[8] M. La Rosa, Z. Physik 21, 333 (1924); Z. Physik 34, 698 (1925).
[9] W. E. Bernheimer, Z. Physik 36, 302 (1926).
[10] H. Thirring, Z. Physik 31, 133 (1925).

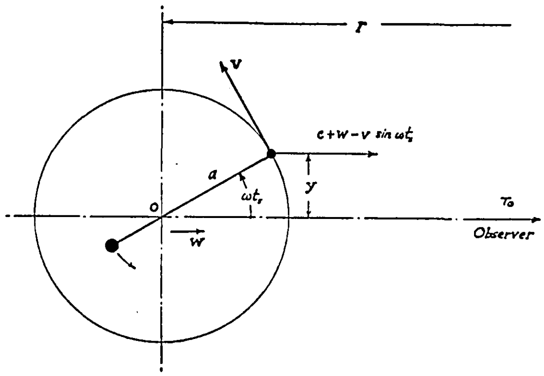

FIG. 1. Circular orbit for a binary star.

velocity of light toward the observer is $(c+w)-v\ \sin\omega t_s$. The time required for this light to reach the observer is

$$\frac{r-a\ \cos\omega t_s}{(c+w)-v\ \sin\omega t_s}.$$

The reading of the observer's clock, when he sees the star in the position of Fig. 1, is

$$t=t_s+\frac{r-a\ \cos\omega t_s}{(c+w)\{1-[v/(c+w)]\ \sin\omega t_s\}}. \tag{1}$$

Since v and w are very small compared with c (less than 0.1 percent), Eq. (1) may be written

$$t\cong t_s+\frac{r-a\ \cos\omega t_s}{c}\left(1+\frac{v}{c}\ \sin\omega t_s\right).$$

Also, $a\ll r$ so

$$t\cong t_s+\frac{r}{c}+\frac{rv}{c}\ \sin\omega t_s. \tag{1a}$$

It is convenient to express this equation in terms of the displacement of the star from its central position:

$$y/a=\sin\omega t_s.$$

Then

$$t=\frac{1}{\omega}\ \sin^{-1}(y/a)+\frac{r}{c}+\frac{vr}{c^2}(y/a)$$

or

$$\theta=\sin^{-1}(y/a)+\Gamma\cdot(y/a), \tag{2}$$

where θ specifies the angular position of the star in its orbit, corrected for the mean velocity of light:

$$\theta=\omega t_s-r/c. \tag{3}$$

A characteristic constant is

$$\Gamma=\frac{2\pi}{\tau}\left(\frac{v}{c}\right)\left(\frac{r}{c}\right). \tag{4}$$

Figure 2 shows the apparent motion of the star, as seen by the distant observer. If $\Gamma=0$, the motion is a true sinusoid, and for $\Gamma<0.1$, the motion departs very little from sinusoidal. As Γ increases, however, the curve is tilted and the function may become multivalued. For example, if $\Gamma=2$, *one* star is seen as θ varies between 0.69 and 2.45 radians; but at $\theta=2.45$, the observer suddenly begins to see *three* stars at different points on the same orbit. This continues until $\theta=3.83$, when the star seems to revert to its single state. This phenomenon would be shown also by the spectrograph. Each spectral line would be single for $0.69<\theta<2.45$ but would appear as three lines for the remainder of the cycle.

Now consider the critical condition. As Γ increases, we arrive at the case of $d\theta/d(y/a)=0$, beyond which the

TABLE I. Visual binaries.

Name	Parallax $p('')$	Distance (light years)	Period (yr)	(v/c)	Γ_E
δ Equ	0.060±.006	54.3	5.70	7.82×10^{-5}	4.68×10^{-3}
κ Peg	0.026±.005	125	11.3	9.77×10^{-5}	6.78×10^{-3}
ϵ Hya	0.020±.005	163	15.3	7.47×10^{-5}	5.00×10^{-3}
42 Com	0.063±.008	51.7	25.9	4.02×10^{-5}	5.05×10^{-4}
85 Peg	0.092±.006	35.4	26.3	3.36×10^{-5}	2.84×10^{-4}
Σ 3121	0.056±.008	58.2	34.3	3.47×10^{-5}	3.70×10^{-4}
ζ Her	0.111±.005	29.4	34.5	3.51×10^{-5}	1.88×10^{-4}
α CMi	0.312±.006	10.4	39.0	3.30×10^{-5}	5.55×10^{-5}
β 416	0.169±.016	19.3	42.2	2.55×10^{-5}	7.32×10^{-5}
μ HerBC	0.111±.006	29.4	43.2	2.70×10^{-5}	1.15×10^{-4}
Krüger 60	0.257±.004	12.7	44.3	2.15×10^{-5}	3.87×10^{-5}
ξ Sco	0.040±.005	81.5	44.7	4.00×10^{-5}	4.58×10^{-4}
Σ 2173	0.052±.006	62.7	46.0	4.41×10^{-5}	3.78×10^{-4}
τ Cyg	0.050±.006	65.2	47.0	3.85×10^{-5}	3.36×10^{-4}
α CMa	0.371±.004	8.78	50.0	4.07×10^{-5}	4.49×10^{-5}
β 648	0.064±.005	50.9	56.6	3.44×10^{-5}	1.94×10^{-4}
ξ UMa	0.146±.006	22.3	59.8	2.85×10^{-5}	6.67×10^{-5}
99 Her	0.042±.006	77.6	63.0	3.76×10^{-5}	2.91×10^{-4}
α Cen	0.758±.010	4.29	78.8	2.94×10^{-5}	1.01×10^{-5}
70 Oph	0.192±.005	17.0	87.7	2.66×10^{-5}	3.24×10^{-5}
γ CrB	0.022±.006	148	87.8	3.76×10^{-5}	3.98×10^{-4}
0 Σ 79	0.027±.004	121	88.9	2.35×10^{-5}	2.01×10^{-4}
ξ Boo	0.168±.007	19.4	153	1.87×10^{-5}	1.49×10^{-5}
Σ 2052	0.055±.006	59.2	132	1.91×10^{-5}	4.64×10^{-5}
o_2 EriBC	0.203±.008	16.0	248	1.36×10^{-5}	5.52×10^{-6}
α Gem	0.076±.004	42.8	306	3.58×10^{-5}	3.14×10^{-5}
η Cas	0.182±.005	17.9	346	1.59×10^{-5}	5.17×10^{-6}

curve becomes multivalued. From Eq. (2),

$$\frac{d\theta}{d(y/a)}=\pm\frac{1}{[1-(y/a)^2]^{\frac{1}{2}}}+\Gamma=0,$$

or

$$\Gamma=\frac{1}{[1-(y/a)^2]^{\frac{1}{2}}}. \tag{5}$$

Figure 2 indicates that this zero slope will occur at $y/a=0$, so Eq. (5) gives for the first critical point,

$$\Gamma_1=1.00. \tag{6}$$

For values of Γ less than unity, no multiple images can occur.

The time effect is also interesting. For $w\ll c$, Eq. (1) becomes

$$t=t_s+\frac{r-a\ \cos\omega t_s}{c-v\ \sin\omega t_s}.$$

Differentiation gives

$$\frac{dt}{dt_s}=1+\frac{\omega v\cos\omega t_s}{[c-v\sin\omega t_s]^2}[r-a\cos\omega t_s]+\frac{\omega a\sin\omega t_s}{c-v\sin\omega t_s}.$$

If $a\ll r$ and $v\ll c$,

$$\frac{dt}{dt_s}\cong 1+\frac{v}{c}\sin\omega t_s+\frac{2\pi}{\tau}\left(\frac{r}{c}\right)\left(\frac{v}{c}\right)\cos\omega t_s,$$

or

$$\frac{dt}{dt_s}\cong 1+\Gamma\cos\omega t_s. \tag{7}$$

Suppose that a star emits toward the observer Q_s joule steradian^{-1} during the time interval Δt_s. The radiation reaches the observer, not in the interval Δt_s but in the interval Δt:

$$\Delta t=\Delta t_s[1+\Gamma\cos\omega t_s]. \tag{8}$$

If the star were not moving, the average radiant pharosage[11] measured by the observer would be

$$D_0=\frac{Q_s}{r^2\Delta t}=\frac{Q_s}{r^2\Delta t_s}\quad (\text{watt m}^{-2}).$$

But with the star in motion, the observed pharosage is

$$D=\frac{Q_s}{r^2\Delta t}=\frac{Q_s}{r^2\Delta t_s[1+\Gamma\cos\omega t_s]}.$$

The ratio is

$$\frac{D}{D_0}=\frac{1}{1+\Gamma\cos\omega t_s}. \tag{9}$$

If $\Gamma=1$, the observer will measure a periodic variation in pharosage ranging from $\frac{1}{2}D_0$ to infinity.

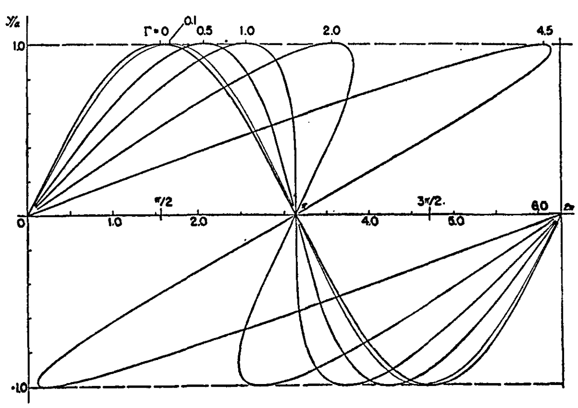

Fig. 2. Apparent behavior of a binary as seen by a distant observer. The distortion of the sinusoidal curve is caused by the difference in the velocity of light from various parts of the orbit (assuming composition of velocities). The effect becomes more pronounced as Γ increases: $\Gamma=(2\pi/\tau)(v/c)(r/c)$. The critical condition occurs at $\Gamma=1$, beyond which there are multiple images.

[11] Moon and Spencer, J. Opt. Soc. Am. **36**, 666 (1946); *Lighting Design* (Addison-Wesley Press, Cambridge, Massachusetts, 1948).

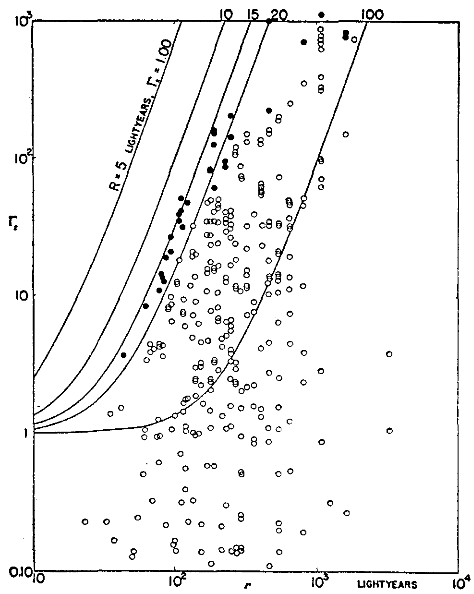

Fig. 3. Γ_E as a function of Euclidean distance for spectroscopic binaries. The black points are listed in Table II. The curves are for $\Gamma_R=1.00$ and space constants of 5, 10, 15, 20, and 100 light years.

The dearth of points below 70 light years is caused partly by the limited number of stars near the earth. The small number of points beyond 10^3 light years is an artifact, produced by lack of parallax data for distant stars. Between 70 and 1000 light years, however, the points, are fairly well distributed; which makes their sharp boundary at the curve $R=15$ rather startling.

Among the hundreds of variable stars that have been measured, nothing like such a variation has ever been observed. One must conclude, therefore, either that the theory is wrong or that Γ is always much less than unity. Perhaps Γ could be as large as 0.1 without making the star noteworthy either by distortion of apparent orbit or by variation in apparent magnitude.

Assuming classical wave theory, one would expect that the apparent frequency of the radiation would vary inversely as the time interval, Eq. (7), or

$$\frac{f}{f_s}=\frac{dt_s}{dt}=\frac{1}{1+\Gamma\cos\omega t_s}. \tag{10}$$

This would result in a huge Doppler effect, as pointed out by Zurhellen.[6] On the quantum basis, however, these violent changes would not be expected. La Rosa[8] and de Sitter[7] are in apparent agreement that Eq. (10) may be disregarded.

The foregoing analysis indicates the peculiar phenomena that must occur with binaries of sufficiently short period observed at sufficiently large distance, assuming composition of velocities. The analysis can be extended to the general case of tilted elliptical orbits with both stars radiating, but the general conclusions remain the same.

3. VISUAL BINARIES

First consider the visual binaries. Of these, Bergmann says:[12]

> "If the speed of light depended on its source, the double stars should give rise to peculiar phenomena. . . . In some cases, we should observe the same component of the double star system simultaneously at different places, and these 'ghost stars' would disappear and reappear in the course of their periodic motions."

To check such statements, we computed values of Γ for visual binaries. The values of Γ were in all cases much below the critical value of unity (Sec. 2), being usually of the order of 10^{-5}, as shown in Table I. We conclude, therefore, that *the data on visual binaries prove absolutely nothing about the constancy of the velocity of light.* Contrary to the statements of Bergmann and others, the Ritz hypothesis leads to no multiple images of visual binaries, nor does it predict multiple spectral lines for these stars.

4. SPECTROSCOPIC BINARIES

A similar analysis can be made of spectroscopic binaries. Data for 282 stars were taken from the Fifth

TABLE II. Some spectroscopic binaries (Γ_R computed for $R=5$ light years).

Lick No.	Name	p (″)	r (light years)	τ (days)	v (km sec^{-1})	Γ_E	Γ_R
5	Σ 12 A	0.017 (dyn)	192	0.842	87.96 92.57	153 161	0.0328 0.0345
6	AO Cas	0.002 (spec.)	1628	3.523	218.5 234.5	772 827	2.81×10^{-4} 3.01×10^{-4}
11	13 Cet A	0.052±0.008	62.6	2.082	37	8.48	0.0478
14	π Cas	0.018 (spec.)	181	1.964	117.3 119.0	82.5 83.7	0.0210 0.0213
23	ζ Phe	0.013 (dyn)	250	1.670	125 180	143 206	0.0140 0.0201
34	X Tri	0.004 (spec.)	815	0.972	110	705	2.02×10^{-3}
52	CC Cas	0.017 (spec.)	192	3.369	141.6 291.8	61.3 126.2	0.0132 0.0272
21	β Aur	0.037±.0004	88.0	3.960	108.9 111.0	18.4 18.8	0.0390 0.0398
149	RC Ma	0.039±.0008	83.4	1.136	24.0	13.5	0.0333
152	B 1945	0.029 (spec.)	112	1.933	94.6 116.8	41.8 51.6	0.0435 0.0536
156	α¹ Gem	0.073±0.003	44.6	2.928	31.88	3.72	0.0539
169	V Pup	0.003 (spec.)	1085	1.455	199 342	1132 1942	1.38×10^{-3} 2.37×10^{-3}
174	B 2227	0.038±0.010	85.7	1.563	30.28	12.7	0.0288
195	o Leo	0.028±0.007	116	1.686	60	31.6	0.0296
229	B 3182	0.026±0.005	125	1.271	63.2	47.5	0.0358
246	ζ U Ma A	0.040±0.005	81.3	1.81	42.0	14.4	0.0383
304	B 4247	0.030 (spec.)	109	2.308	97.4 108.7	35.0 39.1	0.0400 0.0446
345	d Ser	0.014±0.006	232	1.850	90 100	86.1 95.6	0.0105 0.0117
368	+16° 3758	0.041±0.010	79.3	4.812	86.03 86.04	10.8 10.8	0.0309 0.0309
432	B 5579	0.007 (spec.)	463	1.729	109.7	224	3.50×10^{-3}
452	B 5764	0.034±0.012	95.5	2.616	74.8 96.0	20.9 26.8	0.0349 0.0447
—	TX Cnc	0.007	463	0.386	112 217	1038 2010	0.0163 0.0314

[12] P. G. Bergmann, *Introduction to the Theory of Relativity* (Prentice-Hall, Inc., New York, 1942), p. 19.

Catalog[13] by Moore and Neubauer. Distances were calculated from Schlesinger's values[14] of trigonometric parallax. When trigonometric values were not available, spectroscopic or dynamic parallaxes were employed. Most of the computed values of Γ were above the critical value of unity, in some cases reaching values of over a thousand.

The data (except for 60 stars with $\Gamma<0.1$) are plotted in Fig. 3, and a list of selected stars is given in Table II. According to theory (Sec. 2), the high values of Γ must result in peculiar behavior which has not been observed. The conclusion is that either the Ritz hypothesis must be abandoned, as the relativists insist, or a radical modification must be made in the values of distance or velocity. These possibilities will be considered in Sec. 6.

5. CEPHEIDS

With a pulsating star, a maximum velocity v of the stellar surface is measured spectrographically. Evidently the same Γ criterion applies as with binary stars. If $\Gamma>1$, the observer receives light simultaneously from several phases of the pulsation, which should result in multiple spectral lines. No such phenomena are observed.

Table III indicates the results for typical cepheids. The velocity data for 144 cepheids were obtained from Joy.[15] Distances were computed by Shapley's formula,[16] on the assumption of no absorption of light in interstellar space. The distances are generally much larger than those of the previous tables; which tends to give large values of Γ. Unless some other explanation can be found, the cepheids provide a proof even more decisive than that given by the spectroscopic binaries, a proof that the velocity of light does not partake of the velocity of the source.

The long-period variables, such as *o* Ceti, were also investigated. The results showed that, like the visual binaries, these stars provide no information for discriminating between the Ritz and the Einstein hypotheses.

6. POSSIBLE EXPLANATIONS

The fact that Γ exceeds the critical value for many cepheids and spectroscopic binaries does not necessarily prove that the velocity of light is independent of the velocity of the source. Possibilities are as follows:

(1) Velocity of light is constant with respect to the *observer* (Einstein).

(2) Velocity of light is constant with respect to the *source* (Ritz). The derivation, Sec. 2, is considered valid but (a) distances are reduced, (b) velocity of light increases at great distances, or (c) Doppler shift does not give true velocity.

(3) The concept of velocity does not apply to light, so c cannot be added to the velocity of a material body (Palágyi).[17]

If the Ritz hypothesis is accepted, the value of Γ must be kept well below unity. But Γ depends on v, r, and c, according to Eq. (4); so a change in any one of the three may satisfy the criterion $\Gamma<1$.

The most promising of these possibilities is Eq. (2a). Assume that light travels in a Riemannian space. The usual distance r employed by astronomers is unchanged as regards material bodies; but for light, it is replaced by the corresponding Riemannian distance[18] s:

$$s=2R\tan^{-1}(r/2R), \tag{11}$$

where R is the *space constant*[19] (radius of curvature) of the space. The velocity of light, $c=ds/dt$, is constant

TABLE III. Cepheids (Γ_R computed for $R=5$ light years).

Name	r (light years)	τ (day)	v_{max} (km sec^{-1})	Γ_E	Γ_R
SU Cas	1015	1.95	11.0	43.7	6.4×10^{-5}
DT Cyg	790	2.50	8.5	20.5	6.3×10^{-5}
AD Gem	5700	3.79	36.0	413	3.5×10^{-6}
Y Aur	5200	3.86	19.5	200	2.2×10^{-6}
CG Cas	20 500	4.36	21.0	753	1.4×10^{-7}
FF Aql	980	4.47	7.1	11.8	2.0×10^{-5}
δ Cep	460	5.37	19.7	12.9	2.0×10^{-4}
η Aql	730	7.18	20.8	16.1	6.3×10^{-5}
PZ Aql	31 000	8.76	19.5	528	2.8×10^{-8}
ζ Gem	650	10.15	14.2	6.96	3.9×10^{-5}
AP Her	20 000	10.42	20.0	292	5.7×10^{-8}
RW Cam	7400	16.41	31.0	107	4.1×10^{-7}
MZ Cyg	98 000	21.17	29.5	1044	1.7×10^{-9}
RX Lib	38 000	24.95	14.5	169	4.9×10^{-9}

with respect to the source. In essence, therefore, the method of this paper leaves astronomical space unchanged but reduces the time required for light to travel from a star to the earth. We postulate[18] also that Doppler shift gives the Euclidean velocity of the source with respect to the observer, dr/dt. The velocity of light received from a moving source is, according to the Ritz principle, c plus the ds/dt of the source corresponding to the Doppler dr/dt.

On the Euclidean basis,

$$\Gamma_E=\frac{2\pi}{\tau c^2}r\frac{dr}{dt}. \tag{12}$$

The corresponding Γ for Riemannian space is

$$\Gamma_R=\frac{2\pi}{\tau c^2}s\frac{ds}{dt}. \tag{13}$$

[13] J. H. Moore and F. J. Neubauer, "Fifth catalog of the orbital elements of spectroscopic binary stars," Lick Observatory Bull. **20**, No. 521.

[14] F. Schlesinger, *General Catalog of Stellar Parallaxes* (Yale University Observatory, 1935); H. N. Russell and C. E. Moore, *The Masses of the Stars* (University of Chicago Press, Chicago, 1940), Table 53.

[15] A. H. Joy, Astrophys. J. **86**, 363 (1937).

[16] H. Shapley, *Galaxies* (The Blakiston Company, Philadelphia, 1943), p. 62.

[17] M. Palágyi, *Ausgewählte Werke* (J. A. Barth, Leipzig, 1925), Vol. III.

[18] P. Moon and D. E. Spencer, "Riemannian space for astronomy" (to be published).

[19] E. W. Hobson, *The Domain of Natural Science* (Cambridge University Press, Cambridge, 1923), p. 144.

But the Riemannian metric[18] is given by

$$ds = dr/[1+(r/2R)^2],$$

so

$$\frac{\Gamma_E}{\Gamma_R} = \frac{(r/2R)[1+(r/2R)^2]}{\tan^{-1}(r/2R)}; \tag{14}$$

and if Γ_R has the critical value of unity,

$$\Gamma_E = \frac{(r/2R)[1+(r/2R)^2]}{\tan^{-1}(r/2R)}. \tag{14a}$$

TABLE IV. Some peculiar spectroscopic binaries (Γ_R computed for $R=5$ light years).

Lick No.	Name	$p('')$	r (light years)	τ (day)	v (km sec^{-1})	Γ_E	Γ_R
436	δ Cap	0.065±.007	50.2	1.023	65.7	24.6	0.257
196	WU Ma	0.019±.007	171.2	0.334	134 188	524 964	0.157 0.289
273	ι Boo B	0.079±.005	41.2	0.268	115 231	136 272	2.44 4.88
—	VW Cep	0.053	61.4	0.278	75 230	126 386	0.747 2.29

Curves of critical Γ_E *vs* r are plotted in Fig. 3 for several values of R.

7. RIEMANNIAN SPACE

Assuming that light behaves as if astronomical space were Riemannian, we next decide on the space constant R. This constant must be large enough so that the metric remains essentially Euclidean for the entire solar system; yet R must be small enough so that $\Gamma < 1$ for all stars.

TABLE V. Distance and velocity (for $R=5$ light years).

Euclidean distance r	$r/2R$	Riemannian distance s	v_R/v_E
1 light year	0.10	0.997 light year	0.990
4	0.40	3.81	0.860
10	1.0	7.85	0.500
30	3.0	12.5	0.100
100	10.0	14.7	0.99×10^{-2}
10^3	100	15.6	1.0×10^{-4}
10^4	1000	15.7	1.0×10^{-6}
∞	∞	15.71	0

Figure 3 shows that the points seem to have a rather sharply defined limit at $R=15$ light years. Data for some of the spectroscopic binaries whose Γ_R approaches unity for $R=15$ are given in Table II. This radius also satisfies the Γ criterion for visual binaries, cepheids, and long-period variables.

The only exceptions were a few stars (Table IV) with very short periods. One star, δ Cap, appears to have very reliable data, but its point would be near the curve $R=5$ of Fig. 3 rather than $R=15$. No other example of this kind could be found. The spectral lines of the WUMa-stars are so diffuse[20] that these stars may be ignored until further information is available.

There remains the question of pharosage[11] variation, Eq. (9), which requires that Γ_R be considerably less than unity. If $\Gamma_R=0.1$ is used as criterion, Eq. (14) indicates that the curves of Fig. 3 must be translated downward by one logarithmic block. Under these circumstances, it is the $R=5$ curve that satisfies the extreme points rather than the $R=15$ curve. *On the basis of present data, therefore, we tentatively fix R at*

$$R = 5 \text{ light years} = 4.73\times10^{16} \text{ m}.$$

This value of R is sufficiently large so that no departure from Euclidean space can be detected in the solar system. The mean distance of Pluto from the sun is 5.91×10^{12} m or 6.26×10^{-4} light years. Thus

$$r/2R = 6.26\times10^{-5}$$

and

$$dr/ds = 1+(6.26\times10^{-5})^2,$$

TABLE VI. Apparent variation in luminous output.

Lick No.	Name	r (light years)	Eccentricity	m	Γ_R	Max D/D_0	Δm
	Variables						
52	CC Cas	192	0.102	7.3 –7.4	0.0132	1.013	0.014
121	β Aur	88.0	0.00	2.07–2.16	0.0390	1.039	0.042
149	RC Ma	83.4	0.013	5.38–5.98	0.0333	1.033	0.036
	Constant stars						
5	Σ 12 A	192	0.027	6.1	0.0328	1.033	0.036
11	13 Cet A	62.6	0.1	5.6	0.0478	1.048	0.052
14	π Cas	181	0.010	5.02	0.0210	1.021	0.023
23	ζ Phe	250	0.14	4.13	0.0140	1.014	0.015
152	B 1945	112	0.002	5.27	0.0435	1.044	0.047
156	α^1 Gem	44.6	0.002	2.85	0.0539	1.054	0.059
174	B 2227	85.7	0.051	5.67	0.0288	1.029	0.031
195	o Leo	116	<0.02	3.76	0.0296	1.030	0.032
229	B 3182	125	0.00	5.12	0.0358	1.036	0.039
246	ζ U Ma A	81.3	0.541	2.40	0.0383	1.038	0.042
304	B 4247	109	0.00	5.91	0.0400	1.040	0.043
368	+16° 3758	79.3	0.073	6.46	0.0309	1.031	0.033
452	B 5764	95.5	0.00	4.66	0.0349	1.035	0.038

giving a departure from the Euclidean metric of the order of 10^{-7} percent. Even for the nearest stars, the effect of the Riemannian metric is small. With α Centauri, for instance, $r=4.3$ light years and the metric changes by only about 18 percent.

A comparison of Euclidean and Riemannian results is presented in Table V. That a space constant of 5 gives values of Γ_R that are well below the critical values is shown by the final columns of Tables II and III. This conclusion is indicated also by Table VI, which presents additional data on some of the critical spectroscopic binaries of Table II. The maximum value of Γ_R is approximately 0.05, which should give no appreciable distortion of the orbit and very small variation in apparent magnitude.

The seventh column of Table VI lists values of maximum pharosage ratio caused by the variation in Δt. From Eq. (9), the maximum ratio is obtained when

[20] O. Struve, *Stellar Evolution* (Princeton University Press, Princeton, 1950), p. 175.

$\omega t_s = \pi$ and is

$$(D/D_0)_{\max} = 1/(1-\Gamma_R). \tag{9a}$$

The eighth column indicates the corresponding variations to be expected in stellar magnitude:

$$\Delta m = 2.500 \log(D/D_0)_{\max}. \tag{15}$$

For the variable stars, this deviation of one-twentieth magnitude or less would certainly not be detected. But if experimental data should prove that the other stars of Table VI are truly constant in magnitude to better than $0.05m$, then the theory will have to be modified or the space constant reduced below $R=5$.

Since there is no reason for assuming that the velocity of light behaves like an ordinary velocity, the foregoing explanation seems to offer a simple and reasonable world picture that allows all of our ordinary ideas of local space and time to remain unchanged. Einstein's relativity is abandoned.† Velocity of light in free space is always c with respect to the source, and has a value for the observer which depends on the relative velocity of source and observer. True Galilean relativity is preserved, as in Newtonian gravitation.

† The use of the Galilean transformation instead of the Lorentz transformation necessitates a change in Maxwell's equations. This question will be considered in another paper.

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Diffraction of Spherical Vector Waves by an Infinite Half-Plane

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The following work is an extension of an earlier publication on scalar diffraction of light. [K. L. McDonald and F. S. Harris, J. Opt. Soc. Am. 42, 321 (1952)]. The present treatment pertains to the vector light field. A new expression is given for the case of electromagnetic waves emitted by a simple harmonic Hertzian oscillator and diffracted by a thin black infinite half-plane immersed in a homogeneous and isotropic medium. By use of Maggi's transformation the problem is reduced to the evaluation of two integrals. Both integrals are approximated by the same method used in the scalar treatment. Two particular orientations of the oscillator are considered, and the six field components are written out for each case in terms of Fresnel integrals. Energy flow, relative intensity distribution, and polarization are discussed. The relative intensity is shown to agree with scalar predictions in the region of the shadow-boundary-plane, and therefore with experiment. The oscillator is infinitely removed from the diffracting edge, thereby allowing a comparison of the black screen with Sommerfeld's perfectly reflecting screen.

I. INTRODUCTION

In a previous publication[1] there was discussed the problem of diffraction of monochromatic spherical scalar waves incident upon a thin black infinite half-plane. It was shown that the theoretical and experimental intensity distributions agree only when the radius of the point source aperture becomes indefinitely small. There was also shown to exist a slight difference in the diffraction patterns produced by highly reflecting and nonreflecting screens. The question of polarization, however, was fittingly ignored, because we concerned ourselves with a theory which imparted no directional properties to the disturbance.

To obviate this shortcoming, we now assume that light is electromagnetic in character and thus is fully described by Maxwell's electromagnetic equations.

The formulation of our problem, which is truly a classical problem in diffraction, nay be accredited for the most part to F. Kottler.[2] This development is somewhat unorthodox, since it assumes the existence of discrete magnetic charges and currents. The final conclusions, however, do satisfy Maxwell's equations in a region free from electric charges and currents, and its predictions are in excellent agreement with experiment.

The first theoretical work on this problem was concerned with the task of expressing the vectors $\bar{E}$, $\bar{H}$ at any interior point in terms of the values of $\bar{E}$, $\bar{H}$ over an enclosing surface S, analogous to Kirchhoff's method for the scalar disturbance.* By extending Maxwell's equations to admit of the existence of discrete magnetic charges and currents, any prescribed ordinary discontinuity in $\bar{E}$ and $\bar{H}$ over the enclosing "transition" surface may be expressed in terms of the charge and current densities on the surface. By use of electric and magnetic vector and scalar potentials and Kirchhoff's solution of the inhomogeneous wave equation, the values of $\bar{E}$ and $\bar{H}$ at any interior point are thus expressed in terms of the retarded values of $\bar{E}$ and $\bar{H}$ over S.

[1] K. L. McDonald and F. S. Harris, J. Opt. Soc. Am. 42, 321 (1952).

[2] F. Kottler, Ann. Physik 71, 458 (1923).

* The six field components are scalar functions which satisfy the wave equation and therefore Kirchhoff's formula. But the components at any interior point must not only satisfy the wave equation but also Maxwell's equations. The problem is therefore not the integration of a wave equation, scalar or vector, but of the simultaneous system of vector equations relating $\bar{E}$ and $\bar{H}$.