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Abstract

Weber-Maxwell electrodynamics is a modernized, compressed, cleansed and, in many respects, advantageous representation of classical electrodynamics that results from the Liénard-Wiechert potentials. In the non-relativistic domain, it is compatible with both Maxwell's electrodynamics and Weber electrodynamics. It is suitable for all electrical engineering tasks, ranging from electrical machines to radar and high-frequency technologies. Weber-Maxwell electrodynamics also simplifies access to quantum physics and other areas of modern physics, such as optics and atomic physics. Particular advantages of Weber-Maxwell electrodynamics are its simple and fast computability in computer calculations and, as it is based on point charges, in the simulation of plasmas. The latter is particularly important for fusion research. Moreover, Weber-Maxwell electrodynamics is also highly suited to academic and post-primary education, as it allows an easy comprehension of both magnetism and electromagnetic waves. Due to the novelty of Weber-Maxwell electrodynamics, there are currently no articles that summarize its most important aspects. The present article aims to achieve this.

Weber-Maxwell electrodynamics: classical electromagnetism in its most compact and pure form

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Weber-Maxwell electrodynamics is a modernized, compressed, cleansed and, in many respects, advantageous representation of classical electrodynamics that results from the Liénard-Wiechert potentials. In the non-relativistic domain, it is compatible with both Maxwell's electrodynamics and Weber electrodynamics. It is suitable for all electrical engineering tasks, ranging from electrical machines to radar and high-frequency technologies. Weber-Maxwell electrodynamics also simplifies access to quantum physics and other areas of modern physics, such as optics and atomic physics. Particular advantages of Weber-Maxwell electrodynamics are its simple and fast computability in computer calculations and, as it is based on point charges, in the simulation of plasmas. The latter is particularly important for fusion research. Moreover, Weber-Maxwell electrodynamics is also highly suited to academic and post-primary education, as it allows an easy comprehension of both magnetism and electromagnetic waves. Due to the novelty of Weber-Maxwell electrodynamics, there are currently no articles that summarize its most important aspects. The present article aims to achieve this.

Contents

I. Introduction

The history of electrodynamics is characterized by numerous developments and theories that were ultimately unable to prevail in practice [1]. One of these theories is Weber electrodynamics, which was developed in the middle of the 19th century by Carl Friedrich Gauss and Wilhelm Weber. Gauss remains a well-known scientist today. Regrettably, despite having an SI unit named after him and an extensive body of scientific work that was far ahead of its time, this is not the case for Weber [2]–[5].

Weber electrodynamics [6] is a very compact and elegant representation of the scientific knowledge of that time. It takes the form of a single formula known as the Weber force. According to the knowledge of that time, science believed that the Coulomb force could be generalized to moving point charges and magnetism was just a multi-particle effect. It is fascinating that the Weber force actually works very well with direct currents and low-frequency alternating currents and allows some effects to be explained that are difficult to interpret using Maxwell's equations [7]–[10].

The basic concept of Weber electrodynamics demonstrates that the definition of a magnetic field can be avoided, because the Weber force shows that the magnetic force can be interpreted as the sum of the individual forces of all charge carriers within an electrically neutral line current. This does not require the individual charge carriers to have individual magnetic fields [11]–[13]. The Weber force is therefore more than just a mathematical description, it also represents a compression of knowledge and an interpretation of magnetism.

However, the Weber force cannot be used to represent electromagnetic waves, at least not directly and without a transmission medium. This is because the Weber force only depends on the locations and speeds of point charges at the current time. The Weber force is therefore an instantaneous force that propagates between point charges without any time delay.

The inability of Weber electrodynamics to represent electromagnetic free-space waves ultimately led to it being almost completely forgotten after around 1890, i.e., after the introduction of the displacement current in Maxwell's equations by James Clerk Maxwell and the experimental detection of electromagnetic free-space waves by Heinrich Hertz. This is not surprising, as electromagnetic free-space waves fascinated scientists to a great extent around 1900, and many new technologies such as radio communication and radar were developed. For this reason, the majority of scientists were only interested in mathematical formalisms that allowed the study and rationalization of electromagnetic free-space waves. The fact that Weber electrodynamics worked better for electrostatics and magnetostatics than the conglomerate of Maxwell's equations, Lorentz transformation, and Lorentz force was soon forgotten.

Currently, few scientists are familiar with Weber electrodynamics, and Maxwell's electrodynamics is the undisputed stan-

dard theory of electrical engineering. However, the predominance of Maxwell's electrodynamics obscures the fact that it also has certain disadvantages, which will not be discussed here. Instead, this article describes the fundamentals of so-called Weber-Maxwell electrodynamics, which has gradually emerged in the last two years. It is based on the Liénard-Wiechert potentials, i.e., the solution of Maxwell's equations for point charges. By means of a mathematical trick, it is possible to simplify the abstract Liénard-Wiechert potentials – without approximation and without loss of information – to a simple force formula that shows great similarities with the force formula of Weber electrodynamics. Ultimately, this shows that there is a close connection between the two seemingly fundamentally different philosophies, allowing their respective disadvantages to be eliminated.

The objective of this article is to provide a compact summary of the current developmental state of Weber-Maxwell electrodynamics and to show what still needs to be investigated. The article is divided into an introductory section that describes the theoretical foundations of Weber-Maxwell electrodynamics, a section that shows how Weber-Maxwell electrodynamics can be used in practice, and a theoretical section that presents the derivation of Weber-Maxwell electrodynamics from Maxwell's equations and shows how Weber electrodynamics can be derived from Weber-Maxwell electrodynamics. In addition, a proof is included demonstrating that the conservation of momentum is always fulfilled in classical electrodynamics and that the limits of Newtonian mechanics are not exceeded, even in the presence of electromagnetic waves.

II. Equations and properties of Weber-Maxwell electrodynamics

In contrast to Maxwell's electrodynamics, the mathematical basis of Weber-Maxwell electrodynamics does not consist of a set of partial differential equations, rather of a kind of generalized Coulomb law, which also applies in particular and explicitly to arbitrarily accelerated point charges. The formula for the electromagnetic force F that a point charge q_s with the trajectory $r_s(t)$ exerts on another point charge q_d with the trajectory $r_d(t)$ is

$$F = \frac{q_d\, q_s\, \gamma(v)}{4\,\pi\,\varepsilon_0} \left(\frac{(r\,c + r\,v)\left(c^2 - v^2 - r \cdot a\right)}{(r\,c + r \cdot v)^3} + \frac{a\,r}{(r\,c + r \cdot v)^2} \right), \tag{1}$$

in Weber-Maxwell electrodynamics. To shorten the notation, the retarded distance vector

$$r := r_d(\tau) - r_s(\tau), \tag{2}$$

the retarded difference velocity

$$v := \dot{r}_d(\tau) - \dot{r}_s(\tau) \tag{3}$$

and the retarded difference acceleration

$$a := \ddot{r}_d(\tau) - \ddot{r}_s(\tau) \tag{4}$$

are introduced. $\gamma(.)$ is the Lorentz factor.

In addition to the formula of the Weber-Maxwell force (1), one also needs the time τ, which can be calculated iteratively using equation[1]

$$\tau = t - \frac{\|r\|}{c}. \tag{5}$$

The fixed-point iteration converges[2], as long as the difference velocity of the two point charges is lower than the speed of light c at all times.

Time τ in the preceding equations corresponds to the time at which the force has left the charge q_s to reach the charge q_d at time t. The equation (5) shows that the electromagnetic force travels independently of the relative velocity v in every inertial frame at the vacuum speed of light c. This means that in Weber-Maxwell electrodynamics, Einstein's postulates are already fulfilled without Lorentz transformation. This represents a significant practical advantage.

Although somewhat surprising at first glance, Weber-Maxwell electrodynamics is able to correctly represent all aspects of classical electrodynamics, including electromagnetic waves. This will become apparent in the course of this article. Furthermore, it will be shown that all other effects, such as magnetism, Lorentz force, and induction, are also included. This may also be surprising, as the Weber-Maxwell force does not require the definition of a magnetic field. Mathematically, both can be traced back to the fact that the Weber-Maxwell force can be derived from the Liénard-Wiechert potentials, i.e., the solution of Maxwell's equations for point charges, and that it contains classical Weber electrodynamics as a special case. Incidentally, the close connection to the Liénard-Wiechert potentials distinguishes Weber-Maxwell electrodynamics from the approach proposed by Moon and Spencer in an article in 1954 [14].

Another special feature of Weber-Maxwell electrodynamics is that it clearly shows that the usual conservation laws also apply in classical electrodynamics; particularly the conservation of momentum. If the source and the receiver of the force are swapped, the signs of r, v, and a are reversed. This results in the force (1) also changing its sign. It is therefore immediately clear that the Weber-Maxwell force satisfies the principle *action = reaction*. The fulfillment of Newton's third law is an essential prerequisite for the fulfillment of the conservation laws and important for electrical engineering. In contrast, Maxwell's equations together with the Lorentz force obscure this fact.

III. Practical application

A. Direct current and low-frequency alternating current

Weber-Maxwell electrodynamics is based on the premise that all electromagnetic effects arise from the presence and motion of point charges in an otherwise completely empty, absolute, Newtonian space. In some fields of research, such as atomic physics or plasma physics, this is an advantage, since working

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[1]Note that r depends on τ due to definition (2). To solve the equation, Newton's method can be applied.

[2]This can easily be shown by means of the Banach fixed-point theorem.

with fields, Lorentz force, and Lorentz transformation is highly complicated when point charges are involved. Conversely, in electrical engineering, one almost never works with point charges, but with electric currents and voltages. It is therefore necessary to show how the individual force between two point charges can be converted into a force between currents.

A naive approach to calculate the force of a current acting on a test charge would be to insert the trajectories of all electrons and metal ions of a current-carrying wire into the formula (1) and then sum up all the individual forces. This procedure is practical for a computer program as no approximations are required. However, this is not a suitable approach for theoretical estimations and analyses.

For theoretical purposes, it is often useful to first split a current path into very small segments of length dl. The quantity of all negative charge carriers in each segment is then $dl\,\lambda_-$, where λ_- is the number of negative charge carriers per meter conductor length. Accordingly, the total charge of all positive charge carriers in the segment is $dl\,\lambda_+$. The introduction of linear charge densities allows the use of integrals instead of sums. Another significant simplification results from the separation of a current into a direct current component and an alternating current component.

In the case of direct current, the two charge quantities move almost uniformly during the propagation time of the force, i.e., with the constant velocities $\boldsymbol{v}_-$ and $\boldsymbol{v}_+$. As shown in section IV-B, this approximation allows a simplification of the Weber-Maxwell force (1) to the Weber force

$$\boldsymbol{F}_W(q_d, q_s, \boldsymbol{r}, \boldsymbol{v}) = \frac{q_d\,q_s\,\boldsymbol{r}}{4\,\pi\,\varepsilon_0\,r^3}\left(1 + \frac{v^2}{c^2} - \frac{3}{2}\left(\frac{\boldsymbol{r}}{r}\cdot\frac{\boldsymbol{v}}{c}\right)^2\right), \quad (6)$$

provided that v is much lower than the speed of light c. $\boldsymbol{r}$ is in this case only the instantaneous distance vector $\boldsymbol{r} := \boldsymbol{r}_d(t) - \boldsymbol{r}_s(t)$ and $\boldsymbol{v}$ the constant difference velocity. This means that the formula (5) no longer needs to be taken into account for direct current.

The force $d\boldsymbol{F}_{DC}$ that a direct current element exerts on a test charge q_d moving at speed $\boldsymbol{v}$ can therefore be expressed by the equation

$$d\boldsymbol{F}_{DC}(\boldsymbol{r}) = \boldsymbol{F}_W(q_d, dl\,\lambda_-, \boldsymbol{r}, \boldsymbol{v} - \boldsymbol{v}_-) + \\ \boldsymbol{F}_W(q_d, dl\,\lambda_+, \boldsymbol{r}, \boldsymbol{v} - \boldsymbol{v}_+). \quad (7)$$

In metallic conductors, $\boldsymbol{v}_-$ is usually extremely small and $\boldsymbol{v}_+$ can even be zero. For this reason, it is possible to use the first-order Taylor series:

$$d\boldsymbol{F}_{DC}(\boldsymbol{r}) \approx \boldsymbol{F}_W(q_d, dl\,\lambda_-, \boldsymbol{r}, \boldsymbol{v}) - \frac{d\boldsymbol{F}_W(q_d, dl\,\lambda_-, \boldsymbol{r}, \boldsymbol{v})}{d\boldsymbol{v}}\cdot\boldsymbol{v}_- + \\ \boldsymbol{F}_W(q_d, dl\,\lambda_+, \boldsymbol{r}, \boldsymbol{v}) - \frac{d\boldsymbol{F}_W(q_d, dl\,\lambda_+, \boldsymbol{r}, \boldsymbol{v})}{d\boldsymbol{v}}\cdot\boldsymbol{v}_+. \quad (8)$$

With $-\lambda_- = \lambda_+$ this becomes

$$d\boldsymbol{F}_{DC}(\boldsymbol{r}) = -\frac{d\boldsymbol{F}_W(q_d, dl\,\lambda_+, \boldsymbol{r}, \boldsymbol{v})}{d\boldsymbol{v}}\cdot(\boldsymbol{v}_+ - \boldsymbol{v}_-). \quad (9)$$

The calculation of the vector gradient finally results in

$$d\boldsymbol{F}_{DC}(\boldsymbol{r}) = \frac{\mu_0\,q_d\,I_{DC}\,\boldsymbol{r}\left(3\,(\boldsymbol{r}\cdot\boldsymbol{v})\,\boldsymbol{r} - 2\,r^2\,\boldsymbol{v}\right)\cdot dl}{4\,\pi\,r^5}, \quad (10)$$

using the equations $\lambda_+\,(\boldsymbol{v}_+ - \boldsymbol{v}_-) = I_{DC}\,d\boldsymbol{l}/dl$ and $\varepsilon_0\,c^2 = 1/\mu_0$. Here, I_{DC} is the current strength of the direct current in the wire and $d\boldsymbol{l}$ is a vector that has the same length as the wire segment and points in the direction of the current flow.

The equation (10) is Ampère's force law in its original form [15, p. 29]. As Maxwell already demonstrated, this force law leads to the same result for closed current loops as the Biot-Savart law in combination with the Lorentz force [16, p. 162]. However, this does not apply to current loops that are not closed. The presence of wire stubs means that there is no direct current flow. However, alternating current can flow. If its frequency is so low that the wavelength of the emitted electromagnetic wave is considerably longer than the distance of the test charge q_d, there is usually no objection to calculating the force using equation (10), since the approximations used to derive the Weber force are also sufficiently valid here. In this case, the current strength $i(t)$ of the alternating current at current time t can be used for I_{DC}. However, the field oscillates instantaneously in the entire space. This means that electromagnetic waves are not represented correctly by this method.

In contrast to the force law (10), the application of the Biot-Savart law together with the Lorentz force is fundamentally wrong for stubs or point charges because, as Maxwell has shown, it is necessary to presume that the current loop is closed in order to derive the Biot-Savart law. In the general case, however, Maxwell's equations lead to the Weber-Maxwell force (1) and thus ultimately to Ampère's force law (10) and not to the Grassmann force [17]. For this reason, caution is advised.

B. High-frequency alternating current

The application of the Weber force (6) and Ampère's force law (10) is, as has been demonstrated, only possible if the distance between the two point charges q_s and q_d is so small that the distance variation of the point charges during the force propagation from q_s to q_d is almost constant. With alternating current, this condition is only fulfilled if the distance r is much smaller than the wavelength of the electromagnetic wave.

If this condition is not satisfied, another approximation can often be used. The approach for simplification is to exploit the fact that with alternating current, the force-generating charges are at rest in the temporal average. The trajectory of a sinusoidally oscillating point charge q_s, which is stationary in the time average, is

$$\boldsymbol{r}_s(t) = \overline{\boldsymbol{r}}_s + \boldsymbol{s}(t), \quad (11)$$

with

$$\boldsymbol{s}(t) := \hat{s}\,\boldsymbol{e}_s\,\sin(\omega\,t). \quad (12)$$

$\overline{\boldsymbol{r}}_s$ is the time average of the trajectory, $\hat{s}$ the maximum displacement (peak amplitude), $\boldsymbol{e}_s$ the unit vector of the direction of oscillation, and ω the angular frequency.

The special case (11) yields for the definitions (2), (3) and (4):

$$\boldsymbol{r} = \boldsymbol{r}_d(\tau) - (\overline{\boldsymbol{r}}_s + \boldsymbol{s}(\tau)) \quad (13)$$

$$\boldsymbol{v} = \dot{\boldsymbol{r}}_d(\tau) - \dot{\boldsymbol{s}}(\tau) \tag{14}$$

$$\boldsymbol{a} = \ddot{\boldsymbol{r}}_d(\tau) - \ddot{\boldsymbol{s}}(\tau). \tag{15}$$

For a high-frequency alternating current, the peak amplitude $\hat{s}$ is usually very small compared to the distance between q_s and q_d. For this reason, we substitute the equations (13), (14), and (15) into the Weber-Maxwell force (1) and expand the expression into a first-order Taylor series. In this way we obtain

$$\boldsymbol{F} \approx \boldsymbol{F}|_{\hat{s}=0} + \hat{s} \left(\frac{\mathrm{d}}{\mathrm{d}\hat{s}} \boldsymbol{F} \right)\Bigg|_{\hat{s}=0}. \tag{16}$$

For test charges at rest q_d, $\dot{\boldsymbol{r}}_d(\tau)$ and $\ddot{\boldsymbol{r}}_d(\tau)$ are zero and we obtain with $\overline{\boldsymbol{r}} = \boldsymbol{r}_d(\tau) - \overline{\boldsymbol{r}}_s$ the equation

$$\boldsymbol{F}(\overline{\boldsymbol{r}}) \approx \frac{q_d q_s \overline{\boldsymbol{r}}}{4 \pi \varepsilon_0 \overline{r}^3} - \\ \frac{q_d q_s \left(\overline{r}^2 \boldsymbol{s}(\tau) - 3 \overline{\boldsymbol{r}} (\overline{\boldsymbol{r}} \cdot \boldsymbol{s}(\tau)) \right)}{4 \pi \varepsilon_0 \overline{r}^5} - \\ \frac{q_d q_s \left(\overline{r}^2 \dot{\boldsymbol{s}}(\tau) - 3 \overline{\boldsymbol{r}} (\overline{\boldsymbol{r}} \cdot \dot{\boldsymbol{s}}(\tau)) \right)}{4 \pi c \varepsilon_0 \overline{r}^4} - \\ \frac{q_d q_s \left(\overline{r}^2 \ddot{\boldsymbol{s}}(\tau) - \overline{\boldsymbol{r}} (\overline{\boldsymbol{r}} \cdot \ddot{\boldsymbol{s}}(\tau)) \right)}{4 \pi c^2 \varepsilon_0 \overline{r}^3}. \tag{17}$$

It is evident that only the last term in equation (17) is important for high-frequency alternating currents. The term in the first line is compensated in a metallic wire by metal ions at rest. The two lines in the middle are only relevant if the measurement is performed in the immediate vicinity of the conductor. For greater distances $\overline{r}$, only the last term remains. We can therefore provide a rather simple equation for the force $\mathrm{d}\boldsymbol{F}_{AC}$ that an alternating current element exerts on a stationary test charge q_d at a distance $\boldsymbol{r}$:

$$\mathrm{d}\boldsymbol{F}_{AC}(\boldsymbol{r}) \approx -\frac{\mu_0 q_d \, \mathrm{d}l \, \lambda_- \left(r^2 \ddot{\boldsymbol{s}}(\tau) - \boldsymbol{r} (\boldsymbol{r} \cdot \ddot{\boldsymbol{s}}(\tau)) \right)}{4 \pi r^3}. \tag{18}$$

This can be further transformed to

$$\mathrm{d}\boldsymbol{F}_{AC}(\boldsymbol{r}) = \frac{\mu_0 q_d \, \mathrm{d}l \, \lambda_-}{4 \pi r} \left(\frac{\boldsymbol{r}}{r} \times \left(\frac{\boldsymbol{r}}{r} \times \ddot{\boldsymbol{s}}\left(t - \frac{r}{c}\right) \right) \right), \tag{19}$$

exploiting that for this special case $\tau = t - r/c$.

The current strength I_{AC} of an alternating current is defined as the root mean square (RMS) of the instantaneous value $i(t)$ during a cycle, i.e., the direct current strength that would deliver the same energy in the same time. The motion according to equation (12) corresponds to the current

$$i(t) = \lambda_- \hat{s} \omega \cos(\omega t). \tag{20}$$

Therefore, the RMS is[3]

$$I_{AC} = -\frac{1}{\sqrt{2}} \lambda_- \hat{s} \omega. \tag{21}$$

[3]The negative sign is necessary because λ_- represents a negative charge quantity.

If this and the equation (12) are inserted into the equation (19), the force that a wire segment with alternating current exerts on a test charge q_d at a distance $\boldsymbol{r}$ is because of $\mathrm{d}\boldsymbol{l} = \boldsymbol{e}_s \, \mathrm{d}l$:

$$\mathrm{d}\boldsymbol{F}_{AC}(\boldsymbol{r}) = \frac{\mu_0 q_d I_{AC} \omega \, (\boldsymbol{r} \times (\boldsymbol{r} \times \mathrm{d}\boldsymbol{l})) \sin\left(\omega \left(t - \frac{r}{c} \right) \right)}{\sqrt{8} \pi r^3}. \tag{22}$$

Equation (22) is the analogous equation to Ampère's force law (10) for alternating current. However, there are two major restrictions: The first is that the test charge needs to be located at an appropriate distance from the current element, as the equation (22) only represents the far field. Secondly, the test charge q_d must not move too rapidly. The calculations shown here can also be carried out without these restrictions, but the resulting formulas are then considerably longer.

The practical value of the formula (22) is that it provides a relatively simple means by which to estimate the field of the electromagnetic force generated by an alternating current. A reader trained in classical electrodynamics will recognize that the equation (22) corresponds to the far field of a Hertzian dipole.

C. Simulation

By means of the equations (10) and (22), it is possible to obtain an idea of the shape of a electromagnetic field of a certain configuration quickly. However, if one needs accurate results, it is usually necessary to calculate with the complete Weber-Maxwell force (1). For simple tasks (for example, for individual point charges with non-trivial trajectories), such analyses can be carried out quickly and easily with *Mathematica*. If one plots the resulting fields, one recognizes that the Weber-Maxwell force not only represents simple waves, it also contains effects such as the Doppler effect, which is only included in standard electrodynamics if the Lorentz transformation is applied, which makes the calculation considerably more complicated and laborious.

In more complex tasks, large numbers of point charges interact with each other. This means that both the force generated by a point charge and how this force affects every other point charge in the environment need to be calculated. Furthermore, in complex tasks, point charges are usually subject to constraint forces, which must also be taken into account. Complex tasks can therefore only be analyzed satisfactorily with specialized software. As Weber-Maxwell electrodynamics is still very new, there is currently only a single software framework that works with the Weber-Maxwell force. The source code of the framework referred to as *OpenWME* is freely available at *Github*. Compared to other EM solvers, *OpenWME* is extremely fast and powerful. Its speed advantage results from the fact that there is no need to solve Maxwell's equations numerically.

OpenWME contains a number of small applets that demonstrate possible use cases. Figure 1 shows a selection whereby the partial figures represent the corresponding fields at representative times. These were taken from the videos in *OpenWME*. Details, parameters, and further explanatory information can be found in the *OpenWME* documentation.
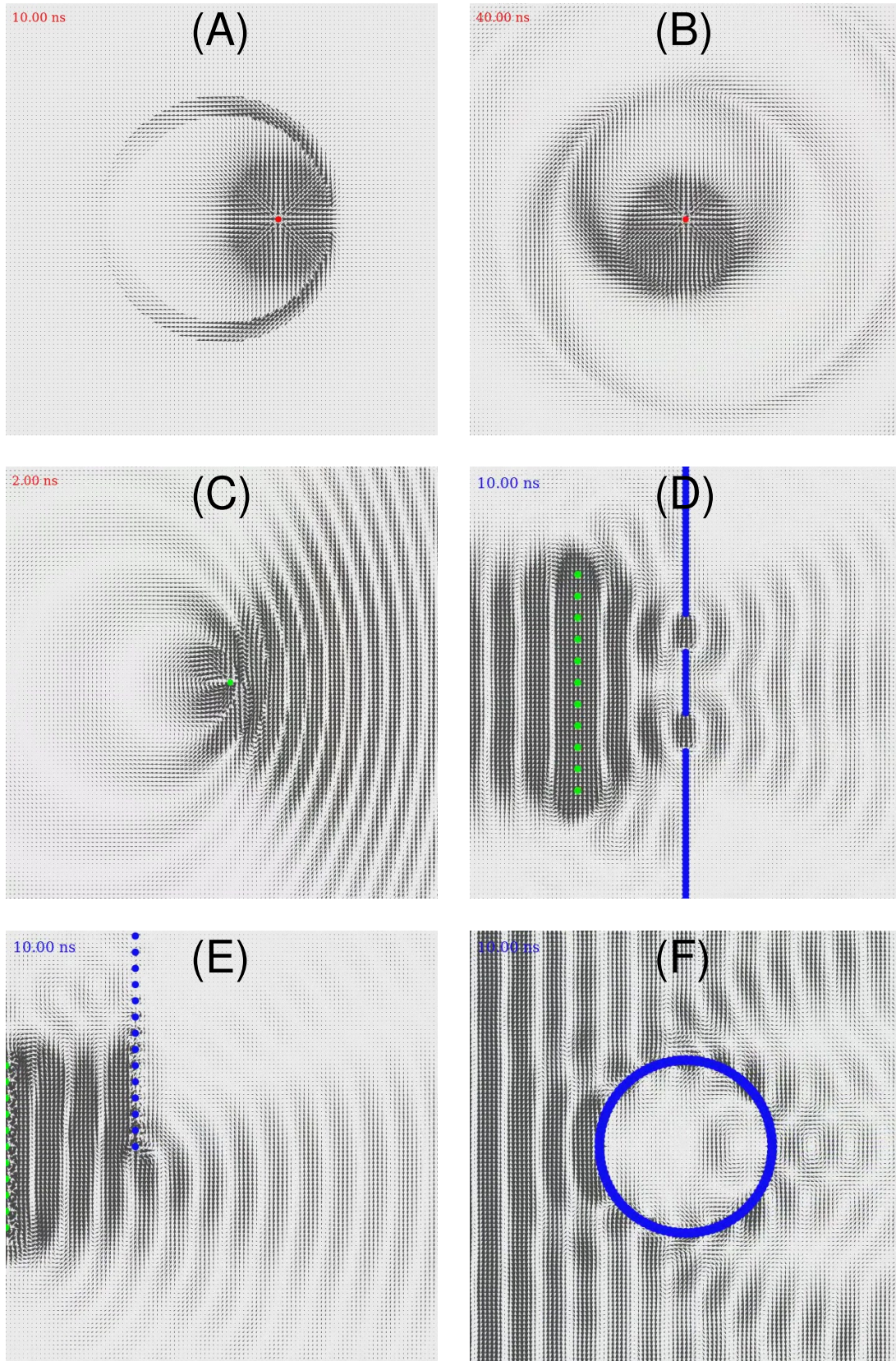
Fig. 1. Calculated fields with OpenWME: (A) Field of an initially stationary point charge that was suddenly accelerated for an instant (bremsstrahlung). (B) Field of a point charge moving on a path corresponding to a lying number eight. (C) Field of a fast-moving Hertzian dipole. (D) Interference at a double slit. (E) Diffraction at a half-plane. (F) An electromagnetic wave coming from the left is deflected around a reflecting surface.

The applets demonstrate that there are hardly any limits regarding the complexity of the tasks. This is somewhat surprising, as Weber-Maxwell electrodynamics does not require differential equations. *OpenWME* thus clearly shows that classical electrodynamics is basically very simple and that its complexity only arises through the interactions of a large number of point charges. Realizing this is particularly important for pupils and students, as they are often not yet able to recognize the simplicity hidden behind complexity.

With the help of Weber-Maxwell electrodynamics, pupils and students can be better enabled to comprehend effects such as diffraction, interference, Lorentz force, and induction in detail and to develop an intuitive understanding of the fundamentals of physics. This is particularly useful for engineering students, as the creative use of the laws of nature requires an intuitive and profound understanding.

IV. PROOFS AND DEDUCTIONS

A. Deduction of the Weber-Maxwell force from Maxwell's equations

Maxwell's electrodynamics is presently still the standard. It has this status because it has performed very well in practice for more than a hundred years. It is therefore necessary to show how Maxwell's electrodynamics and Weber-Maxwell electrodynamics are related. In the following section, the Weber-Maxwell force is consequently deduced from Maxwell's equations.

Maxwell's electrodynamics in a vacuum consists of four differential equations [18] [19]

$$\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}, \tag{23}$$

$$\nabla \cdot \boldsymbol{B} = 0, \tag{24}$$

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}, \tag{25}$$

$$\nabla \times \boldsymbol{B} = \mu_0 \, \boldsymbol{j} + \frac{1}{c^2} \frac{\partial \boldsymbol{E}}{\partial t} \tag{26}$$

and the supplemental formula

$$\boldsymbol{F}_M = q_d \, \boldsymbol{E} + q_d \, \boldsymbol{v} \times \boldsymbol{B}. \tag{27}$$

The formula (27) is known as the Lorentz force. It provides the force that is exerted by an electric field $\boldsymbol{E}$ and a magnetic field $\boldsymbol{B}$ on a point-shaped test charge q_d that is located at $\boldsymbol{r}$ at time t. The fields $\boldsymbol{E}$ and $\boldsymbol{B}$ are in turn auxiliary quantities that can be calculated by inserting the charge density ρ and current density $\boldsymbol{j}$ following from the problem description into the system of differential equations (23) to (26).

$\boldsymbol{v}$ represents a velocity. Usually, $\boldsymbol{v}$ is the speed of the test charge q_d in the laboratory frame of reference, i.e., the frame in which the field-generating device is located. Since electrical currents in metallic conductors consist of a large number of charge carriers with various speeds that only move extremely slowly, $\boldsymbol{v}$ is in electrical engineering practically always identical to the difference velocity between the test charge q_d and the field-generating wires and metallic conductors. This means that a test charge that is at rest in relation to the field-generating device is not affected by the magnetic field $\boldsymbol{B}$. In this case, the Lorentz force (27) can be simplified to

$$\boldsymbol{F}_M = q_d \, \boldsymbol{E}. \tag{28}$$

In the following, we assume that not only the test charge q_d, but also the field-generating charge q_s is a point charge. The charge density of this point charge is

$$\rho = q_s \, \delta(\boldsymbol{r} - \boldsymbol{r}_s(t)), \tag{29}$$

where $\boldsymbol{r}_s(t)$ is the location of the point charge q_s at time t. For the current density,

$$\boldsymbol{j} = \dot{\boldsymbol{r}}_s(t) \, \rho \tag{30}$$

applies.

Maxwell's equations can be transferred into a wave equation. To obtain this equation, we first differentiate Maxwell's fourth equation (26) with respect to time t. This results in

$$\nabla \times \frac{\partial \boldsymbol{B}}{\partial t} = \mu_0 \, \frac{\partial \boldsymbol{j}}{\partial t} + \frac{1}{c^2} \, \frac{\partial^2 \boldsymbol{E}}{\partial t^2}. \tag{31}$$

Then, we insert the third Maxwell equation (25) and obtain

$$\frac{1}{c^2} \, \frac{\partial^2 \boldsymbol{E}}{\partial t^2} = -\mu_0 \, \frac{\partial \boldsymbol{j}}{\partial t} - \nabla \times (\nabla \times \boldsymbol{E}). \tag{32}$$

Based on the identity $\nabla \times (\nabla \times \boldsymbol{E}) = \nabla (\nabla \cdot \boldsymbol{E}) - \nabla^2 \boldsymbol{E}$ and using the first Maxwell equation (23), the equation (28), and $\mu_0 = 1/(\varepsilon_0 \, c^2)$, we get the wave equation

$$\left(\frac{1}{c^2} \, \frac{\partial^2}{\partial t^2} - \nabla^2 \right) \boldsymbol{F}_M = -\frac{q_d}{\varepsilon_0} \left(\frac{1}{c^2} \, \frac{\partial \boldsymbol{j}}{\partial t} + \nabla \rho \right). \tag{33}$$

The wave equation (33) is solvable. The approach is described in Section IV in [20], but it can also be found in certain textbooks on theoretical electrical engineering or physics, e.g. in [18] or [19]. The solution reads

$$\boldsymbol{F}_M = -q_d \left(\frac{\partial}{\partial t} \boldsymbol{A} + \nabla \, \Phi \right), \tag{34}$$

with

$$\Phi = \frac{q_s \, c}{4 \, \pi \, \varepsilon_0 \, (c^2 \, (t - \tau) - \dot{\boldsymbol{r}}_s(\tau) \cdot (\boldsymbol{r} - \boldsymbol{r}_s(\tau)))}, \tag{35}$$

$$\boldsymbol{A} = \frac{1}{c^2} \, \dot{\boldsymbol{r}}_s(\tau) \, \Phi \tag{36}$$

and

$$\tau = t - \frac{1}{c} \, \|\boldsymbol{r} - \boldsymbol{r}_s(\tau)\|. \tag{37}$$

The potentials Φ and $\boldsymbol{A}$ are known as Liénard-Wiechert potentials in the center-of-momentum frame of the test charge q_d. In the current scientific literature, the Liénard-Wiechert potentials are always the final result. The fact that it is possible to simplify them further by means of a mathematical trick was not known [21]. This method is explained below, and it is used to derive the Weber-Maxwell force from the Liénard-Wiechert potentials.

First, we calculate the derivatives of the potentials Φ and $\boldsymbol{A}$ and obtain the equations

$$\nabla \Phi = \frac{q_s \, c \, h_2(\tau) \, \nabla \tau + q_s \, c \, \dot{\boldsymbol{r}}_s(\tau)}{4 \pi \, \varepsilon_0 \, h_1(\tau)^2} \tag{38}$$

and

$$\frac{\partial}{\partial t} \boldsymbol{A} = \frac{q_s \, [h_2(\tau) \, \dot{\boldsymbol{r}}_s(\tau) - h_1(\tau) \, \ddot{\boldsymbol{r}}_s(\tau)] \, \frac{\partial \tau}{\partial t} - q_s \, c^2 \, \dot{\boldsymbol{r}}_s(\tau)}{4 \pi \, \varepsilon_0 \, c \, h_1(\tau)^2} \tag{39}$$

using the auxiliary quantities

$$h_1(\tau) := (\boldsymbol{r} - \boldsymbol{r}_s(\tau)) \cdot \dot{\boldsymbol{r}}_s(\tau) - c^2 \, (t - \tau) \tag{40}$$

and

$$h_2(\tau) := c^2 - \dot{\boldsymbol{r}}_s(\tau) \cdot \dot{\boldsymbol{r}}_s(\tau) + (\boldsymbol{r} - \boldsymbol{r}_s(\tau)) \cdot \ddot{\boldsymbol{r}}_s(\tau). \tag{41}$$

Now, we insert the equations (38) and (39) into the equation (34) and obtain

$$\boldsymbol{F}_M = -\frac{q_d \, q_s}{4 \pi \, \varepsilon_0} \frac{c^2 \, \frac{h_2(\tau)}{h_1(\tau)} \, \nabla \tau + \left[\frac{h_2(\tau)}{h_1(\tau)} \, \dot{\boldsymbol{r}}_s(\tau) - \ddot{\boldsymbol{r}}_s(\tau) \right] \, \frac{\partial \tau}{\partial t}}{c \, h_1(\tau)}. \tag{42}$$

The remaining derivatives in equation (42) can be derived by applying the differential operators to both sides of the equation (37). This gives us

$$\nabla \tau = \frac{\boldsymbol{r} - \boldsymbol{r}_s(\tau) + (\boldsymbol{r} - \boldsymbol{r}_s(\tau)) \cdot \dot{\boldsymbol{r}}_s(\tau) \, \nabla \tau}{c \, \|\boldsymbol{r} - \boldsymbol{r}_s(\tau)\|} \tag{43}$$

and

$$\frac{\partial \tau}{\partial t} = 1 + \frac{(\boldsymbol{r} - \boldsymbol{r}_s(\tau)) \cdot \dot{\boldsymbol{r}}_s(\tau)}{c \, \|\boldsymbol{r} - \boldsymbol{r}_s(\tau)\|} \frac{\partial \tau}{\partial t}. \tag{44}$$

By solving the equations using equation $\|\boldsymbol{r} - \boldsymbol{r}_s(\tau)\| = c \, (t - \tau)$, and the definition (40), we obtain the surprisingly simple equations

$$\nabla \tau = \frac{\boldsymbol{r} - \boldsymbol{r}_s(\tau)}{h_1(\tau)} \tag{45}$$

and

$$\frac{\partial \tau}{\partial t} = -\frac{c^2 \, (t - \tau)}{h_1(\tau)}. \tag{46}$$

These can now be used in equation (42).

Finally, let us assume that the center-of-momentum frame of the test charge q_d is moving along the trajectory $\boldsymbol{r}_d(t)$ and that the velocity difference between q_s and q_d is much lower than the speed of light in a vacuum c. In this case, the use of non-relativistic dynamics and the Galilean transformation are allowed. We now use this to generalize the force to a moving test charge q_d.

Since $\boldsymbol{r}$ disappears in the center-of-momentum frame of the test charge, we first perform the substitution

$$\boldsymbol{r} \to \boldsymbol{0}. \tag{47}$$

Only the source charge q_s appears to be moving in the center-of-momentum frame of the test charge, namely with the trajectory $\boldsymbol{r}_s(t) - \boldsymbol{r}_d(t)$. For this reason, we now carry out the substitutions

$$\boldsymbol{r}_s(\tau) \to -\boldsymbol{r} := \boldsymbol{r}_s(\tau) - \boldsymbol{r}_d(\tau), \tag{48}$$

$$\dot{\boldsymbol{r}}_s(\tau) \to -\boldsymbol{v} := \dot{\boldsymbol{r}}_s(\tau) - \dot{\boldsymbol{r}}_d(\tau) \tag{49}$$

and

$$\ddot{\boldsymbol{r}}_s(\tau) \to -\boldsymbol{a} := \ddot{\boldsymbol{r}}_s(\tau) - \ddot{\boldsymbol{r}}_d(\tau). \tag{50}$$

This and $c \, (t - \tau) = r$ finally give us

$$\boldsymbol{F}_M = \frac{q_d \, q_s}{4 \pi \, \varepsilon_0} \left(\frac{(\boldsymbol{r} \, c + r \, \boldsymbol{v}) \, (c^2 - v^2 - \boldsymbol{r} \cdot \boldsymbol{a})}{(r \, c + \boldsymbol{r} \cdot \boldsymbol{v})^3} + \frac{\boldsymbol{a} \, r}{(r \, c + \boldsymbol{r} \cdot \boldsymbol{v})^2} \right). \tag{51}$$

A comparison of the force $\boldsymbol{F}_M$ with the Weber-Maxwell force (1) reveals that the equation

$$\boldsymbol{F} = \gamma(v) \, \boldsymbol{F}_M \tag{52}$$

applies. The force calculated using Maxwell's equations is therefore identical to the Weber-Maxwell force except for a scalar renormalization factor $\gamma(v)$ that only depends on the difference velocity v.

The introduction of the Lorentz factor appears to be arbitrary. However, it is a reasonable measure, as this ad-hoc assumption eliminates a number of equations. In addition, the Lorentz factor hardly plays any role in the case of electromagnetic waves, since transmitters and receivers often move very slowly in relation to each other. Ultimately, the purpose of the Lorentz factor is only to achieve compatibility with Weber electrodynamics. This in turn integrates the very important findings of Carl Friedrich Gauss and Wilhelm Weber, i.e., that magnetism is only a multi-particle effect. In other words, the added Lorentz factor renders the Lorentz force (27) and the magnetic field superfluous. The following section IV-B will discuss this even further.

B. Deduction of the Weber force from the Weber-Maxwell force

The Weber-Maxwell force (1) is very general, which sometimes makes calculations unnecessarily complicated. In the case of direct current, a major simplification is possible because a force-generating point charge q_s usually moves so slowly that the difference trajectory is almost a straight line during the time period between the emission of the force at time τ and the arrival at the receiver of charge q_d at time t.

This means that the distance vector $\boldsymbol{s} := \boldsymbol{r}_d(t) - \boldsymbol{r}_s(t)$ between q_d and q_s at time t can be expressed by equation

$$\boldsymbol{s} = \boldsymbol{r} + \boldsymbol{v} \, (t - \tau). \tag{53}$$

Furthermore, the acceleration $\boldsymbol{a}$ disappears and the velocity $\boldsymbol{v}$ is a constant. The Weber-Maxwell force can therefore be simplified to

$$\boldsymbol{F} = \frac{q_d \, q_s \, \gamma(v) \, (\boldsymbol{r} \, c + r \, \boldsymbol{v}) \, (c^2 - v^2)}{4 \pi \, \varepsilon_0 \, (r \, c + \boldsymbol{r} \cdot \boldsymbol{v})^3}. \tag{54}$$

Because $c^2 - v^2 = c^2 / \gamma(v)^2$, this can be further rearranged and we obtain

$$\boldsymbol{F} = \frac{q_d \, q_s \, \left(\frac{\boldsymbol{r}}{r} + \frac{\boldsymbol{v}}{c} \right)}{4 \pi \, \varepsilon_0 \, \gamma(v) \, r^2 \, \left(1 + \frac{\boldsymbol{r}}{r} \cdot \frac{\boldsymbol{v}}{c} \right)^3}. \tag{55}$$

Because of equation (5), $t - \tau = r/c$. If we insert this into the equation (53), we get

$$\frac{s}{r} = \frac{r}{r} + \frac{v}{c}. \tag{56}$$

Equation (55) thus becomes

$$F = \frac{q_d\,q_s\,s}{4\,\pi\,\varepsilon_0\,\gamma(v)\,r^3\left(1 + \frac{r}{r}\cdot\frac{v}{c}\right)^3}. \tag{57}$$

In addition, equation (56) becomes

$$\frac{r}{r}\cdot\frac{s}{r} = 1 + \frac{r}{r}\cdot\frac{v}{c} \tag{58}$$

and equation (57) can be transformed into

$$F = \frac{q_d\,q_s\,s}{4\,\pi\,\varepsilon_0\,\gamma(v)\left(\frac{r}{r}\cdot s\right)^3}. \tag{59}$$

Now, the only disturbing term is $r/r \cdot s$. For this term, the equation

$$\frac{r}{r}\cdot s = \frac{s - v\,(t-\tau)}{c\,(t-\tau)}\cdot s = \frac{s^2}{c\,(t-\tau)} - \frac{s\cdot v}{c} \tag{60}$$

applies due to equation (53) and $t - \tau = r/c$. From equation (5) follows

$$t - \tau = \frac{r}{c} = \frac{\|s - v\,(t-\tau)\|}{c}, \tag{61}$$

i.e.,

$$t - \tau = \frac{s^2}{s\cdot v + \sqrt{(s\cdot v)^2 + s^2\,(c^2 - v^2)}}. \tag{62}$$

This can be inserted into equation (60) and we get

$$\begin{aligned}
\frac{r}{r}\cdot s &= \frac{1}{c}\,\sqrt{(s\cdot v)^2 + s^2\,(c^2 - v^2)} \\
&= \sqrt{s^2 - \frac{1}{c^2}\|s\times v\|^2}.
\end{aligned} \tag{63}$$

This allows equation (59) to be transformed into

$$F = \frac{q_d\,q_s\,s}{4\,\pi\,\varepsilon_0\,\gamma(v)\left(s^2 - \frac{1}{c^2}\|s\times v\|^2\right)^{3/2}}. \tag{64}$$

This equation now only depends on the current distance vector s between q_d and q_s at time t. Quantities that depend on the past time τ are no longer included, which significantly simplifies the practical use of the equation in calculations.

Equation (64) is not only a special solution of Maxwell's equations but also an alternative representation of the classical Weber force in Weber electrodynamics. To show this, we perform the substitution $v \to u\,v$ and expand the term into a Taylor series. This yields

$$F = \frac{q_d\,q_s\,s}{4\,\pi\,\varepsilon_0\,s^3}\left(1 - \frac{s^2\,v^2 - 3\,\|s\times v\|^2}{2\,s^2\,c^2}\,u^2 + O(u)^3\right). \tag{65}$$

Now, we can set $u = 1$, which reverses the substitution $v \to u\,v$. By means of formula

$$\|s\times v\|^2 = s^2\,v^2 - (s\cdot v)^2, \tag{66}$$

we ultimately obtain the approximation

$$F = \frac{q_d\,q_s\,s}{4\,\pi\,\varepsilon_0\,s^3}\left(1 + \frac{v^2}{c^2} - \frac{3}{2}\left(\frac{s}{s}\cdot\frac{v}{c}\right)^2\right), \tag{67}$$

which corresponds to the Weber force without acceleration terms. This demonstrates that the essence of Weber electrodynamics is included in Weber-Maxwell electrodynamics.

However, it also becomes evident that the acceleration terms in classical Weber electrodynamics are artifacts and that Weber electrodynamics is only valid under certain conditions, namely when

- the transit time of the force from q_s to q_d is so small that both point charges move almost uniformly within this time interval; and
- the differential velocity between the two point charges is very small compared to the speed of light.

In electrical engineering, this is the case with direct currents and low-frequency alternating currents. A low frequency means that the distance between the current and the measuring location is much smaller than the wavelength. If this condition is not met, Weber electrodynamics is unsuitable and Weber-Maxwell electrodynamics is required. The fact that classical Weber electrodynamics is incorrect for accelerated point charges has also been demonstrated experimentally [22], [23].

C. Proof of the conservation of momentum

The question of whether the conservation of momentum is satisfied or violated in Maxwell's electrodynamics is not easy to answer. In Weber-Maxwell electrodynamics, however, providing a proof for conservation of momentum is very straightforward. First, we can verify that the force (1) exerted by a point charge q_s on another point charge q_d is equal in magnitude to the force exerted by the point charge q_d on the point charge q_s. The direction, on the other hand, is exactly inverse. Ultimately, the equation

$$F_{ik} = -F_{ki} \tag{68}$$

applies to any pair q_i and q_k of point charges, which is known as Newton's 3rd law. From this, it follows immediately that, in an isolated system consisting of n point charges, the total momentum p of all point charges is invariant.

The proof is simple. The temporal change of the total momentum p of the system is the sum of the temporal momentum changes of all point particles:

$$\dot{p} = \sum_{k=1}^{n} \dot{p}_k. \tag{69}$$

The temporal momentum change $\dot{p}_k$ of the kth point charge is equal to the sum of all forces acting on this point particle, i.e.,

$$\dot{p}_k = \sum_{i=1}^{n} F_{ki}, \tag{70}$$

provided we define that $F_{ii} := 0$. This gives

$$\dot{p} = \sum_{k=1}^{n}\sum_{i=1}^{n} F_{ki}. \tag{71}$$

Because of equation (68),

$$\dot{\boldsymbol{p}} = \sum_{k=1}^{n} \sum_{i=1}^{n} \boldsymbol{F}_{ki} = -\sum_{k=1}^{n} \sum_{i=1}^{n} \boldsymbol{F}_{ik} \qquad (72)$$

follows. Formal renaming of the summation indices leads to

$$\dot{\boldsymbol{p}} = \sum_{k=1}^{n} \sum_{i=1}^{n} \boldsymbol{F}_{ki} = -\sum_{i=1}^{n} \sum_{k=1}^{n} \boldsymbol{F}_{ki} = -\sum_{k=1}^{n} \sum_{i=1}^{n} \boldsymbol{F}_{ki}, \qquad (73)$$

which can only be true if $\dot{\boldsymbol{p}}$ is exactly zero. This shows that the total momentum $\boldsymbol{p}$ must be a conserved quantity.

Furthermore, it demonstrates that the sum of all forces acting on the point charges disappears in an isolated system. Consequently, electromagnetic waves themselves cannot have any momentum. Instead, momentum is a property that only matter can possess. The electromagnetic force is just the mediator. However, if we only consider a subsystem consisting of an electromagnetic wave and the receiver of the force, it appears as if the electromagnetic wave has momentum, since it ultimately causes a momentum change at the receiver. However, we must not forget that somewhere else the transmitter of the wave is experiencing a compensating momentum change at the same time. Electromagnetic waves therefore only have momentum if we consider non-isolated systems.

V. Unresolved issues

As has become clear up to this point, Weber-Maxwell electrodynamics is a fully-fledged theory of electromagnetism in the non-relativistic regime. However, it explicitly does not apply to very high difference velocities. The question of how Weber-Maxwell electrodynamics can be generalized to this domain is still completely open. It may be sufficient to use relativistic dynamics without the Lorentz transformation. Conversely, it might also be necessary to integrate the special theory of relativity into Weber-Maxwell electrodynamics in a way that seems suitable.

However, the relativistic range is not of great interest for electrical engineering as it does not work with such high velocities. Moreover, Weber-Maxwell electrodynamics correctly reproduces the relativistic effects in the non-relativistic regime, even without the Lorentz transformation. However, if Weber-Maxwell electrodynamics is to be applied to atomic physics, a more in-depth study of this subject cannot be avoided.

In the context of atomic physics, whether existing theories that use the concept of the magnetic field contain systematic errors must also be clarified. Magnetic fields are particularly problematic for point charges because, as has been shown, they can only be meaningfully defined for closed current loops. Furthermore, it has long been known that Maxwell's equations are overdetermined [18]. In fact, neither the second Maxwell equation nor the cross product term of the Lorentz force is required to derive the Weber-Maxwell force in section IV-A from the Maxwell equations.

Another open point concerns the conservation of energy. Currently, the conservation of energy for Weber-Maxwell electrodynamics can only be proven if the point charges are moving sufficiently linearly. It is possible to specify a potential energy formula for the force (64), which in turn can then be used in the proof of the conservation of energy [24]. The conservation of angular momentum can currently also only be shown for the force (64).

However, these questions are more of theoretical interest. Of greater practical importance is the further development of the usability of Weber-Maxwell electrodynamics for computer simulations. To this end, it would be useful if the Weber-Maxwell force could be generalized to surface and volume elements. At present, complex objects still have to be modeled with point charges and Hertzian dipoles. This seems unnecessarily time-consuming, as it can be assumed that force formulas can also be found for surface and volume elements. Their use would then both improve the quality of simulations and reduce their resource requirements and necessary computing time.

Another open question is how Weber-Maxwell electrodynamics can be used in plasma physics. It is apparent that the Weber-Maxwell force should be particularly suitable for plasma physics as it avoids the cumbersome and problematic use of magnetic fields. It is likely that Weber-Maxwell electrodynamics might quickly lead to new results and successes in fusion research.

Furthermore, the question arises as to why the Weber-Maxwell force has the form that it has. The mere appearance of the Weber-Maxwell force gives rise to the suspicion that the electromagnetic forces could be transmitted by force carriers, which – on the one hand – are emitted like projectiles at random speeds by the force-generating point charges and – on the other hand – are only accepted by the force-absorbing point charges if they move slower than the speed of light in their center-of-momentum frames. This is certainly only a hypothesis for now, and one that still needs to be examined with suitable experiments. However, the Weber-Maxwell force shows that the Lorentz transformation is not necessarily the only approach to explaining Einstein's postulates and relativistic effects.

A further remark is addressed to the community of scientists working on gravitational research: Weber-Maxwell electrodynamics demonstrates that it is possible to describe complicated relativistic wave phenomena without differential equations. Something similar could also be possible in the field of gravitation.

And finally, it is necessary to develop new textbooks that make Weber-Maxwell electrodynamics available to students of electrical engineering and physics. The high degree of clarity and simplicity of Weber-Maxwell electrodynamics suggests that it might even be included in high-school physics lessons. This would also require the adaptation and extension of existing textbooks.

VI. Conclusions

This article has shown that it is relatively straightforward to combine Maxwell's classical electrodynamics with Weber's

interpretation of magnetism. It is interesting to note that this synthesis results in a theory that works completely without a magnetic field, avoids the Lorentz transformation, and requires only the two basic equations (1) and (5).

It is also interesting that Weber-Maxwell electrodynamics does not use differential equations. This represents a clear advantage. On the one hand, it considerably increases the speed of computer simulations, as it is no longer necessary to rely on numerical solution methods for partial differential equations. On the other hand, it makes classical electrodynamics easier to learn, which is a decisive advantage, not only for pupils and students, but also for practicing electrical engineers.

Although some questions remain unanswered, it can be concluded that Weber-Maxwell electrodynamics is equivalent to Maxwell's electrodynamics, but clearly surpasses it in terms of practical usability. Its performance is not only supported by the theoretical analyses in this article, but also demonstrated by means of the new software framework *OpenWME*. This not only shows that the electrostatic and magnetostatic aspects are reproduced correctly, but also that all electrodynamic effects, such as diffraction, interference, scattering, and bremsstrahlung, can be reproduced correctly. The latter results from the fact that the Weber-Maxwell force (1) is basically just an alternative – but more usable – representation of the Liénard-Wiechert potentials.

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