AAPT PHYSICS EDUCATION | AMERICAN JOURNAL of PHYSICS

Teaching of transcendence in physics

Stanley L. Jaki

Citation: American Journal of Physics **55**, 884 (1987); doi: 10.1119/1.14946
View online: http://dx.doi.org/10.1119/1.14946
View Table of Contents: http://scitation.aip.org/content/aapt/journal/ajp/55/10?ver=pdfcov
Published by the American Association of Physics Teachers

[36]L. S. Feldt, in *Encyclopedia of Statistical Sciences* (Wiley, New York, 1982), Vol. 4, p. 417.

[37]D. Hestenes (private communication).

[38]In a recent study (Ref. 40), 323 students took a standard full semester P201 course with the present innovations, except that only a 53-student test-group fraction took the SDI labs (31% of lab time, 10% of total instructional time) while the remaining 270-student control group took only standard labs. Pre- and post-course H^2-mechanics exam scores were test group (42.1%, 77.3%); control group (43.5%, 72.8%). A statistical t-test analysis indicates a probability $p = 0.105$ that the difference (77.3 − 72.8) is merely the result of chance.

[39]S. Tobias, Phys.Today **38**, 61 (June 1985); Change **18**, 36 (March/April, 1986); in Ref. 34, p. 11; Phys. Teach. (in press).

[40]S. Tobias and R. R. Hake, "Professors as physics students: What can they teach us?" (preprint, 1987).

[41]In addition to informal verbal comments to instructors, we obtained written responses and comments at the start of the fourth week of the course using a student evaluation method described by R. R. Hake and J. C. Swihart "Diagnostic Student Computerized Evaluation of Multi-component Courses (DISCOE)," Teaching and Learning, Indiana Univ., Jan. 1979 (available on request).

[42]A. Bork, Phys. Today **34**, 24 (Sept. 1981); A. B. Arons, Science **224**, 1051 (1984).

[43]This attitude generates its own negation. By refusing to engage in or support "remedial" instruction to promote cognitive development of undergraduates, who form the pool of precollege teachers, university faculty apply a "degenerative signal into" (an educational) "feedback loop" [Ref. 31a,d,e (Sec. 1.16)].

[44]R. W. Wood, *How to Tell the Birds from the Flowers* (Paul Elder, San Francisco, 1907; Dodd, Mead, New York, 1917; Dover, New York, 1959), p. 2.

[45]Adapted from Ref. 31e, pp. 3-13 to 3-15 (see also Ref. 13). A high inertia 50 lb (23 kg) block is more instructive, but high humidity with consequent vapor condensation on the glass may force use of lower-mass blocks.

[46]Available from Central Scientific Company, 11222 Melrose Ave., Franklin Park, Illinois 60131-1364 [(312) 451-0150].

Teaching of transcendence in physics

Stanley L. Jaki
Department of Physics, Seton Hall University, South Orange, New Jersey 07079

(Received 31 October 1986; accepted for publication 25 November 1986)

Efforts aimed at showing that modern physics points to a truly transcendental factor as the explanation of the universe should be welcomed by those who have urged the teaching of physics in a broad cultural context. Those efforts may profit from the following guidelines: avoid the antiontological basis of the Copenhagen interpretation of quantum mechanics; make much of the reality of the universe and its enormous degree of specificity as revealed by general relativity and the cosmic background radiation; exploit Gödel's incompleteness theorems against any grand unified theory proposed as if it were true *a priori* and necessarily; and realize that the design argument always presupposes the validity of the cosmological argument.

Of the three nouns in the title of this article, the word teaching may serve as the least controversial starting point. At any rate, the teaching of physics was the practical target of Dr. Pollard's article,[1] which has already prompted a rejoinder in this Journal.[2]

Those who have urged the teaching of physics in its broader cultural context, of which religion is an integral part, should, for the sake of consistency alone, welcome Dr. Pollard's suggestion that physics be taught with reference to the transcendence transpiring through it. The transcendence Dr. Pollard had in mind was implied in his introductory reference to his serving with Dr. Berger of *Rumors of Angels* fame on the advisory board of the Center of Theological Inquiry in Princeton that was mistakenly referred to as the Center of Technological Inquiry. At the end of his article Dr. Pollard openly spoke of the transcendence that only an inference to a creator can evoke, a frankness befitting a Christian clergyman-physicist and the author of books dealing with the relation of physics to theology.

The strategy Dr. Pollard followed in his article may be unwelcome not only for reasons relating to Weltanschauung, religious and philosophical, but also for reasons relating to teaching. His broader strategy is an account of the overriding importance that the statistical method achieved in physics since the time of Boltzmann. Dr. Pollard seems to overlook the fact that just because that method has generally been taken for a renunciation of causality, at least on the atomic level, the truth about causality and about the nature of physical reality did not thereby become the function of majority opinion, however overwhelming. Precisely because these questions are of fundamental importance in all human reasoning, the teaching about them in physics should remain very cautious. Otherwise, the physics teacher may simply indoctrinate his students in a particular climate of opinion. Worse, he would thereby promote a repetition of a pattern in the history of physics, namely, the befogging of the mental vision of physicists taking the prevailing climate of opinion for indisputable verity. The categorical assertion, on mechanistic grounds, of the existence of the ether by classical physicists[3] may be

an instructive mirror in which to see the equally categorical rejection, on nonmechanistic grounds, of causality by modern physicists.

Of no less importance, if quantum mechanics is taken for the abolition of ontological causality, how can any finding of physics be expected to nudge anyone toward making a causal, that is, ontological, inference to the existence of an order transcendent not only to a superspace "enveloping" ordinary physical reality, but also to a creator ontologically infinitely above that superspace? In short, if the teaching of transcendence in physics is to be introduced with lengthy references to the history of modern physics, philosophical clarity will not thereby become a negligible commodity. Philosophy shall remain the sole arbiter if the same transcendence is buttressed with reports about the ontological status accorded to superspace by prominent physicists, regardless of their number or even the number of Nobel Prizes they may claim to themselves. That geometry as an ideal world has an existence of its own receives no more support from the very esoteric geometry used in today's physics than the support Plato could claim from his inchoate "Euclidean" geometry. That real matter, let alone entire universes, can be generated by geometry and mathematics, however recondite, is a claim to which physics students ought not to be made heedless devotees. Otherwise the physics teacher lowers his status to that of a dutiful public relations agent.

It is again a dubious tactic in teaching transcendence in physics if an analogy is used about which it is then later admitted that it has nothing to do with the kind of transcendence to be really evoked by the physics teacher. For, as Dr. Pollard admits, the "transcendence" of superspace has nothing in it that could not be fully treated by physics.[4] As to the other transcendence, however, he states that physics has no competence whatsoever about it.[5] Possibly Dr. Pollard wanted to stress the existence of a mental realm over the strictly physical that in itself may be a good propedeutics in this age heavily preoccupied with purely empirical data. Yet one wonders whether a philosophically inarticulate Platonism will help where even its most sophisticated brands have failed to produce more than regularly recurring fascination, which is very different from demonstration.

Here a point is reached that is of utmost importance in enlightening physics students about transcendence lurking behind their subject matter. The demonstration or proof of transcendence can only be a strictly philosophical proof that is very different from a proof in mathematics or geometry, or in mathematical physics for that matter. In the latter, as E. Meyerson showed long ago,[6] proofs are identity relations, tautologies in a sense, however precious. As to physics, which deals with real experiments, it cannot as much as provide a proof of the existence of the plainest laboratory instruments. Their existence is merely assumed on grounds that are philosophical but hardly ever discussed before physics students. They are educated to deal with supreme skill with the mathematical or quantitative aspects of individual things and of their aggregates. As happens all too often, the greater that skill of theirs the more apt they are to take those properties for the things themselves. In the process they can grow woefully insensitive to the profound meaning of the predicate "is," although they continually rely on it. Thus a few years after graduation they can become so boastful about the existence-creating power of their equations as to declare that "the universe is the last free lunch"[7] and receive rounds of applause for wit and wisdom.

This leads directly to Dr. Pollard's more specific strategy or arguments—he would call them suggestions—that indicate a transcendental order, or rather a being, a creator, a mind responsible for the actual mathematical order embedded in the physical realm. His "suggestions" are essentially an argument from design that has been around for the past dozen or so years under the label anthropic principle. In essence the principle states that in its earliest state the universe was limited to a specific and narrow set of quantitative properties and that those initial conditions predetermine the formation of galaxies, of stars, of planets, of the relative abundance of chemical elements, and of the eventual appearance of organic life, including man himself. As many others before him, Dr. Pollard casts this line of thought into a probabilistic framework. There the actual universe is seen as a most unlikely case of a very large number of other possibilities or universes. Was there a design, which is always a choice, operative in the background on behalf of a single, definite set of parameters embodied in our universe?

By elaborating on the improbability of our universe, a call for a positive answer to that question can be presented with considerable plausibility. Even more graphically can be presented the extreme improbability that the evolution of life on still purely hypothetical other planetary systems would even remotely match the one it followed on the Earth. After all, as Dr. Pollard pointedly recalled, even on our Earth evolution branched off in three directions once the *Pangea* had broken into Afro-Eurasia, America, and Australia. While concrete estimates of improbability can be made very suggestive, they can never serve as basis for rigorous philosophical inference. Much less can they provide a firm grasp on reality itself, let alone on the kind of transcendental level, or the creator, which Dr. Pollard had in mind.

As a matter of fact, the universe, which has been the classic stepping stone to that level, is not spotted through probability calculations in the cosmology of General Relativity. In that theory, which provided in the history of science the first contradiction-free account of the totality of gravitationally interacting matter, the universe is taken for an indisputable entity. Einstein took the universe also in a quasitranscendental sense germane to a pantheistic perspective as he spoke of his "cosmic religion."[8] The decisive point here has, of course, nothing to do either with Einstein's religious proclivities or with his philosophical utterances about the cosmos. It should, however, be interesting to recall here a few details: He hastened to assure a friend, M. Solovine, around 1950, that his cosmology would not turn him into a believer. In the same context he voiced his dismay about the possible exploitation of his cosmology by "Pfaffen" (a German pejorative for priests) on behalf of the cosmological argument.[9] Long before that he was considerably upset when he heard of de Sitter's demonstration of the instability of the Einsteinian universe.[10] Einstein seemed to perceive that a universe subject to overall change would hardly measure up to the eternal changelessness usually ascribed to an ultimate entity. He hoped that the expanding universe would turn out to be a perpetually oscillating universe.[11] For Einstein the universe was the ultimate also in the sense that he considered, as did Bertrand Russell, any inference transcending the universe as wholly unjustified.[12] But as often happens with advocates of such

restrictions, Einstein too came close to taking his own mind to be the final intelligibility. Thus he hoped to construct a final theory of the physical universe so perfect and definitive that even the good Lord could not improve on it.[13] That in his moments of bold *a priori* speculations he still left the last word to experiments and observations[14] was due to his elemental respect, befitting a first-rate physicist, for the empirically real.

Underlying and running through all these details was Einstein's unshaken conviction about the validity of the notion of the universe as part and parcel of the cosmology of General Relativity. It is that validity that is the decisive point from the perspective of a transcendence embedded in modern physics. To see this it should be enough to think of a principal step in Kant's strategy through which he wanted to destroy the rationality of the cosmological argument. The strategy consisted in his claim that since science cannot show whether the universe is finite or infinite, continuous or atomistic, temporal or eternal, the notion of the universe is unscientific and therefore philosophically invalid.[15] Believing as he did that he had thereby discredited the stepping stone to transcendence, Kant felt confident that the rational approach to God was forever blocked because, as he rightly argued, the argument from design was but a variant on the cosmological argument. That Kant left the road open to God along the practical reason, that is, on the basis of emotive premises, was hardly a true concession. The complete autonomy of man as a rational being, or his radical immanence which was Kant's chief aim to demonstrate, could not be disputed on the basis of "pure" reason by which the Age of Enlightenment set so great a store.

The notion and reality of the universe as embedded in the cosmology of General Relativity would be a hard nut for a latter-day Kant to crack. He would have just as hard a task on hand in face of the overall temporal changes to which the universe is subject. With relatively no more than the very meager training in science that he had, today he would be reduced to reading popularizations of cosmology in which for the past 10 years fairly frequent references were made to a conclusion reached by R. Tolman in 1934 about the ever-diminishing energy amplitude of subsequent cosmic oscillations.[16] With his acumen, Kant would perceive that such an oscillating model is in fact but a variation on the simply expanding model that powerfully evokes a cosmic process from a beginning to an end. He might, of course, take for victims of elementary fallacy all those who on the basis of the expansion of the universe would speak of the moment of creation. That moment is the emergence of the universe out of nothing that is inaccessible to physics where all inferences are from one empirical state to another strictly empirical state. The inability of physics to move, say, beyond the state corresponding to Planck's time, is a purely operational impasse that in no way should be construed as a glimpse of the first moment of the universe, let alone of the "nothing" that "precedes" it.

A latter-day Kant (who stands for many physicists sympathetic to an idealist philosophy) would find very upsetting the main result of research on the 3 K cosmic background radiation, if he were to exorcize the specter of transcendence from modern physics. That research shows the universe to be enormously specific from its earliest phases on. Such specificity is a far cry from the three-dimensional erstwhile homogeneity in which a Laplace, a Herbert Spencer, and many others, who fell under the sway of the mythical Kant–Laplace nebular hypothesis, saw as the origin of the actual universe. Since that homogeneity appeared so natural to ordinary perception, it was readily taken for that primordial stage that precisely because of its "naturalness" evoked no transcendental perspectives.

Modern scientific cosmology, as far as it remains in the Einsteinian tracks, provides two holds for anyone interested in transcendence. One is the validity of the notion of the universe as a totality of consistently interacting things. The other is its breathtaking specificity or limitedness to a narrow set of physical parameters. It takes no expertise in science to realize that such an entity is one that could have been otherwise. The expertise needed to recognize this is philosophical in a very elementary sense. It is again that kind of expertise or sensitivity that would make one infer at this point the existence of a creator, an infinite being (with respect to intellect, power, and will) that alone can account for the existence of the specific totality of specific things. The reason for making that inference is an appreciation of the *principle of unrestricted rational resolve to account for existence.*

The inference cannot be evaded by tracing this very specific universe to another that has to be no less specific, and those two a third, and so forth. Such a procedure can easily become an exercise in infinite regress. It can also lay bare a serious flaw in one's definition of the universe. The tracing of a universe to another is conceivable only if the two are in interaction, but in that case they form a single universe. If they are not interacting (actually or potentially) then they are mutually unknowable. (The plurality of universes is poor English at best.) Those who refuse to make an inference to anything or anyone beyond the universe as its cause on the ground that the universe is the last stage that can empirically be reached, should consider that their grasp of the universe as such implies much more than purely empirical considerations. Kant clearly saw this when he claimed that the universe, in addition to soul and God, was one of the three main items of traditional metaphysics. As to those who would rather take lightly the reality of universe within the perspective of science, they would do well to ponder the voicing by K. R. Popper, an antimetaphysician, of the age-old truth that "all science is cosmology."[17]

Another avenue that has been tried to forestall the inference from the universe to its creator is the effort to show that this incredibly specific universe is what it is and cannot be anything else. Several prominent physicists[18] tried during the last half a century to come up with such a theory. They would not stop by wondering at the stunning measure to which one mathematical theory fits far more the physical universe than all the others.[19] They would rather try to find some quality in that mathematical theory that would show its necessary validity at the exclusion of all the others. They might have desisted had they reflected on the implications of Godel's incompleteness theorems according to which no nontrivial set of arithmetical propositions (the simplest propositions in mathematics) can have its proof of consistency within itself.[20] Therefore, a cosmological theory will have in its mathematical part (which is anything but trivial) an incompleteness. That a theory with such a defect can be called necessary is still to be demonstrated. At any rate, such a theory implies the claim that no other physical phenomenon or interaction can exist except the ones already known and predicted by that theory. In view of the ever new and unexpected phenomena revealed through physical research, the claim in question may be bordering on plain hubris.

This is not to suggest that a complete account of all physical phenomena is impossible. Those who hold that the Creator arranged everything according to measure, number, and weight,[21] and created man's mind to the image of His rationality, can most confidently expect such a theory to come in the long run at least. But they may be no less confident that even such a theory will not be a "necessary" one. It will not deprive the universe of its contingency, a term about which an important note is necessary. Contingency can mean at least two very different things in English.[22] It may mean a so-called chance event, that is, its unpredictability. It can also mean a strict dependence on a choice that can obviously be made only by a person or free being. Now if an infinitely perfect Being is postulated as the creative cause of the universe, then his choice must have built-in consistency. A true God cannot play dice with the universe by changing its laws, its parameters, at the spur of the moment. Such a postulate is in full harmony with the stunning measure of consistency revealed by modern science about the totality of things or universe. This is why predictions can be made across a range in space and time that stretches over 60 or so magnitudes. Such predictions in physics would be a mere fantasy if chance reigned because the combination of probabilities at every phase and at every major sector of the universe can only decrease the likelihood of any event. Clearly one ought not to speak of the basic randomness of the universe and at the same time extol the economy of thought in the coherence revealed by physics among the various parts and components of the universe.[23]

This is still to be perceived by most proponents and admirers of the second main type of modern cosmological speculations, a type that would have been abhorrent to Einstein. The type is a carrying to its ultimate implications the counter ontology of the Copenhagen interpretation of quantum mechanics. According to that interpretation, an interaction that cannot be measured exactly cannot take place exactly.[24] The interpretation implies a clear equivocation between two meanings attached to "exactly": one is purely operational, the other is clearly ontological. This equivocation, to which prominent attention was called as early as 1930,[25] still has to be recognized in physics teaching also in reference to transcendence. For in the inflationary theory of the universe a cosmic twist is given to the long fashionable assertion that the energy with which a radium atom disintegrates at a given moment is uncaused to the extent to which it cannot be measured, and therefore the uncaused energy portion (that is always equivalent to mass) is to be accounted for by mere mathematics. The origin of the universe is now viewed in the same fashion. The phrase, "the universe is the last free lunch," may or may not be more than a flippant boasting about one's prowess with the mathematical techniques of quantum mechanics. But, if it is to be taken at face value, any conscientious physics teacher could only be grateful for a clue to the possible ideological motivation behind it. He may also find it instructive to think of a passage in Koestler's *Darkness at Noon* in which the Stalinist supervisor of political prisoners bluntly reminds his prominent victim that "as long as chaos dominates the world, God is an anachronism."[26]

At any rate, the inflationary universe, in which many universes are produced randomly, has already provoked efforts to find some interaction among those universes. One more evidence that physics cannot live without a universe that is one and is the very opposite of randomness that, if it is total, is the very opposite to oneness. The perception that absolute chance is unthinkable may help one perceive the presence of some steady parameters beneath all agglomerates of stochastic variables, so useful in physics.

If the teaching of physics is a serious matter, the art of doing physics ought not to be presented as a facile game with words and formulas. Yet this facile game is visible everywhere. Its most consistent result is solipsism, mostly covert, but at times unabashedly open. The covert form is a mistaken form of Berkelian perceptionism grafted on Platonist idealism. The anthropic principle is often cast into that form. A case in point is the headline, "I am, therefore the universe exists," introductory to a discussion of a recent book on that principle.[27] Those who speak of a self-exciting universe certainly reveal the excitement they feel about their own mental prowess. If it allegedly can start an entire universe, it is still to display as much effectiveness as to open a bank account by merely thinking hard about the geometry of the deposit and the teller's window. The danger of the covert forms of solipsisms in physics is far greater for the average physics student or physicist, most of whom know philosophy only by hearsay, than the overt unabashed solipsism, say, of an Eddington and some others of lesser stature. Curiously, Eddington never stopped thinking of transcendence—he even lived a life of prayer. This is only one more example of the inconsistency typical of human beings. For consistently practiced solipsism is the only rigorous, unanswerable, though literally self-defeating means whereby the specter of transcendence can be barred from physics or from any other fields of broad and sustained inquiry.

[1]W. G. Pollard, Am. J. Phys. **52**, 877 (1984).

[2]J. Rosen, Am. J. Phys. **54**, 700 (1986).

[3]Maxwell's statement that "the ether is certainly the largest body of which we have any knowledge," was but an echo of countless other categorical assertions made by prominent physicists throughout the 18th and 19th centuries. A good number of them are quoted in my *The Relevance of Physics* (University of Chicago Press, Chicago, 1967), pp. 79–85.

[4]Reference 1, p. 878.

[5]Reference 1, p. 881.

[6]E. Meyerson, *Identity and Reality* (1907; Dover, New York, 1962).

[7]A widely publicized remark of Dr. A. H. Guth.

[8]For his relevant statements, see R. W. Clark, *Einstein: The Life and Times* (Crowell, New York, 1971), pp. 465–66.

[9]Quoted in full in English translation in my Gifford Lectures, *The Road of Science and the Ways to God* (University of Chicago Press, Chicago, 1978), pp. 192–93.

[10]See Einstein's letter of 22 June 1917, to W. de Sitter, quoted in C. Kahn and F. Kahn, Nature **257**, 454 (1975).

[11]His statement made in a note to Z. Phys. **16**, 228 (1927) was a corrective to his earlier rejection in Z. Phys. **11**, 326 (1922) of A. Friedmann's paper of an oscillating variant of Einstein's universe.

[12]Reference 9, pp. 193 and 276.

[13]See his contribution to *Festschrift Prof. Dr. A. Stodola überreicht* (Orell Füssli Verlag, Zurich, 1929), p. 127.

[14]Without empirical verification even his General Theory of Relativity would "turn into mere dust and ashes," was a remark of Einstein in a lecture given in Prague in 1920, a remark kept for posterity by H. Feigl. See Ref. 9, p. 260.

[15]Kant set forth this reasoning in the antinomies of his *Critique of Pure Reason*.

[16]R. Tolman, *Relativity, Thermodynamics and Cosmology* (Cambridge, U.P., New York, 1934), pp. 439–44.

[17]K. R. Popper, *The Logic of Scientific Discovery* (1959; Harper Torchbooks, New York, 1968), p. 15.

[18]For details, see my *Cosmos and Creator* (Scottish Academic P., Edinburgh, 1980), pp. 45–48.

[19]In speaking of the measure of that fitness in his article, "The Unreasonable Effectiveness of Mathematics in the Natural Sciences," Commun. Pure Appl. Math. **13**, 12 (1960), E. P. Wigner, contrary to Dr. Pollard's claim, does not seem to go beyond that wondering.

[20]For the past 20 years I have repeatedly called attention to this implication of Gödel's theorems, especially in publications mentioned in Refs. 3 and 18.

[21]A phrase from the Book of Wisdom (11:20), which, according to E. Curtius, well known for his studies on medieval literature, was the most often quoted biblical passage throughout the Middle Ages.

[22]Very instructive in this respect can be a perusal of the entries, contingency and contingent, in the Greater Oxford Dictionary of the English Language.

[23]In addition to this inconsistency, J. Rosen offers another in his rejoinder to Dr. Pollard's article as he speaks of philosophy both as a mere fashion and as something very important in the work of a physicist. Positions of such latitude are as unconvincing as as they are refractory to any strict argument.

[24]A further development of that point can be found in my article, "Chance or Reality: Interaction in Nature versus Measurement in Physics," Philosophia (Athens) **10-11**, 85-105 (1980–81); reprinted in my book *Chance or Reality and Other Essays* (University Press of America, Lanham, MD and Intercollegiate Studies Institute, Bryn Mawr, PA, 1986), pp. 1–21.

[25]The prominent forum was *Nature*, the British scientific weekly read all over the world, which carried in its 27 Dec. 1930, issue (p. 995) a letter by J. E. Turner, who wrote in a way of conclusion: "But every argument that, since some change cannot be 'determined' in the sense of 'ascertained,' it is therefore not 'determined' in the absolutely different sense of 'caused,' is a fallacy of equivocation."

[26]A. Koestler, *Darkness at Noon* (1941; New American Library, New York, 1961), p. 134.

[27]See T. Ferris' review in The *New York Times Book Review* (16 Feb. 1986, p. 20) of *The Anthropic Cosmological Principle* by J. D. Barrow and F. J. Tipler (Oxford U.P., New York, 1986).

How not to focus a small source with a single lens

William S. Heaps
NASA/Goddard Space Flight Center, Laboratory for Atmospheres, Greenbelt, Maryland 20771

(Received 15 April 1985; accepted for publication 26 November 1986)

It has been observed that when focusing a small finite source onto a fixed image plane with a single lens, the focus position does not necessarily correspond to the minimum imaged spot size. A simple analysis of this situation illuminates conditions under which it may arise.

I have had considerable success for a number of years at determining the focal point for a small finite source and a fixed lens by moving a target along the optical axis and noting the position of minimum spot diameter. Obviously the minimum spot diameter obtains at the focal plane of the combination. Recently, while aligning the detection optics for a laser radar system, I encountered a slightly different situation in which the source-to-image plane distance was fixed and the system was to be focused by moving the lens. I had positioned a ground glass screen at the desired focus and began to adjust the lens in a threaded mount seeking the minimum spot diameter. I was somewhat embarassed in front of several collegues when the adjustment range of the lens was exhausted without a clear-cut minimum diameter being obtained. A hasty check of the system dimensions and design calculations revealed no obvious error. I next suspected that the lens had been supplied with an incorrect focal length. When the focal length had been measured and found correct, I began a detailed analysis of the optical design. The explanation proved to be the shocking (at least to me) fact that the minimum spot diameter is not necessarily obtained by focusing. The following analysis shows the conditions under which this situation may arise.

Figure 1 depicts a small source of radius h located a distance L from a target screen. The distance L shall be held constant. A lens of radius r and focal length f is located a distance x from the screen. It is assumed that the lens is strong enough to produce a focus on the screen ($f < L/4$). The object radius at the focus is represented by h' and the radius of the spot on the target screen is represented by h''. We shall proceed to solve for the value of h' and of h'' as functions of x. Next, we shall determine the value of x for which h'' is minimum and solve for this minimum value. Finally, we shall determine the conditions under which this minimum h'' is less than the spot radius h' when the source is imaged on the screen.

Using the thin lens formula one has

$$1/(L-x) + 1/q = 1/f, \tag{1}$$

where q is the distance from the lens to the focused image of the source. Strictly speaking, this formula is not correct in the presence of a spherical aberration that displaces the minimum spot diameter to a position between the paraxial ray image and the marginal ray image.[1] The effect under investigation can be much more severe than this. Solving for q one obtains

$$q = f(L-x)/(L-x-f), \tag{2}$$

$$h'/q = h/(L-x) \tag{3}$$