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(*b*. Venice, Italy, 19 January 1879; *d*. New York, N. Y., 6 June 1943)

mathematics.

Fubini was the son of Lazzaro Fubini, who taught mathematics at the Scuola Macchinisti in Venice, and Zoraide Torre. At the age of seventeen, after brilliantly completing secondary studies in his native city, he entered the Scuola Normale Superiore di Pisa, where Dim and Bianchi were among his teachers. In 1900 he defended a thesis on Clifford's parallelism inelliptical spaces, the results of which rapidly became classic because of their inclusion in the 1902 edition of Luigi Bianchi's treatise on differential geometry. Fubini remained in Pisa for another year to complete work on the diploma allowing him to teach at the university level. The important memoir that he wrote in this connection deals with the fundamental principles of the theory of harmonic functions in spaces of constant curvature, a subject quite different from that of his doctoral thesis.

Placed in charge of a course at the University of Catania toward the end of 1901, Fubini soon won the competition for nomination as full professor. From Catania he went to the University of Genoa and then, in 1908, to the Politecnico in Turin. There he taught mathematical analysis, and at the same time, at the University of Turin, higher analysis. In 1938 Fubini was forced to retire under the racial laws promulgated by the Fascist government. The following year, at the invitation of the Institute for Advanced Study at Princeton, he immigrated to the United States and was welcomed among the institute's members. His prudent decision to seek voluntary exile was in part dictated by his concern for the future of his two sons, both engineers. Already in poor health, he continued to teach at New York University until he died of a heart ailment at the age of sixty-four.

A man of great cultivation, fundamentally honorable and kind, Fubini possessed unequaled pedagogic talents. His witty banter and social charm made him delightful company; he was small in stature, and his voice was vigorous and pleasant. Deeply imbued with a sense of family, he wished toward the end of his life legally to add Ghiron—the maiden name of his wife, whom he had married in 1910—to his own. Those works on mathematical subjects designed to be of use to engineers resulted from his own interest in watching over his sons' studies. With regard to Luigi Bianchi, Fubini's gratitude was the equal of his respect and admiration for Bianchi as a model for both his life and his work. Upon Bianchi's death in 1928, Fubini succeeded him as coeditor of the *Annali di matematica pura ed applicata*, a position that he held until 1938. A member of several Italian scientific academies, Fubini received the royal prize of the Lincei in 1919.

Fubini was one of Italy's most fecund and eclectic mathematicians. His contributions opened new paths for research in several areas of analysis, geometry, and mathematical physics. Guided by an ever-alert geometric intuition and possessed of an absolute mastery of all the techniques of calculation, he was able to follow leads that had barely been glimpsed. His technical mastery often permitted him to discover simpler demonstrations of such theorems as those of Bernstein and Pringsheim on the development of Taylor series.

In analysis Fubini did work on linear differential equations, partial differential equations, analytic functions of several complex variables, and monotonic functions. He also studied, in the calculus of variations, the reduction of Weierstrass' integral to a Lebesgue integral; the possibility of expressing every surface integral by two simple integrations, and the converse; and the manner of deducing from the existence of $\delta^n f/\delta x^n$ and $\delta^n f/\delta y^n$ the existence of lower-order derivatives of the function $f(x,y)$. In addition, Fubini determined, with regard to the minimum-value principle, the limit of a series of functions that take on given values on the contour of a domain, by supposing that the corresponding Dirichlet integrals tend toward their lower limit; he also indicated how his procedure could be applied to the calculus of variations. Finally, he investigated nonlinear integral equations and those with asymmetric kernels.

In the field of discontinuous groups, Fubini studied linear groups and groups of movement on a Riemannian variety in order to establish their criteria of discontinuity, as well as to prove the existence of fundamental domains and to indicate the method of constructing them. He examined functions admitting of such groups, as well as the automorphic harmonic functions in a space of n dimensions, in this way generalizing certain theorems of Weierstrass. For continuous groups, he established the conditions required in order to be able to attribute a metric to them.

In the field of non-Euclidean spaces Fubini, in his thesis on Clifford's notion of parallelism, introduced sliding parameters, which made possible the transposition to elliptical geometry of certain results of ordinary differential geometry, such as Frenet's formulas and the determination of couples of applicable surfaces. His work on the theory of harmonic functions in spaces of constant curvature contains an extension of the Neumann method and of the Appell and Mittag-Leffer theorems.

The most extensive field that Fubini cultivated was that of differential projective geometry, for which he elaborated general procedures of systematic study that still bear his name. The difficulties to be surmounted in order to pass from classical to projective differential geometry arise mainly from mathematical techniques and their use. To succeed in this endeavor, Fubini utilized absolute differential calculus and certain contravariant differentials. First he defined the local application of two varieties with respect to a Lie group; then he introduced the "projective linear element" as the quotient of two covariant differential forms and demonstrated that the necessary and sufficient condition for a projective application is the equality of these elements. He envisaged homogeneous coordinates normalized from a variable point on the surface or hypersurface, and he defined the "projective normals," the "projective geodesics," and the more general geodesics. In a Euclidean space the transformation by affinity of a surface of constant curvature is characterized by its second projective normal's being extended to infinity. These fundamental investigations of metric, or affine, geometry, which were pursued by other researchers, are collected in *Geometria proiettiva differenziale* and *Introduction à la géométrie projective différentielle des surfaces*, both written in collaboration with Eduard Čech.

Fubini's contributions to mathematical physics are varied. They began during World War I with theoretical studies on the accuracy of artillery fire and then turned to such problems in acoustics and electricity as anomalies in the propagation of acoustic waves of large amplitude, the pressure of acoustic radiation, and electric circuits containing rectifiers. Fubini was also interested in the equations of membranes and vibrating diaphragms. The mathematical aspects of the engineering sciences likewise occupied his attention. A work on engineering mathematics and its applications appeared posthumously in 1954. Finally, one must note his textbooks—courses in analysis and collections of problems which have been used by many generations of students—to appreciate fully the many-faceted work of Fubini, one of the most luminous and original minds in mathematics during the first half of the twentieth century.

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