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Brian Leftow (2005)

GÖDEL, KURT

(1906–1978)

Kurt Gödel, a logician, was born in Brno, in what is now the Czech Republic, and educated at the University of Vienna, where he became privatdozent in 1933. In 1940 he joined the Institute for Advanced Study in Princeton, New Jersey, where he remained for the rest of his career. Following David Hilbert, Gödel was instrumental in establishing mathematical logic as a fundamental branch of mathematics, achieving results such as the incompleteness theorems that have had a profound impact on twentieth-century thought. In philosophy, by contrast, he represents the path not taken. Of his few writings in this area, including posthumous publications, those that focus on the more immediate ramifications of his own (and closely related) mathematical work have had the greatest impact.

GÖDEL'S INFLUENCE

A close student of the history of philosophy, Gödel follows Plato, Gottfried Wilhelm Leibniz, and Edmund Husserl as opposed to the more fashionable Aristotle, Immanual Kant, and Ludwig Wittgenstein. (On Kant, however, see Gödel 1946/9 and 1961.) Methodologically, two patterns in his thinking stand out. First, a tendency to move from the possible to the actual is reflected in his Leibnizian ontological argument for the existence of god (Gödel 1970). He relies here on the S5 modal principle, (possibly necessarily P $\supset$ necessarily P). It can also, arguably, be discerned in his mathematical Platonism—because the distinction between the possible and the actual, relevant to material being, collapses in the formal realm of mathematics (see Yourgrau 1999). Finally, in relativistic cosmology (Gödel 1949, 1946/9) he concludes from the possible existence of rotating universes, where time is merely ideal, to its ideality in the actual world.

Second, he is preoccupied with probing mathematically the limits of formal methods in representing intuitive concepts. In his first incompleteness theorem, for example, by applying an ingenious arithmetization of metamathematics to a formal system of arithmetic, Gödel was able to construct a formula expressing its own unprovability, and thus to prove (as he made explicit later) the indefinability within the system of the intuitive concept of arithmetic truth (see Feferman 1984). Along the same lines one may view his results in cosmology as demonstrating the limits of the theory of relativistic space-time in representing the intuitive concept of time, although here, interestingly, his response was to abandon the intuitive concept (see Yourgrau 1999).

From a broader perspective Gödel isolates two basic philosophical worldviews: one with a "leftward" direction, toward skepticism, materialism, and positivism, the other inclined toward "the right," toward spiritualism, idealism, and theology (or metaphysics; Gödel 1961). He puts empiricism on the left and a priorism on the right and points out that although mathematics, qua a priori science, belongs "by its nature" on the right; it too has followed the spirit of the times in moving toward the left—as witnessed by the rise of Hilbert's formalism. With Gottlob Frege, Gödel resists this trend, pointing to his incompleteness theorems as evidence that "the Hilbertian combination of materialism and aspects of classical mathematics … proves to be impossible" (1961, p. 381).

FREGE AND GÖDEL

Frege's mathematical philosophy is held together by two strands that may appear to be in tension with one another: on one side his Platonism and conceptual realism, on the other his conception of arithmetic as analytic (that is, as resting on definitions and the laws of logic) and his "context principle" (which seems to put our sentences—hence language—at the center of his philosophy). This second aspect of Frege's thought, via Bertrand Russell and Wittgenstein, helped persuade the positivists of the Vienna Circle (whose meetings Gödel attended) that mathematics is without content, a mere matter of (more or less arbitrary) linguistic conventions concerning the syntax of (formal) language. This conclusion was, however, rejected by both Frege and Gödel (1944, 1951, 1953–59), Frege hoping, contra Kant, "to put an end to the widespread contempt for analytic judgments and to the legend of the sterility of pure logic" (1884, p. 24; see also 1879, p. 55). Gödel, for his part, insists that "'analytic' does not mean 'true owing to our definitions,' but rather 'true owing to the nature of the concepts occurring therein'" (1951, p. 321). (See Parsons, 1994.)

Frege and Gödel are in further agreement against the spirit of the times, that the fundamental axioms of mathematics should be not simply mutually consistent but (nonhypothetically) true. They also reject Hilbert's conception of axiom systems as "implicit definitions," with Gödel insisting that a formal axiomatic system only partially characterizes the concepts expressed therein. Indeed, his Incompleteness Theorem makes the point dramatically: "Continued appeals to mathematical intuition are necessary … for the solution of the problems of finitary number theory.… This follows from the fact that for every axiomatic system there are infinitely many undecidable propositions of this type" (1947 [1964], p. 269). And it is in human ability—if indeed humans possess it—to intuit new axioms in an open-ended way that Gödel sees a possible argument to the effect that minds are not (Turing) machines (Gödel 1951; Wang 1996).

What kind of intuitions, however, are these? Gödel does, it is true, employ a Kantian term here, but he does not mean concrete immediate individual representations, and on just this point he faults Hilbert: "What Hilbert means by '*Anschauung*' is substantially Kant's space-time intuition.… Note that it is Hilbert's insistence on *concrete* knowledge that makes finitary mathematics so surprisingly weak and excludes many things that are just as incontrovertibly evident to everybody as finitary number theory" (1958 [1972], p. 272, n. b). (See also 1947 [1964], p. 258.) Note, further, that mathematical intuition, though a form of a priori knowledge, does not ensure absolute certainty, which Gödel rejects (Wang 1996); rather, as with its humbler cousin, sense perception, it too may attain various degrees of clarity and reliability (see Gödel 1951, his remarks on Husserl in 1961, and Parsons 1995, 1995a).

THE GÖDEL PHILOSOPHY

Frege and Hilbert, then, serve as useful coordinates in mapping Gödel's philosophy, in its tendency to "the right." What if one chooses Albert Einstein as a third coordinate? Note first that "idealistic" in the title of Gödel (1949) is not a gesture toward a subjective philosophy such as George Berkeley's. (In his final years, he became sympathetic with Husserl's later idealism, which does not exclude objectivism. See van Atten and Kennedy 2003.) Rather, Gödel is pointing to the classic Platonic distinction between appearance and reality. Though the world may appear (to the senses) as if temporal, this is in fact an illusion. Only reason—here, mathematical physics—can provide a more adequate cognition of reality (i.e., of Einstein-Minkowski space-time). Gödel makes a sharp distinction between intuitive time, which lapses, and the temporal component of space-time. By his lights, already in the special theory of relativity (STR) intuitive time has disappeared, because "the existence of an objective lapse of time means … that reality consists of an infinity of layers of 'now' which come into existence successively" (Gödel 1949, pp. 202–203), whereas the relativity of simultaneity in the STR implies that "each observer has his own set of 'nows,' and none of these various systems of layers can claim the prerogative of representing the objective lapse of time" (p. 203).

These observations, however, rely on the equivalence of all "observers" or reference frames in the STR, whereas in the general theory of relativity (GTR), of which the STR is an idealized special case, the presence of matter and the consequent curvature of space-time permit the introduction of privileged observers, in relation to which one can define a "world time" (which, one may say, objectively lapses). Gödel's discovery is that there exist models of the GTR—the rotating universes—where, provably, no such definition of a world time is possible. In particular, these worlds permit time travel, in the sense that, "for *every* possible definition of a world time one could travel into regions of the universe which are past according to that definition," and "this again shows that to assume an objective lapse of time would lose every justification in these worlds" (1949 p. 205). The idea here is clearly that if a time has "objectively lapsed," it no longer exists and so is not *there* to be revisited (in the future). Hence, by contraposition, if it can be revisited, it never did objectively lapse in the first place.

To describe the Gödel universe as static, however, as opposed to our own, would be misleading. The time traveler's rocket ship, for example, would move at a speed of at least $1/\sqrt{2}$ of the velocity of light! It would seem to observers, just as in this world, to be moving at great speed, and in general the denizens of Gödel's universe may well experience time much as we do in the actual world. Indeed, that is why Gödel moves from the mere possible existence of the Gödel universe to the ideality of time in the actual world, because "if the experience of the lapse of time can exist without an objective lapse of time, no reason can be given why an objective lapse of time should be assumed at all" (p. 206; see Yourgrau 1999).

Here, then, is another example of the Janus-faced quality of Gödel's thinking, presaged already in his arithmetization of metamathematics—contributing mathematically to "the left" while at the same time, as he sees it, pointing to "the right."

See also Gödel's Incompleteness Theorems; Logic, History of; Mathematics, Foundations of.

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GÖDEL'S INCOMPLETENESS THEOREMS

The axiomatic method is at the heart of mathematics. The work of mathematicians is to derive the consequences of axioms. According to Euclid, axioms are evidently true, and deduction from them is a powerful method of learning new truths. The rise of non-Euclidean geometry disrupted the carefree connection between truth and proof and led many modern thinkers to adopt the formalistic attitude that the mathematician's sole endeavor is to work out the consequences of axioms, taking no professional interest in inquiring what, if anything, the axioms are true of.

In 1931 Kurt Gödel proved a deep theorem that showed that deduction from axioms cannot be all there is to mathematical understanding. Gödel showed that, for whatever system of truths of number theory we choose to regard as axiomatic, there will be statements of basic arithmetic that we can recognize as true even though they are not consequences of the axioms. That there are truths not derivable from our axioms is hardly surprising; nobody ever promised us omniscience. What is surprising is that there are arithmetical statements *we can recognize as true* even though they are not derivable, so that no system of axioms we can write down fully captures our arithmetical understanding. Moreover this situation holds not only for systems of axioms we are capable of producing today but also for whatever systems we may devise in the future.