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Graham Priest (2005)

LOGIC, SYMBOLIC

See *Logic, History of*

LOGIC, TRADITIONAL

In logic, as in other fields, whenever there have been spectacular changes and advances, the logic that was current in the preceding period has been described as "old" or "traditional," and that embodying the new material has been called "new" or "modern." The Stoics described themselves as "moderns" and the Aristotelians as devotees of the "old" logic, in the later Middle Ages the more adventurous writers were called *moderni,* and since the latter part of the nineteenth century the immensely expanded logic that has developed along more or less mathematical lines ("mathematical logic," "symbolic logic," "logistics") has been contrasted with the "traditional" logic inherited from the sixteenth and seventeenth centuries. In every case the logic termed "old" or "traditional" has been essentially Aristotelian, but with a certain concentration on the central portion of the Aristotelian *corpus,* the theory of categorical syllogism—the logic of Aristotle himself having been rather less circumscribed than that of the "tradition," especially of the sixteenth to the nineteenth century.

THE LOGIC OF TERMS

To begin with the categorical syllogism, an inference, argument, or syllogism (traditionally, all arguments are assumed to be syllogistic) is a sequence of propositions (premises followed by a conclusion), such as "All animals are mortal; all men are animals; therefore, all men are mortal." Propositions, in turn, are built up from terms—for example, "animals," "mortals," "men." The traditional order of treatment, therefore, begins with the study of terms (or, in writers with a psychological or epistemological bias, ideas) and goes on to the study of propositions (or judgments), concluding with that of syllogisms (or inferences).

The terms from which the propositions principally studied in the traditional logic are built up are common nouns (*termini communes*), such as "man" and "horse," although some attention is also paid to singular terms, such as "Socrates," "this man," and "the man next door." Much of the traditional theory is devoted to the arrangement of common nouns in an order of comprehensiveness, and here a distinction is made between two aspects of their functioning—their "extension" (as the logicians of Port-Royal called it) or "denotation" (John Stuart Mill) and their "intension" (Sir William Hamilton), "comprehension" (Port-Royalists), or "connotation" (Mill). The extension or denotation of a common noun is the set of individuals to which it applies, its intension or connotation the set of attributes that an individual must possess for the common noun to be applicable to it. Thus, the connotation of the term *man* consists of the attributes of being an animal, being rational, and perhaps possessing a certain bodily form; its denotation consists of all objects that possess these attributes.

Broadly, the connotation of a term is its meaning, the denotation its application. The analysis of the meaning of a term is described as definition, and the breaking up of the set of objects to which it applies into subsets is described as division. The subsets of the set of individuals to which a given term applies are called the species of the genus denoted by the given term. The attribute that marks off a particular species from others of the same genus is called its differentia. The species is said by scholastic logicians to "fall under" its genus, and the standard way of defining a species is by giving its genus and its differentia.

The ordering of terms into species and genera is often thought of as having an upper and a lower limit. The upper limit, or *summum genus,* will be a broad category such as "thing" (*substantia*)—horses are animals, animals are organisms, organisms are bodies, bodies are things. More abstract terms will come to an end in more abstract categories, such as "quality" or "relation" (scarlet is a species or kind of red, red is a color, color is a quality). The *infima species,* or lower limit, is a more difficult concept. Man, for example, is commonly given as an *infima species,* but are not men divisible into, for instance, dark-haired and fair-haired men? This is answered, from the point of view of intension, by dividing the attributes of an individual into those that constitute its essence or nature and those that are merely accidental, and genuine species are said to be marked off by "essential" attributes only; further subdivisions differentiated by "accidental" attributes, such as the color of a man's hair, are not counted as genuine species. This distinction is not recognized by some writers. Gottfried Wilhelm Leibniz counted all attributes of an individual as essential, so that someone would not be *that* individual if he were in the least respect different from what he is. At the other extreme, Mill said that "individuals have no essences," although he had a use for the term *essence* in connection with general terms: It is of the essence of being a man, for example, to be an animal, if being an animal is one of the attributes commonly employed in fixing the application of the word *man.*

An allied doctrine of Mill's is that the proper names of individuals, by contrast with common nouns, have no connotation, only denotation. We may not be able to think of a named individual without thinking of him as having certain attributes, but the purpose of a proper name is not to convey the fact that he has those attributes but only to identify him as *that* individual. This view has been criticized by various writers, on the ground, among others, that we cannot identify an object at all without knowing at least its *infima species.* Mill has also been criticized for using the same term, "denotation," both for the application of a common noun and for what is named by a proper name.

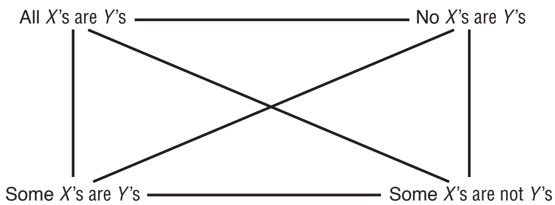
Common terms can be simple or complex. Some kinds of complexity are of logical interest—for example, the conjunctive combination exemplified by "blind man" (i.e., what is both blind and human) and the disjunctive combination exemplified by "man-or-beast." This kind of complexity is of interest because, for one thing, it links up with the previous topic, a blind man being a species (in the broad though not the narrow sense) of man and a man being a species (again in the broad sense) of man-or-beast (i.e., of animal). Again, the term "son-of-Philip" is compounded of the relative expression "son of" and the proper name "Philip," and this, too, links with the preceding topic, a son of Philip being a species (in the broad sense) of son. But the logical behavior of complex terms of these types is a topic of modern rather than traditional logic. Even traditional logic, however, has something to say about negative terms, such as "non-man" (i.e., what is not human), as will be shown in what follows.

The distribution of terms is a subject that will be more intelligible after propositions and syllogisms have been considered.

THE LOGIC OF PROPOSITIONS

OPPOSITION. The division of traditional logic called the logic of propositions is not to be confused with what is now called the propositional calculus. The propositional calculus studies the logical behavior of propositions formed from simpler propositions by means of various connectives (for example, "Either all men are liars or no men are"), as opposed to propositions formed not from other propositions but from terms (for example, "No men are liars"). The traditional logic of propositions or judgments, on the other hand, is chiefly concerned with the classification and simpler interrelations of precisely the second class of propositions, although it normally also touches on "compound" or "hypothetical" propositions, without going beyond their simplest types and the simplest inferences involving them.

Propositions not compounded of other propositions are called categorical. This word has the force of "unconditional," the implied contrast being with forms like "*If* all that the Bible says is true, all men are mortal" or "*Either* not all that the Bible says is true, *or* all men are mortal." Categoricals have a *subject term* and a *predicate term* ("men" is the subject term and "mortal" the predicate term of "All men are mortal") and are subdivided in two main ways—according to quantity, into universals ("All men are mortal," "No men are mortal") and particulars ("Some men are mortal," "Some men are not mortal"), and according to quality, into affirmatives ("All men are mortal," "Some men are mortal") and negatives ("No men are mortal," "Some men are not mortal"). These are often displayed in a square, with universals at the top, particulars at the bottom, affirmatives on the left, negatives on the right:



Universal affirmatives are called A-propositions, particular affirmatives I-propositions, universal negatives E-propositions, and particular negatives O-propositions (the vowels being taken from the words *affirmo* and *nego*). Two other "quantities" are commonly mentioned, namely *singular* and *indefinite.* Singular propositions, such as "Socrates is mortal," are a genuinely distinct type, which we shall touch upon at appropriate points; indefinites, such as "Men are mortal," seem merely to be universals or particulars in which the quantity is left unstated. The expressions other than terms which enter into these forms are called "syncategorematic"; they are divided into the signs of quantity "all" and "some" and the copulas "is" or "are" and "is not" or "are not." ("No" is both a sign of quantity and a sign of negation.)

These types of propositions—A, E, I, and O—are the traditional "four forms," and as a preliminary to logical manipulation it is customary to restate given sentences in some standard way that will make their quantity and quality immediately evident. The forms given above, with "all," etc., and with plural common nouns for terms, are the most widely used, but it is in some ways less misleading to use "every," etc., and the terms in the singular—"Every *X* is a *Y,*" "No *X* is a *Y,*" "Some *X* is a *Y,*" "Some *X* is not a *Y.*" What is important is to understand that "some" means simply "at least one"; "Some men are mortals" or "Some man is a mortal" must be understood as neither affirming nor denying that more than one man is a mortal and as neither affirming nor denying that all men are (i.e., "some" does *not* mean "only some").

A square of the type shown earlier is called a *square of opposition,* and propositions with the same terms in the same order may be "opposed" in four ways. Universals of opposite quality ("Every *X* is a *Y,*" "No *X* is a *Y*") are said to be *contraries*; these cannot be jointly true. Particulars of opposite quality ("Some *X* is a *Y,*" "Some *X* is not a *Y*") are said to be *subcontraries*; these cannot be jointly false. Propositions opposed only in quantity are said to be *subalterns,* the *subalternant* universal implying (without being implied by) the *subalternate* particular ("Every *X* is a *Y*" implies "Some *X* is a *Y,*" and "No *X* is a *Y*" implies "Some *X* is not a *Y*"). Propositions opposed in both quantity and quality ("Every *X* is a *Y*" and "Some *X* is not a *Y,*" and "No *X* is a *Y*" and "Some *X* is a *Y*") are *contradictories*; they cannot be jointly true or jointly false—the truth of a given proposition implies the falsehood of its contradictory; its falsehood implies the contradictory's truth.

EQUIPOLLENCE. Closely connected with the theory of opposition is that of the equipollence of propositions with the same terms in the same order but with negative particles variously placed within them. Since contradictories are true and false under reversed conditions, any proposition may be equated with the simple denial of its contradictory. Thus, "Some *X* is not a *Y*" has the same logical force as "Not every *X* is a *Y,*" and, conversely, "Every *X* is a *Y*" has the force of "Not (some *X* is not a *Y*)," or, to give it a more normal English expression, "Not any *X* is not a *Y.*" Similarly, "Some *X* is a *Y*" has the force of "Not (no *X* is a *Y*)" and "No *X* is a *Y*" that of "Not (some *X* is a *Y*)"—that is, "Not any *X* is a *Y.*" Also, since "no" conveys universality and negativeness at once, "No *X* is a *Y*" has the force of "Every *X* is not-a-*Y,*" and, conversely, "Every *X* is a *Y*" has the force of "No *X* is not-a-*Y.*" Writers with an interest in simplification have seen in these equivalences a means of dispensing with all but one of the signs "every," "some," and "no." Thus the four forms may all be expressed in terms of "every," as follows: "Every *X* is a *Y*" (A), "Every *X* is not-a-*Y*" (E), "Not every *X* is not-a-*Y*" (I), "Not every *X* is a *Y*" (O).

Of singular propositions all that need be said at this point is that they divide into affirmatives ("Socrates is mortal," "This is a man," "This man is mortal") and negatives ("Socrates is not mortal," etc.) and that when their subject is formed by prefixing "this" to a common noun (as in "This man is mortal"), the singular form is implied by the corresponding universal ("Every man is mortal") and implies the corresponding particular ("Some man is mortal"). Some of the traditional logicians attempted to assimilate singular propositions to particulars, some to assimilate them to universals, but these attempts are not very impressive, and it is one of the few merits of the Renaissance logician Peter Ramus that he and his followers treated them consistently as a type of their own.

CONVERSION OF PROPOSITIONS. With regard to pairs of propositions of the same form and with the same terms, but in reverse order—for example, "No *X* is a *Y*" and "No *Y* is an *X*"—these are sometimes equivalent and sometimes not. Where they are, as in the case just given, they are said to be *converses* of one another, and the forms are said to be convertible. E and I are convertible; A and O are not. That every man is an animal, for example, does not imply that every animal is a man, and that some animal is not a horse does not imply that some horse is not an animal. Conversion, the inference from a given proposition to its converse ("Some men are liars; therefore, some liars are men"), is a type of immediate inference—that is, inference involving only one premise (as opposed, for instance, to syllogisms, which have two). Other immediate inferences are those from a given proposition to an "equipollent" form in the sense of the preceding section (for example, "Every man is mortal; therefore, not any man is not") and from a subalternant universal to its subalternate particular ("Every man is mortal; therefore, some man is mortal").

The conversion just described is "simple" conversion; with universals (even A, though it is not "simply" convertible) there is also a conversion *per accidens,* or *subaltern* conversion—that is, a legitimate inference to the corresponding particular form with its terms transposed. Thus, although "Every man is an animal" does not imply that every animal is a man, it does imply that some animal is.

Other forms of immediate inference arise when negative terms are introduced. The simultaneous interchange and negation of subject and predicate is called *conversion by contraposition,* or simply contraposition. It is a valid process with A's and O's, not with E's and I's. ("Every man is an animal" implies "Every non-animal is a non-man"—whatever is not an animal is not a man—and "Not every animal is a man" implies "Not every non-man is a non-animal," but "No horse is a man" does not imply "No non-man is a non-horse"; "Some *X* is a *Y*" is true and "Some non-*Y* is a non-*X*" false if the *X*'s and the *Y*'s overlap and between them exhaust the universe.) All of the four forms may be "obverted" (Alexander Bain's term)—that is, have their quality changed and the predicate negated ("Every *X* is a *Y*" implies "No *X* is a non-*Y*," "No *X* is a *Y*" implies "Every *X* is a non-*Y*," and similarly with the particulars). A variety of names are given to the results of repeated successive obversion and conversion.

THE LOGIC OF SYLLOGISM

A categorical syllogism is the inference of one categorical proposition, the conclusion, from two others, the premises, each premise having one term in common with the conclusion and one term in common with the other premise—for example:

Every animal is mortal;
Every man is an animal;
Therefore, every man is mortal.

The predicate of the conclusion (here "mortal") is called the major term, and the premise that contains it (here written first) the major premise. The subject of the conclusion ("man") is the minor term, and the premise that contains it (here written second) the minor premise. The term common to the two premises ("animal") is the middle term.

FIGURES AND MOODS. Syllogisms are divided into four figures, according to the placing of the middle term in the two premises. In the first figure the middle term is subject in the major premise and predicate in the minor; in the second figure predicate in both; in the third figure subject in both; in the fourth predicate in the major and subject in the minor. The following schemata, with *P* for the major term, *S* for the minor, and *M* for the middle, sum up these distinctions:

Figure 1	Figure 2	Figure 3	Figure 4
$M - P$	$P - M$	$M - P$	$P - M$
$S - M$	$S - M$	$M - S$	$M - S$
$S - P$	$S - P$	$S - P$	$S - P$

Within each figure, syllogisms are further divided into *moods,* according to the quantity and quality of the propositions they contain.

Not all of the theoretically possible combinations of propositions related as above constitute valid syllogisms, sequences in which the third proposition really follows from the other two. For example, "Every man is an animal; some horse is an animal; therefore, no man is a horse" (mood AIE in Figure 2) is completely inconsequent (even though all three propositions happen in this case to be true). During the Middle Ages those syllogistic moods that are valid acquired certain short names, with the mood indicated by the vowels, and all of them were put together in a piece of mnemonic doggerel, of which one of the later versions is the following:

Barbara, Celarent, Darii, Ferioque prioris;
Cesare, Camestres, Festino, Baroco secundae;
Tertia *Darapti, Disamis, Datisi, Felapton,*
Bocardo, Ferison habet. Quarta insuper addit
Bramantip, Camenes, Dimaris, Fesapo, Fresison.

Here Bocardo, for example, means the mood OAO in Figure 3, of which an illustration (C. S. Peirce's example) would be

Some patriarch (viz., Enoch) is not mortal;
Every patriarch is a man;
Therefore, some man is not mortal.

There is also a group of moods (Barbari and Celaront in Figure 1, Cesaro and Camestrop in Figure 2, Camenop in Figure 4) in which a merely particular conclusion is drawn although the premises would warrant our going further and making the conclusion universal (the "subaltern" moods). The Ramists added special moods involving singulars (if we write S and N for affirmative and negative singulars, we have ASS and ESN in Figure 1, ANN and ESN in Figure 2 and SSI and NSO in Figure 3). It may be noted that every syllogism must have at least one universal premise, except for SSI and NSO in Figure 3—the so-called expository syllogisms, for example, "Enoch is not mortal; Enoch is a patriarch; therefore, not every patriarch is mortal." Moreover, every syllogism must have at least one affirmative premise, and if either premise is negative or particular, the conclusion must be negative or particular, as the case may be ("the conclusion follows the weaker premise," as Theophrastus put it, negatives and particulars being considered weaker than affirmatives and universals).

REDUCTION. The mnemonic verses serve to indicate how the valid moods of the later figures may be "reduced" to those of Figure 1—that is, how we may derive their conclusions from their premises without using any syllogistic reasoning of other than the first-figure type. (This amounts, in modern terms, to proving their validity from that of the first-figure moods taken as axiomatic.) In the second-figure mood Cesare, for example, the letter *s* after the first *e* indicates that if we *simply convert* the major premise we will have a pair of premises from which we can deduce the required conclusion in Figure 1, and the initial letter *C* indicates that the first-figure mood employed will be Celarent. An example of a syllogism in Cesare (EAE in Figure 2) would be

> No horse is a man;
> Every psychopath is a man;
> Therefore, no psychopath is a horse.

This conclusion may equally be obtained from these premises by proceeding as follows:

No horse is a man—*s*⟶No man is a horse;
Every psychopath is a man → Every psychopath is a man;
Therefore, no psychopath is a horse.

Here the right-hand syllogism, in which the first premise is obtained from the given major by simple conversion and the second is just the given minor unaltered, is in the mood Celarent in the first figure. Festino "reduces" similarly to Ferio, and Datisi and Ferison (in the third figure) reduce to Darii and Ferio, though in the third-figure cases it is the minor premise that must be simply converted. Darapti and Felapton reduce to Darii and Ferio by conversion of the minor premise, not simply, but *per accidens* (this is indicated by the *s* of the other moods being changed to *p*).

Camestres (Figure 2) and Disamis (Figure 3) are a little more complicated. Here we have not only an *s,* for the simple conversion of a premise, but also an *m,* indicating that the premises must be transposed, and a further *s* at the end because the transposed premises yield, in Figure 1, not the required conclusion but rather its converse, from which the required conclusion must be obtained by a further conversion at the end of the process. An example in Disamis would be the following:

> Some men are liars;
> All men are automata;
> Therefore, some automata are liars.

If we convert the major premise and transpose the two, we obtain the new pair

> All men are automata;
> Some liars are men,

and from these we may obtain in the first-figure mood Darii not immediately the conclusion "Some automata are liars" but rather "Some liars are automata," from which, however, "Some automata are liars" does follow by simple conversion.

Baroco and Bocardo are different again. In both of them neither premise is capable of simple conversion, and if we convert the A premises *per accidens* we obtain pairs IO and OI, and there are no valid first-figure moods with such premises—in fact, no valid moods at all with two particular premises. We therefore show that the conclusion follows from the premises by the device called reductio ad absurdum. That is, we assume for the sake of argument that the conclusion does not follow from the premises—that is, that the premises can be true and the conclusion false—and from this assumption, using first-figure reasoning alone, we deduce impossible consequences. The assumption, therefore, cannot stand, so the conclusion does after all follow from its premises.

Take, for example, the following syllogism in Baroco (AOO in Figure 2):

> Every man is mortal;
> Some patriarch (viz., Enoch) is not mortal;
> Therefore, some patriarch is not a man.

Suppose the premises are true and the conclusion is not. Then we have

> (1) Every man is mortal;
> (2) Some patriarch is not mortal;
> (3) Every patriarch is a man.

(This is the contradictory of the conclusion.) But from (1) and (3), in the first-figure mood Barbara, we may infer

(4) Every patriarch is mortal.

However, the combination of (2) and (4) is impossible. Hence, we can have both (1) and (2) only if we drop (3)—that is, if we accept the conclusion of the given second-figure syllogism.

It is possible to "reduce" all the second-figure and third-figure moods to Figure 1 by this last method, and although this procedure is a little complicated, it brings out better than the other reductions the essential character of second-figure and third-figure reasoning. Figure 1 is governed by what is called the *dictum de omni et nullo,* the principle that what applies to all or none of the objects in a given class will apply or not apply (as the case may be) to any given member or subclass of this class. As Immanuel Kant preferred to put it, first-figure reasoning expresses the subsumption of cases under a rule—the major premise states some affirmative or negative rule ("Every man is mortal," "No man will live forever"), the minor asserts that something is a case, or some things are cases, to which this rule applies ("Enoch and Elijah are men"), and the conclusion states the result of applying the rule to the given case or cases ("Enoch and Elijah are mortal," "Enoch and Elijah will not live forever"). Hence, in Figure 1 the major premise is always universal (that being how rules are expressed) and the minor affirmative ("Something *is* a case").

Second-figure reasoning also begins with the statement of a rule ("Every man is mortal") but in the minor premise denies that we have with a given example the result which the rule prescribes ("Enoch and Elijah are *not* mortal," "Enoch and Elijah *will* live forever") and concludes that we do not have a case to which the rule applies ("Enoch and Elijah cannot be men"). It combines, in effect, the first-figure major with the contradictory of the first-figure conclusion to obtain the contradictory of the first-figure minor (compare the "reduction" of Baroco). A second-figure syllogism, in consequence, must have a universal major, premises opposed in quality, and a negative conclusion. Its practical uses are in refuting hypotheses, as in medicine or detection ("Whoever has measles has spots, and this child has no spots, so he does not have measles"; "Whoever killed *X* was a person of great strength, and *Y* is not such a person, so *Y* did not kill *X*").

In the third figure we begin by asserting that something or other does not exhibit the result which a proposed rule would give ("Enoch and Elijah are *not* mortal," "Enoch and Elijah *will* live forever"), go on to say that we nevertheless do have here a case or cases to which the rule would apply if true ("Enoch and Elijah *are* men"), and conclude that the rule is not true ("Not all men are mortal," "Some men do live forever"). A third-figure syllogism, consequently, has an affirmative minor (the thing is a case) and a particular conclusion (the contradictory of a universal being a particular); its use is to confute rashly assumed rules, such as proposed scientific laws.

This rather neat system of interrelations (first clearly brought out by C. S. Peirce) concerns only the first three figures; it was not until the later Middle Ages, in fact, that a distinct fourth figure was recognized. The common division of figures assumes that we are considering completed syllogisms, with the conclusion (and its subject and predicate) already before us; however, the question Aristotle originally put to himself was not "Which completed syllogisms are valid?" but "Which pairs of premises will yield a syllogistic conclusion?" Starting at this end, we cannot distinguish major and minor premises as those containing, respectively, the predicate and subject of the conclusion. Aristotle distinguished them, in the first figure, by their comparative comprehensiveness and mentioned what we now call the fourth-figure moods as odd cases in which first-figure premises will yield a conclusion wherein the "minor" term is predicated of the "major." Earlier versions of the mnemonic lines accordingly list the fourth-figure moods with the first-figure ones and (since the premises are thought of as being in the first-figure order) give them slightly different names (Baralipton, Celantes, Dabitis, Fapesmo, Frisesomorum).

DISTRIBUTION OF TERMS. Terms may occur in A-, E-, I-, and O-propositions as distributed or as undistributed. The rule is that universals distribute their subjects and particulars distribute their predicates, but what this means is seldom very satisfactorily explained. It is often said, for example, that a distributed term refers to all, and an undistributed term to only a part, of its extension. But in what way does "Some men are mortal," for example, refer to only a part of the class of men? Any man whatever will do to verify it; if any man whatever turns out to be mortal, "Some men are mortal" is true. What the traditional writers were trying to express seems to be something of the following sort: A term t is distributed in a proposition $f(t)$ if and only if it is replaceable in $f(t)$, without loss of truth, by any term "falling under it" in the way that a species falls under a genus. Thus, "man" is distributed in

Every man is an animal;
No man is a horse;
No horse is a man;
Some animal is not a man,

since these respectively imply, say,

Every blind man is an animal;
No blind man is a horse;
No horse is a blind man;
Some animal is not a blind man.

On the other hand, it is undistributed in

Some man is keen-sighted;
Some man is not disabled;
Every Frenchman is a man;
Some keen-sighted animal is a man,

since these do not respectively imply

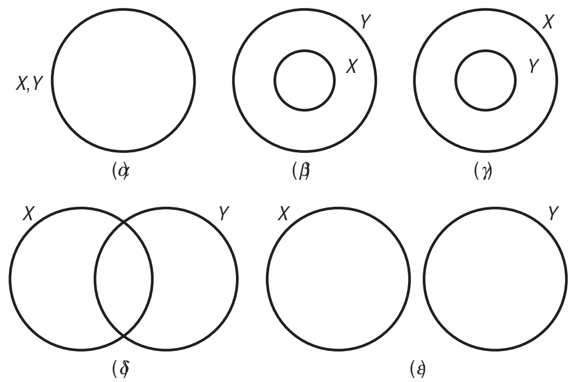
Some blind man is keen-sighted;
Some blind man is not disabled;
Every Frenchman is a blind man;
Some keen-sighted animal is a blind man.

In this sense A- and E- propositions do distribute their subjects and E- and O-propositions their predicates. John Anderson pointed out that the four positive results above may be established syllogistically, given that all the members of a species (using the term widely) are members of its genus—in the given case, that all blind men are men. From "Every man is an animal" and "Every blind man is a man," "Every blind man is an animal" follows in Barbara; with the second example the syllogism is in Celarent, with the third in Camestres, with the fourth in Baroco. Note, however, that the mere prefixing of "every" to a term is not in itself sufficient to secure its "distribution" in the above sense; for example, "man" is not distributed in "Not every man is disabled," since this does not imply "Not every blind man is disabled."

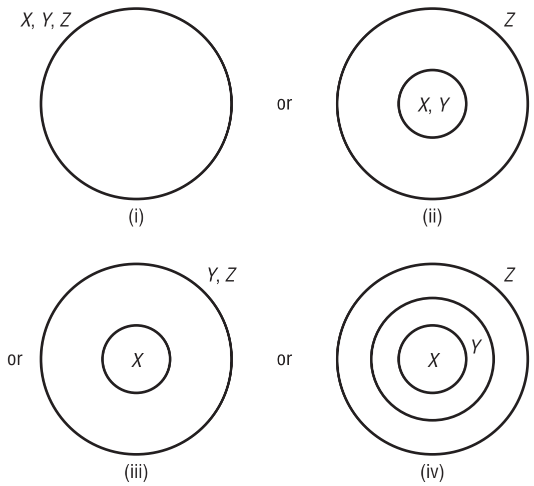
For a syllogism to be valid the middle term must be distributed at least once, and any term distributed in the conclusion must be distributed in its premise (although there is no harm in a term's being distributed in its premise but not in the conclusion). Many syllogisms can quickly be shown to be fallacious by the application of these rules. "Every man is an animal; every horse is an animal; therefore, every horse is a man," for example, fails to distribute the middle term "animal," and it is clear that any second-figure syllogism with two affirmative premises would have the same fault (since in the second figure the middle term is predicate twice, and affirmatives do not distribute their predicates). Other special rules for the different figures, such as that in Figures 1 and 3 the minor premise must be affirmative, can be similarly proved from the rules of distribution together with the rules of quality (that a valid syllogism does not have two negative premises, and that a conclusion is negative if and only if one premise is). Logicians have endeavored to prove some of these rules from others and to reduce the number of unproved rules to a minimum.

EULER'S DIAGRAMS. One device for checking the validity of syllogistic inferences is the use of certain diagrams attributed to the seventeenth-century mathematician Leonhard Euler, although their accurate employment seems to date rather from J. D. Gergonne, in the early nineteenth century.

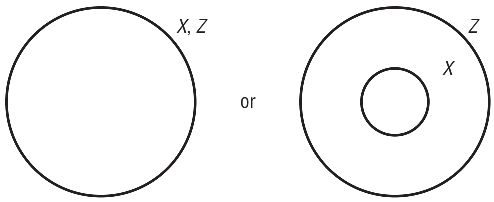
From the traditional laws of opposition and conversion it can be shown that the extensions of any pair of terms *X*, *Y* will be related in one or another of five ways: (α) every *X* is a *Y* and every *Y* is an *X*, that is, their extensions coincide; or (*β*) every *X* is a *Y*, but not every *Y* is an *X*, that is, the *X*'s form a proper part of the *Y*'s; or (*γ*) every *Y* is an *X*, but not every *X* is a *Y*, that is, the *Y*'s form a proper part of the *X*'s; or (*δ*) some but not all *X*'s are *Y*'s and some but not all *Y*'s are *X*'s, that is, the *X*'s and *Y*'s overlap; or (*ϵ*) no *X*'s are *Y*'s and so no *Y*'s are *X*'s, that is, the *X*'s and *Y*'s are mutually exclusive. These five cases are represented by the following diagrams:



"Every *X* is a *Y*" (A) is true if and only if we have either (*α*) or (*β*); "Some *X* is not a *Y*" (O) if and only if we have either (*γ*) or (*δ*) or (*ϵ*); "No *X* is a *Y*" (E) if and only if we have (*ϵ*); and "Some *X* is a *Y*" (I) if and only if we have either (*α*) or (*β*) or (*γ*) or (*δ*). From these facts it follows that A and O are in no case true together and in no case false together, and similarly for E and I; that I is true in every case in which A is and also in two cases in which A is not, and similarly for O and E; that A and E are in no case true together but in two cases are both false; and that O and I are in no case both false but in two cases are both true. After working out analogous truth conditions for the forms with reversed terms, we will see that they are the same for the two I's and the two E's (showing that these are simply convertible) but not for the two A's and the two O's (showing that these are not). Given which of the five relations holds between X and Y and which between Y and Z, we can work out by compounding diagrams what will be the possible relations between X and Z. For example, if we know that every X is a Y and every Y a Z, then we must have either $(\alpha)XY$ and $(\alpha)YZ$ or $(\alpha)XY$ and $(\beta)YZ$ or $(\beta)XY$ and $(\alpha)YZ$ or $(\beta)XY$ and $(\beta)YZ$; that is, we must have



Inspection will show that for X and Z we have in every case either



so in every case every X is a Z. Hence, Barbara is valid.

When employing this procedure it is essential to consider all the possible cases involved. Barbara is not validated, for example, by considering case (iv) alone, as popular expositions of this method sometimes suggest.

POLYSYLLOGISMS, ENTHYMEMES, AND INDUCTION. In an extended argument the conclusion of one inference may be used as a premise of another, and the conclusion of that as premise of a third, and so on. In presenting such an argument we may simply omit the intermediate steps and list all the premises together. For example, the sequence of categorical syllogisms "Every *X* is a *Y*, and every *Y* is a *Z*, so every *X* is a *Z*; and every *Z* is a *T*, so every *X* is a *T*" may be condensed to "Every *X* is a *Y*, every *Y* is a *Z*, and every *Z* is a *T*; therefore, every *X* is a *T*." Such a condensed chain of syllogisms is called a polysyllogism or sorites. The theory of chains of two syllogisms was thoroughly studied by Galen, as reported in an ancient passage unearthed by Jan Łukasiewicz. Galen showed that the only combinations of the Aristotelian three figures that could be thus used were 1 and 1, 1 and 2, 1 and 3, and 2 and 3. His discovery of these four types of compound syllogism was misunderstood by later writers as an anticipation of the view that single syllogisms may be of four figures.

Even when it is not a conclusion from other premises already stated, one of the premises of an inference may often be informally omitted (for example, "Enoch and Elijah are men; therefore, Enoch and Elijah are mortals"). Such a truncated inference is often called an enthymeme. This is not Aristotle's own use of the term, though he did mention that a premise is often omitted in the statement of an enthymeme in his sense. An Aristotelian enthymeme is a merely probable argument—that is, one in which the conclusion does not strictly follow from the premises but is merely made more likely by them. When the claim made for an argument is thus reduced, the normal rules may be relaxed in certain directions; in particular, the second and third figures may be used to yield more than merely negative results. Thus, Figure 2 may be used not only to prove that something is not a case falling under a given rule but also to suggest that it is one—to use a modern example:

> Any collection of particles whose movement is accelerated will occupy more space than it did;
>
> A heated gas will occupy more space than it did;
>
> Therefore, a heated gas may be a collection of particles whose movement is accelerated.

Figure 3 may be similarly used not only to prove that some rule does not hold universally but also to suggest that it does hold universally—for instance:

> *X, Y, Z* are all of them white;
> *X, Y, Z* are all of them swans;
> Therefore, perhaps all swans are white.

If the second premise here is strengthened to "*X, Y, Z* are all the swans there are," the conclusion will follow without any "perhaps" (of course, the new premise is in this case a false one, and the conclusion is also false). The form of inference

> *X, Y, Z,* etc., are all of them *P*'s;
> *X, Y, Z,* etc., are all the *S*'s there are;
> Therefore, all *S*'s are *P*'s

was called by Aristotle "induction"; more accurately, he used this term for a similar passage from all the subspecies to their genus ("The *X*'s, the *Y*'s, and the *Z*'s are all of them *P*'s and are all the *S*'s; therefore, …"). He observed that the "conversion" of the second premise to "All the *S*'s are the *X*'s, the *Y*'s, and the *Z*'s" will turn such an induction into a syllogism in Barbara.

The term *induction* being extended in the more recent tradition to cover the merely probable inference given just previously, we distinguish Aristotelian induction by calling it "formal" or "perfect" induction or (as W. E. Johnson called it) "summary" induction. The Figure 2 type of merely probable inference is one of the things meant by the term "argument from"—or "by"—"analogy" (or just "analogy"); C. S. Peirce called it "hypothesis."

SKEPTICAL CRITICISMS OF SYLLOGISTIC REASONING. In the latter part of the nineteenth century, under the influence of J. S. Mill, textbooks of the traditional type came to have two main divisions, "formal" or "deductive" logic (dealt with more or less as above) and "inductive" logic or "scientific method." With the details of inductive logic we are not concerned here, but we may glance at the view of some writers that merely probable induction and analogy are the only genuine types of reasoning, "formal" or syllogistic reasoning being useless or spurious because it is inevitably circular, assuming in the premises what it sets out to prove as the conclusion.

The second-century skeptic Sextus Empiricus suggested that in the syllogism "Every man is an animal; Socrates is a man; therefore, Socrates is an animal," the only way to establish the major premise is by induction; however, if the induction is incomplete the examination of a new instance—for example, of Socrates—might prove it false, and if it is complete the conclusion ("Socrates is an animal") must already have been used in establishing it. This argument was repeated by such writers as George Campbell, in the eighteenth century, who supplemented it with another, to cover the case in which the major is established not by induction but simply by definition or linguistic convention: "Of course every man is an animal, for being an animal is part of what we mean by being a man." In this case it is the minor premise, "Socrates is a man," that cannot be established without first establishing the conclusion (that he is an animal). The same point was urged by another Scottish philosopher, Thomas Brown. It is allied to an argument used by Sextus to show not that syllogism is circular but that the major premise is superfluous. If, he said, every man is an animal because it follows from an object's being a man that it is an animal, then the allegedly enthymematic "Socrates is a man; therefore, Socrates is an animal" must be valid as it stands.

Richard Whately, answering Campbell's arguments in the early nineteenth century, complained that Campbell had confined himself to examples in which the syllogistic argument was indeed superfluous and countered them with some in which it was not—for example, the case of some laborers, ignorant of the fact that all horned animals are ruminant, digging up a skeleton which they, but not a distant naturalist, could see to be horned, the laborers and the naturalist thus separately providing premises which were both required to obtain the conclusion that the skeleton was of a ruminant animal. Whately admitted that the sense in which we may make a "discovery" by drawing a syllogistic conclusion is different from that in which we make a discovery by observation, but it can be a genuine discovery none the less; there are "logical" as well as "physical" discoveries.

After Whately, J. S. Mill took up the argument, but it is not entirely clear what side he was on. Sometimes he treated a universal major as already asserting, among other things, the conclusion:

> Whoever pronounces the words, All men are mortal, has affirmed that Socrates is mortal, though he may never have heard of Socrates; for since Socrates, whether known to be so or not, really is a man, he is included in the words, All men, and in every assertion of which they are the subject. (*System of Logic,* Book II, Ch. 3, p. 8, note)

"Included in the *meaning* of the words," he must have meant (for it is obvious that neither Socrates the man nor "Socrates," his name, forms any part of the words "All men"), but this contradicts Mill's own insistence that the meaning of general terms like "men" lies wholly in their "connotation" and that "All men are mortal" means that wherever the attributes of humanity are present, mortality is present, too. He rightly chided Brown, who thought that the meaning of "Socrates is mortal" (like that of "Socrates is an animal") is already contained in the minor premise "Socrates is a man," for failing to distinguish the actual connotation of "man" (i.e., the attributes by which its application is determined) from other attributes (such as mortality) which we may empirically discover these to be attended with, but his own view in the passage cited is similarly negligent.

Mill's main point, however, is different and more defensible. When careful and extensive observation warrants the conclusion that, say, all men are mortal, and we then observe that the duke of Wellington is a man and conclude that he is therefore mortal, we have in effect an induction followed by a syllogism. Mill pointed out that if this procedure is justified at all, the introduction of the syllogistic major is superfluous. For if the original body of evidence really does warrant the inference that all men are mortal, it is certainly sufficient to warrant the inference that the duke of Wellington is mortal, given that he is a man. In other words, if we really are justified in the move from particular observations to the general proposition, and from there to new particulars, we would be equally justified in moving directly "from particulars to particulars."

What the syllogistic major does, Mill argued, is simply to sum up in a single formula the entire class of inferences to new particulars which the evidence warrants. That is, "All men are mortal" means, in effect, that if we ever find anyone to be a man we are justified in inferring, from the observations we have previously amassed, that he is mortal. "The conclusion is not an inference drawn *from* the formula"—that is, from "All men are mortal" thus understood—"but an inference drawn *according* to the formula" (ibid., p. 4). Mill here anticipated Gilbert Ryle's treatment of "lawlike statements" as "inference licenses" and echoed Sextus's point that it is inconsistent to require that such licenses be added to the premises of the inferences they permit, since what they license is precisely the drawing of the conclusion from those premises.

Mill in fact here shifted the discussion from Sextus's first skeptical "topic" to his second—from the charge of circularity to the question of what distinguishes a rule of inference from a premise. On this point more was said later in the nineteenth century by C. S. Peirce. Peirce, like Mill, distinguished sharply between the premise or premises from which, and the "leading principle" according to which, a conclusion is drawn. He also noted, as did Mill, that what is traditionally counted as a premise may function in practice as a "leading principle." But it need not, and, indeed, what is traditionally counted as a "leading principle" (say the *dictum de omni et nullo*) may sometimes be, conversely, treated in practice as a premise. Certainly, since *all men are mortal* (leading principle 1), we are justified in inferring the mortality of Socrates (or the duke of Wellington, or Elijah) from his humanity. But equally, since *all members of any class are also members of any class that contains the former as a subclass* (leading principle 2), we are justified in inferring the mortality of Socrates from his being a man *and* from men's being a subclass of mortals. For the very same reason (that all members of any class are also members of any class that contains the former as a subclass) we are justified in inferring the mortality of Socrates from his being a member of a subclass of the class of mortals *and* from the membership of any member of a class in all classes of which it is a subclass. In this last example we have one and the same proposition functioning as a premise and as a leading principle in the same inference (not merely, like "All men are mortal" in the preceding two examples, as a leading principle in one and a premise in another); to be capable of this, Peirce thought, is the mark of a "logical" leading principle.

It is not certain that Peirce's method of distinguishing "logical" from other sorts of "leading principles" will bear inspection. However, he seems to have established his basic point, that what it would be fatal to require in all cases—the treatment of a leading principle as a premise—we may safely permit in some. There may be useful and valid reasoning about subjects of all degrees of abstraction, including logic itself.

HYPOTHETICAL AND DISJUNCTIVE SYLLOGISMS. Traditional textbooks, aside from developing the theory of categorical propositions and syllogisms, have a brief appendix mentioning "hypothetical" (or "conditional") and "disjunctive" propositions and certain "syllogisms" to which they give rise.

"Hypothetical" syllogisms are divided into "pure," in which premises and conclusion are all of the form "If p then q" (notably the syllogism "If p then q, and if q then r; therefore, if p then r," analogous to Barbara), and "mixed," in which only one premise is hypothetical and the other premise and the conclusion are categorical. The mixed hypothetical syllogism has two valid "moods":

(1) *Modus ponendo ponens*: If p then q, and p; therefore, q.

(2) *Modus ponendo tollens*: If p then q, but not q; therefore, not p.

In both these moods the hypothetical premise is called the major, the categorical the minor. *Ponere,* in the mood names, means to affirm, *tollere* to deny. In (1), by affirming the antecedent of the hypothetical we are led to affirm its consequent; in (2), by denying its consequent we are led to deny its antecedent. The fallacies of "affirming the consequent" and "denying the antecedent" (i.e., of doing these things *to start with,* in the minor premise) consist in reversing these procedures—that is, in arguing "If p then q, and q; therefore, p" and "If p then q, but not p; therefore, not q."

"Disjunctive" syllogisms—that is, ones involving "Either-or" propositions—have the following two "mixed" moods:

(3) *Modus tollendo ponens*: Either *p* or *q,* but not *p*; therefore, *q* (or, but not *q*; therefore, *p*).

(4) *Modus ponendo tollens*: Either *p* or *q,* and *p*; therefore, not *q* (or, and *q*; therefore, not *p*).

Mood (4) is valid only if "Either *p* or *q*" is interpreted "exclusively"—that is, as meaning "Either *p* or *q* but not both"—whereas (3) is valid even if it is interpreted as "Either *p* or *q* or both." There is also a *modus tollendo ponens* with the simple "Not both *p* and *q*" as major and the rest as in (4).

DILEMMAS. Hypothetical and disjunctive premises may combine to yield a categorical conclusion in the *dilemma,* or "horned" syllogism (*syllogismus cornutus*), with its two forms:

(5) *Constructive*: If *p* then *r,* and if *q* then *r,* but either *p* or *q*; therefore, *r.*

(6) *Destructive*: If *p* then *q,* and if *p* then *r,* but either not *q* or not *r*; therefore, not *p.*

These basic forms have a number of variations; for instance, *q* in (5) may be simply "not *p,*" making the disjunctive premise the logical truism "Either *p* or not *p*"; or *p* may imply *r* and *q* imply *s,* giving as conclusion "Either *r* or *s*" rather than the categorical *r*; or the disjunctive premise may be conditionalized to "If *s* then either *p* or *q,*" making the conclusion "If *s* then *r.*"

A typical dilemma is that put by Protagoras to Euathlus, whom he had trained as a lawyer on the understanding that he would be paid a fee as soon as his pupil won a case. When the pupil simply engaged in no litigation at all, Protagoras sued him for the fee. His argument was "If Euathlus wins this case, he must pay my fee by our agreement, and if he loses it he must pay it by the judge's decision (for that is what losing this case would mean), but he must either win or lose the case; therefore, in either case he must pay."

"Escaping between the horns" of a dilemma is denying the disjunctive premise; for example, Euathlus might have argued that he would neither win nor lose the case if the judge refused to make any decision. "Taking a dilemma by the horns" is admitting the disjunction but denying one of the implications, as Euathlus might have done by arguing that if he won he would still not be bound by the agreement to pay Protagoras, because this was not the sort of case intended in the agreement. "Rebutting" a dilemma is constructing another dilemma drawing upon the same body of facts but leading to an opposite conclusion. This is what Euathlus did, arguing that if he won the case he would be dispensed from paying by the judge's decision, and if he lost it the agreement would dispense him, so either way he was dispensed from paying. Rebuttal, however, is possible only if one of the other moves (though it may not be clear which) is also possible, for a single set of premises can lead by equally valid arguments to contradictory conclusions only if they contain some fault in themselves.

Dilemmatic reasoning obtains a categorical conclusion from hypothetical and disjunctive premises; the Port-Royalists pointed out that we may also obtain hypothetical conclusions from categorical premises. For in any categorical syllogism we may pass directly from one of the premises to the conclusion stated not categorically but conditionally on the truth of the other premise; for instance, from "Every man is mortal" we may infer that if Socrates is a man he is mortal, and from "Socrates is a man" that if every man is mortal Socrates is, and similarly with all other syllogisms. This "rule of conditionalization" is much used in certain modern logical systems.

TRADITIONAL AND MODERN LOGIC

Not only the "rule of conditionalization" but the whole subject of hypothetical and disjunctive reasoning fits more comfortably into modern than into traditional logic, being an inheritance from the Stoics, the first "modern" logicians, rather than from Aristotle. Traditionalists have often been worried at its finding any place at all in their general *corpus* and have sometimes attempted to justify it by "reducing" hypothetical and disjunctive propositions and syllogisms to "categorical" ones.

Disjunctives, to begin with, may be eliminated as a distinct form by equating "Either *p* or *q*" with the conditional "If not *p* then *q,*" and the conditional form does sometimes look as if it might be a mere verbal variant of the categorical universal. This last is especially true where the conditional is introduced not by the plain "if" but by "if ever" or "if any"; "If ever a gas is heated it expands" and "If any gas is heated it expands" seem simply variants of "Every heated gas expands." But here the antecedent and consequent of the conditional are not, as J. N. Keynes put it, complete propositions with an "independent import"—"it expands" is not on its own a comprehensible sentence; the "it" refers back to the heated gas of the antecedent. Keynes suggested that the term *conditional* be used for precisely this type of "If-then" statement and the term *true hypothetical* confined to cases in which the antecedent and consequent do have "independent import," such as "If Socrates is damned, then there is no justice in heaven." And the representation of "true hypotheticals" as categorical universals is not easy.

In modern logic, from the Stoics through some of the medieval *moderni* to the "logisticians" of our own century, "the stone which the builders rejected has been made the head of the corner." "Pure hypotheticals," together with other forms in which entire propositions are linked by various "connectives," have been made the subject of the most elementary part of logic, the propositional calculus. Aristotelian universals and particulars are built out of these forms (by means of prefixes called "quantifiers") rather than vice versa. (Details are given in the entries Logic, Modern and Russell, Bertrand, section on logic and mathematics.) The essential procedure is to read "Every *A* is a *B*" as "For every individual *x*, if *x* is an *A* then *x* is a *B*" and "Some *A* is a *B*" as "For some individual *x*, *x* is an *A* and *x* is a *B*." Here, instead of a Keynesian "conditional" being explained as a categorical universal in disguise, the explanation is reversed, and the components which, as Keynes said, are "not propositions of independent import" are represented as "propositional functions" in which the place taken in a genuine proposition by an individual name is taken by a variable ("bound" by the initial quantifier "for all *x*"). But the "if" which links these components is the very same "if" which in the "pure hypotheticals" of the propositional calculus links genuine propositions. This "if" is not explained in terms of anything else (except perhaps other connectives) but is taken as fundamental.

In this way the traditional themes are not banished from modern logic but are incorporated into a much larger subject. When the Aristotelian forms are thus interpreted, however, their laws seem to require modification at some points. In particular, the A-form "For any *x*, if *x* is an *A* then *x* is a *B*" does not seem to imply the I-form "For some *x*, *x* is an *A* and *x* is a *B*," for the former does not imply that any *x* in fact is an *A* (it says only that if any *x* is an *A* it is a *B*), whereas the latter does imply this (if some *x* both is an *A* and is a *B*, then that *x* is at least an *A*). This eliminates inference by subalternation and whatever else in the traditional theory depends on it, such as subaltern conversion and syllogisms, like Darapti, which require this for reduction to Figure 1.

Modern logic, however, is not at all monolithic in character, and the sketch just given is a little stylized, depicting modern logic not as a living discipline but rather as a new "tradition" that has displaced the old and against which there are already dissentient voices that give the older tradition a measure of justification (rather like that accorded to pre-Copernican astronomy by the more radical forms of relativity theory). We cannot go back to the prison that would confine all logic to the Aristotelian syllogism, but it is possible to defend (*a*) something like the view that the form "Every *X* is a *Y*" is more fundamental than either "For all *x*, *f*(*x*)" or "If *p* then *q*" and (*b*) the traditional ignoring (in inference by subalternation, etc.) of terms that have no application.

As to (*a*), we now know how to define both "for all *x*" and "if" in terms of a single undefined logical operator which amounts to "for all *x*, if"; for we can take as our fundamental logical complex the form "Anything such that α is such that β" and read "If *p* then *q*" as the special case of this in which α and β are "propositions with independent import," and "For all *x*, β" as the special case in which α is logically true anyway (for instance, in which it has the form "Anything such that β is such that β") and so can he ignored as a "condition" of β's truth. C. S. Peirce—at almost every point the most imaginative and flexible of the "moderns," although he died in 1914—always regarded some such reduction as possible in principle and saw the difference between the "terms" out of which categorical propositions are constructed and the "propositions" out of which we construct hypotheticals as a point of little logical importance.

Peirce, moreover, gave a highly modern justification for the traditional view that within syllogistic logic only the first figure is strictly necessary. Traditional methods of "reducing" other figures to the first do indeed involve another form of inference, namely conversion, and although this can be represented as a kind of enthymematic syllogism, it comes out as syllogism that is already in the second and third figures. For we do it by letting the term *B* be the same as *A* in the two syllogisms

No *C* is a *B* (i.e., an *A*);
Every *A* is a *B* (i.e., an *A*);
Therefore, no *A* is a *C*

(Cesare, Figure 2) and

Every *B* (i.e., *A*) is an *A*;
Some *B* (i.e., *A*) is a *C*;
Therefore, some *C* is an *A*

(Datisi, Figure 3). The replacement of *B* by *A* turns the universal affirmative premise into the logical truism "Every *A* is an *A*," which can be dropped, and the conclusion into the converse of the remaining premise.

We can, however, derive second-figure syllogisms from first-figure ones by a variant of the reductio ad absurdum method, employing nothing but Barbara in its terminal and propositional forms, the forms

(*a*) Every *A* is a *B*, and every *B* is a *C*; therefore, every *A* is a *C*; and

(*b*) If *p* then *q*, and if *q* then *r*; therefore, if *p* then *r*,

together with freedom to rearrange our premises and to "conditionalize" and "deconditionalize" conclusions, that is, to make such passages as that from (*a*) to, and to (*a*) from,

(*c*) Every *A* is a *B*; therefore, if every *B* is. a *C* then every *A* is a *C*

and from (*b*) to, and to (*b*) from,

(*d*) If *p* then *q*; therefore, if (if *q* then *r*) then if *p* then *r*.

As a special case of (*d*) we have

(*e*) If every *B* is a *C* then every *A* is a *C*; therefore, if (if every *A* is a *C* I am much mistaken) then if every *B* is a *C* I am much mistaken.

Forms (*c*) and (*e*) will take us from the premise to the conclusion of

(*f*) Every *A* is a *B*; therefore, if (if every *A* is a *C* I am much mistaken) then if every *B* is a *C* I am much mistaken.

But "If *X* then I am very much mistaken" just amounts to "Not *X*," and (*f*) therefore amounts to

(*g*) Every *A* is a *B*; therefore, if not every *A* is a *C*, not every *B* is a *C*,

that is, a conditionalized form of Bocardo, Figure 3.

The equation of "Not *X*" with "If *X* then I am much mistaken" is Peirce's variant, at this point, of one account of denial. It makes it possible to present the other traditional forms as complexes of "if" and "every" (and "if" and "every," as was shown, are basically the same form of linkage), as follows:

Not every *X* is a *Y* (O) = If every *X* is a *Y* I am much mistaken.

No *X* is a *Y* (E) = Every *X* is not-a-*Y* = Every *X* is such that if it is a *Y* I am much mistaken.

Some *X* is a *Y* (I) = Not (no *X* is a *Y*) = If every *X* is such that if it is a *Y* I am much mistaken, then I am much mistaken.

Syllogisms, in all figures, involving these forms are derivable from Barbara by methods similar to that used to obtain Bocardo above, although the derivations will often be more complicated than the one given. For some of them we require Barbara in yet another form besides (*a*) and (*b*) above, namely the mixed terminal and propositional

Every *X* is a *Y*; therefore, anything such that if it is a *Y*, then *p*, is such that if it is an *X*, then *p*,

and a kind of terminal principle of *modus ponens*,

Whatever is an *X* is a thing such that if its being an *X* implies that *p*, then *p*.

Modern logic will not admit that Barbara gives us all the logic there is, but its techniques do bring out anew the extreme fecundity of this ancient form.

Turning now to the failure of certain traditional forms of inference when terms without application are employed, there have been two more recent lines of attack on the view that traditional logic is simply "wrong" in accepting such forms as "Every *X* is a *Y*; therefore, some *X* is a *Y*." One, used by Łukasiewicz, is formalistic in character; it is a mistake, Łukasiewicz says, to interpret the traditional propositional forms in terms of modern quantification theory in the ways above indicated, or in any other ways. If we just take them as they stand, without interpretation, we can find a rigorous symbolism for them and show that the traditional laws form a self-consistent system; worries about their interpretation are extralogical. T. J. Smiley, on the other hand, thinks the interpretation of the traditional forms in quantification theory worth attempting but points out that quantification theory, as now developed, offers us wider choices of interpretation than was once thought. For quantification theory now handles cases of the form "For all *x*, *f*(*x*)" in which the range of the variable *x* is restricted to objects of some particular sort, each sort of object having its own type of variable. We need not, therefore, interpret "Every man is mortal," say, in the standard modern way as "For any individual object *x*, if that object is human it is mortal" but may read it, rather, as "For any *human* individual *m*, that human individual is mortal" (with no "ifs" about it). This interpretation, when embedded in a suitable theory of "many-sorted" quantification, will yield all the traditional results.

See also Negation.

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LOGIC, TRANSCENDENTAL

See *Kant, Immanuel*

LOGICAL ATOMISM

See *Analysis, Philosophical; Russell, Bertrand Arthur William; Wittgenstein, Ludwig Josef Johann*

LOGICAL EMPIRICISM

See *Logical Positivism*

LOGICAL FORM

One can use sentences to present arguments, some of which are valid. Sentences are complex linguistic expressions that exhibit grammatical structure. And the grammatical properties of sentences need not be obvious. As discussed in this entry, certain arguments seem to be valid because the relevant premises and conclusions exhibit nonobvious logical structure. But this raises questions about what logical structure is and how it is related to grammatical structure.

PATTERNS OF REASONING

An ancient thought is that premises and conclusions have parts and that valid arguments exhibit valid forms, like the following: **Q** if **P**, and **P**; so **Q**. One can say that the variables (in bold) range over propositions, leaving it open for now what propositions are: sentences of some (perhaps unspoken) language, abstract states of affairs, or whatever. One can also assume that declarative sentences can be used, in contexts, to indicate or express propositions. But each sentence of English is presumably distinct from the potential premise/conclusion indicated with that sentence in a given context. Different speakers can use *I swam today* at different times to indicate various propositions, each of which could be expressed in other languages. Nonetheless, propositions seem to be sentence-like in some respects, especially with regard to being composite.

The conclusion of (1)

(1) Chris swam if Pat swam, and Pat swam; so Chris swam.

is evidently part of the first premise, which has the second premise as another part. But simple propositions, without propositional parts, also seem to have structure. Aristotelian schemata like the following are valid: Every *P* is *D*, and every *S* is a *P*; so every *S* is *D*. The italicized variables are intended to range over predicates—logical analogs of nouns, adjectives, and other classificatory terms (like *politician*, *deceitful*, and *senator*). Simple propositions appear to have subject-predicate structure; where a subject can consist of a predicate and a quantifier (indicated with a word like *every*, *some*, or *no*).

Medieval logicians explored the hypothesis that all propositions are composed of simple propositions and a few special elements, indicated with words like *or* and *only*. While they expected some differences between grammatical and propositional structure, the idea was that sentences reflect the important aspects of logical form. The medieval logicians also made great strides in reducing Aristotelian schemata to more basic inferential principles: one concerning replacement of a predicate with a less restrictive predicate, as in *Rex is a brown dog, so Rex is a dog*; and one concerning converse examples, like *Rex is not a dog, so Rex is not a brown dog*.

Nonetheless, traditional logic/grammar was inadequate. If Juliet kissed Romeo, then Juliet kissed someone. And predicates containing quantifiers were problematic.