Zermelo-Fraenkel set theory. That form of axiomatic set theory that avoids the paradoxes of set theory by dropping the axiom of abstraction and substituting for it a set of axioms about set-existence.

Boruch A. Brody (1967)

LOGIC AND THE FOUNDATIONS OF MATHEMATICS

A very detailed account of main developments of logic will be found in *Logic, History of*. Brief explanations of many of the terms commonly used by logicians will be found in *Logical Terms, Glossary of*. The Encyclopedia also features the following articles dealing with questions in logic and the foundations of mathematics: *Artificial and Natural Languages*; *Combinatory Logic*; *Computability Theory*; *Computing Machines*; *Decision Theory*; *Definition*; *Existence*; *Fallacies*; *Geometry*; *Gödel's Theorem*; *Identity*; *Infinity in Mathematics and Logic*; *Laws of Thought*; *Logical Paradoxes*; *Logic Diagrams*; *Logic Machines*; *Many-Valued Logics*; *Mathematics, Foundations of*; *Modal Logic*; *Negation*; *Number*; *Questions*; *Semantics*; *Set Theory*; *Subject and Predicate*; *Synonymity*; *Syntactical and Semantical Categories*; *Types, Theory of;* and *Vagueness*. See "Logic" and "Mathematics, Foundations of," in the index for entries on thinkers who have made contributions in this area.

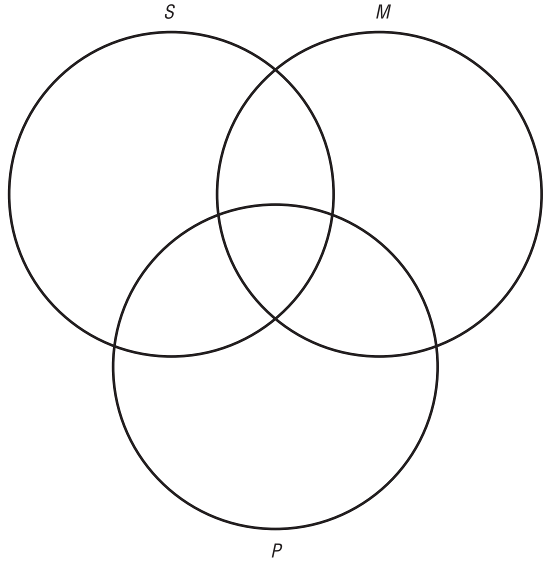
LOGIC DIAGRAMS

"Logic diagrams" are geometrical figures that are in some respect isomorphic with the structure of statements in a formal logic and therefore can be manipulated to solve problems in that logic. They are useful teaching devices for strengthening a student's intuitive grasp of logical structure, they can be used for checking results obtained by algebraic methods, and they provide elegant demonstrations of the close relation of logic to topology and set theory.

Leonhard Euler, the Swiss mathematician, was the first to make systematic use of a logic diagram. Circles had earlier been employed, by Gottfried Wilhelm Leibniz and others, to diagram syllogisms, but it was Euler who, in 1761, first explained in detail how circles could be manipulated for such purposes. Euler's contemporary Johann Heinrich Lambert, the German mathematician, in his *Neues Organon* (1764) used straight lines, in a manner similar to Euler's use of circles, for diagramming syllogisms.

FIGURE 1

Venn Diagram Applied to a Syllogism



VENN DIAGRAMS

The Euler and Lambert methods, as well as later variants using squares and other types of closed curves, are no longer in use because of the great improvement on their basic conception which was introduced by the English logician John Venn. The Venn diagram is best explained by showing how it is used to validate a syllogism. The syllogism's three terms, *S, M,* and *P,* are represented by simple closed curves—most conveniently drawn as circles—that mutually intersect, as in Figure 1. The set of points inside circle *S* represents all members of class *S,* and points outside are members of class not-*S*—and similarly for the other two circles. Shading a compartment indicates that it has no members. An X inside a compartment shows that it contains at least one member. An X on the border of two compartments means that at least one of the two compartments has members.

Consider the following syllogism:

Some *S* is *M.*

All *M* is *P.*

Therefore, some *S* is *P.*

The first premise states that the intersection of sets *S* and *M* is not empty. This is indicated by an X on the bor-