LOGIC IN THE ISLAMIC WORLD [ADDENDUM]

For more on everything in the entry, see especially Hans Daiber's *Bibliography of Islamic Philosophy* (1999). Few scholars would now accept that Arabic logic is "entirely Western"; it grew out of Greek texts, but developed differently from both Hellenistic and Latin logic.

TRANSMISSION OF GREEK LOGIC TO THE ARABS

Research on the translation of the books of the *Organon* and their attendant commentaries is presented in summary essays in Goulet (1989–2003, pp. 502ff).

THE SCHOOL OF BAGHDAD

The leading representative of the textual Aristotelianism of Baghdad was al-Fārābī, and much of his extant work is now either edited or translated (see Lameer 1994).

AVICENNA AND HIS INFLUENCE

The many new editions, translations, and studies of Avicenna are listed by Jules L. Janssens (1999). An attempt to deal philosophically with his modal syllogistic is made by Paul Thom (2003, chapter 4 and idem). See also his essay "Logic and Metaphysics in Avicenna's Modal Syllogistic" (forthcoming).

LOGICIANS OF ANDALUSIA

Averroes, though without much influence in the Islamic world, is the most acute of the Andalusian logicians. See Thom (2003, chapter 5) for a philosophical treatment of his later modal syllogistic.

QUARREL OF THE EASTERN AND WESTERN SCHOOLS

There certainly were major differences among the post-Avicennan logicians, but Nicholas Rescher's use of "Eastern" and "Western" schools to gather them into opposing camps is misleading (see Street 2004, pp. 567ff).

FINAL PERIOD

One cannot assume the tradition ossified because its most common genre became the commentary. The task ahead is to read and appraise the profusion of texts written from the 900s until after the colonial invasions of the nineteenth century. For a study of the attitudes to logic in this period, see Khaled El-Rouayheb's "Sunni Muslim Scholars on the Status of Logic, 1500–1800" (2004).

See also al-Fārābī; Averroes; Avicenna; Islamic Philosophy; Rescher, Nicholas.

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Tony Street (2005)

MEDIEVAL (EUROPEAN) LOGIC

Although some elementary work was done in the ninth and tenth centuries it was not until the end of the eleventh century that medieval logic really began to develop a character of its own. It started as glosses and commentaries on some of a small number of texts that had survived from antiquity. These included Boethius's translations of Porphyry's *Isagoge*, Aristotle's *Categories* and *De interpretatione*, and two works written by Boethius himself, a treatise, *De Topicis Differentiis*, on topical inference based on the work of Themistius and Cicero, and another, *De divisione*, devoted to the various forms of division employed in logic. In the thirteenth century these works were collectively known as the *logica vetus*.

In addition logicians at the beginning of the twelfth century possessed Boethius's very extensive commentaries on the *Isagoge*, *Categories*, and *De interpretatione*, his two-part epitome of Aristotle's *Prior Analytics*, 1–7, *Introductio ad syllogismos categoricos* and *De syllogismo categorico*, his treatise on hypothetical syllogisms, *De*

hypotheticis syllogismis, and his commentary on Cicero's *Topica*, *In Topica Ciceronis*.

Also important in the early development of logic were Marius Victorinus's *De diffinitionibus*, Saint Augustine's *De dialectica*, and, at least in the ninth and tenth centuries, *De decem categoriae*, a fourth-century Latin translation of a Greek paraphrase of Aristotle's *Categories* attributed to Augustine. In addition, Priscian's *Institutiones grammaticae*, with the eleventh- and twelfth-century glosses on it known as the *Glossulae*, were an important influence in the twelfth century on the development of philosophical semantics and in particular of theories of the substantive verb *to be*.

Boethius's translations of Aristotle's *Sophistical Refutations*, *Topics*, and *Prior Analytics* were recovered before the middle of the twelfth century. Along with the translation of the *Posterior Analytics* made then by James of Venice they provided logicians with what was distinguished from the *logica vetus* as the *logica nova*. Apart from the *Sophistical Refutations*, however, it was not until the beginning of the thirteenth century that the works of the *logia nova* had a significant impact on the development of logic. Although some parts of Avicenna's logical works were translated into Latin, unlike other areas of philosophy, Arabic writing had little impact on the development of logic.

From the middle of the twelfth century logicians developed their discipline in various ways and produced works characteristic of what would much later be referred to as the *logica modernorum*. These dealt, for example, with the properties of terms, and in particular the theory of supposition, syncategorematic words, modality, *obligationes, insolublia*, consequences, and sophisms of various kinds, each of which is discussed in this entry.

THE BOETHIAN BACKGROUND

Based as it was upon the texts of the *logica vetus* medieval logic included a great deal that has to do with ontology and philosophical semantics rather than with logic more narrowly construed as the theory of valid argumentation. Boethius gave medieval logicians much of their terminology but his commentaries on Aristotle and even more so his own works are essentially elementary, often confused, and sometimes inconsistent. It was these, however, which provided twelfth-century logicians with the material from which they constructed their new formal and philosophical logics. In particular, the remarkable developments they made in theory of inference had their beginnings in reflection on Boethius's *De Topicis Differentiis* and *De hypotheticis syllogismis*.

TOPICAL INFERENCE. Medieval logic at least in the first half of the twelfth century was characterized by an intense interest in conditional propositions and in the nature of topical inference as formulated by Boethius in *De Topicis Differentiis*. Logicians at this time were not generally concerned to regiment arguments into the modes and figures of the categorical syllogism but everywhere they classified inferences in accordance with lists of topics, based upon those given by Boethius.

In his treatise Boethius proposes to show how arguments may be discovered to settle any given question. What has to be found, he claims, is what Cicero, in his *Topica*, calls an "*argumentum*"—defined as a "reason which brings conviction where something is in doubt." An argument (*argumentatio*) is the expression in speech or writing of the proof of a conclusion constructed with the required *argumentum*. A *locus*, or topic, is the "site," or "source," of *argumenta* (*Diff. Top*. I, 1174D).

Argumenta are invoked by Boethius to warrant the enthymematic inference of a categorical conclusion from categorical premises or the direct proof of a conditional proposition. In each case what is needed is a principle that is not itself provable, called by Boethius a *maximal proposition*, and a relevant fact about the items mentioned in the conclusion. For example, by appealing to the maximal proposition "a genus is predicated of whatever its species is predicated" and the truth that animal is the genus of human being we may either infer from the premiss that Socrates is a human being the conclusion that he is an animal or, directly, the corresponding conditional.

The various relationships which Boethius holds may exist between the predicate and subject of a true categorical proposition or between the antecedent and consequent of a true conditional provide him with his *loci* (*Diff. Top*. II, 1186C). With each *locus* there are associated all the maximal propositions warranting inferences which may be made on the basis of that relationship. The enthymeme above, for example, would be characterized as holding "from species," that is, in virtue of the relationship in which a species stands to its genus.

Boethius gives the lists and classifications of the *loci* provided by both Themistius and Cicero. They are divided into those which are intrinsic, that is, having to do only with the things themselves about which a question is asked, and those which are extrinsic, having no such connection with them. (*Diff. Top*. II, 1186D) Examples of intrinsic *loci* are that from species, given above, and that from what is defined, for which one maximal proposition is: "of that of which what is defined is not

predicated, the definition is not predicated." Examples of extrinsic *loci* are that from authority, which justifies inferences from the authority of the majority of people, or the relevant experts, and *loci* from various kinds of opposition.

Argumenta drawn from the locus from authority are not necessary according to Boethius but they are probable in the sense of being generally convincing. Where Aristotle had taken probability and necessity to be properties of the premises and conclusion of a dialectical syllogism, however, Boethius takes them to characterize the nature of the inference from the premiss, or premisses, to the conclusion of an argument and the corresponding connection between the antecedent and consequent of a conditional (*Diff. Top.* I, 1180C).

THE THEORY OF CONDITIONAL PROPOSITIONS. In *De Topicis Differentiis*, Boethius classifies conditional propositions according to the quality of the antecedent and consequent. He accepts what we would now call a principle of contraposition and so maintains that a topical relationship warrants a conditional of the form "if something's an *A*, then it's a *B*" if and only if it warrants one of the form "if something's not a *B*, then it's not an *A*," where *A* and *B* are general terms such as "human being" and "animal." Conditionals of the form "if something's an *A*, then it's not a *B*" are true, he maintains, only for items which are "opposites," that is, opposed exclusively but not exhaustively. For example, "if something's a human being, then it's not a donkey." Those of the form "if something's not an *A*, then it's a *B*" hold only for items which are "immediates," that is, opposed exclusively and exhaustively. For example, "if something's not well, then it's ill" (*Diff. Top. I*, 1179C).

With *De hypotheticis syllogismis* Boethius provided twelfth-century logicians with an account of the logic of certain conditional and disjunctive propositions but neither he nor any other ancient source provided them with what we would recognize as a propositional logic. Boethius had no clear understanding of the nature of either propositionality or propositional operation (Martin 1991).

In his general treatment of compound propositions in his long commentary on *De interpretatione*, Boethius thus denies that the copulative conjunction "and" does anything other than punctuate a list (*2 In Peri. Herm.*, 5, 109). In the same work he also explicitly rejects the Stoic practice of preposing a negative particle to a categorical proposition as ambiguous between the negation of the subject and predicate terms (*2 In Peri. Herm.*, 10, 261–2).

Without a notion of propositionality, Boethius has no notion a propositional form or of the substitution of propositional contents into propositional contexts to obtain new contents of arbitrary complexity. In *De hypotheticis syllogismis* he thus lists all the various kinds of hypothetical syllogism which he accepts for each different quality of the component categorical propositions. There, just as everywhere else where Boethius employs it, the negative particle preposed to a conditional never takes the whole of the following conditional proposition for its scope but always acts only on the consequent.

Boethius designates a conditional as affirmative if its consequent is affirmative and negative if it is negative no matter what the quality of the antecedent (*Hyp. Syll.* 1.9.6). The only compound propositions he considers are simple conditionals and disjunctions, that is those whose components are both categorical, and compound conditionals of which one or more component is a simple conditional. The most complex form of conditional he considers has simple conditionals for both its antecedent and consequent. These compound conditionals, again, have nothing to do with propositional logic as it is now understood. Conditionalized instances of contraposition, for example, are not true instances of the form since Boethius requires for the truth of "if (if something's an *A*, then it's a *B*), then (if something's a *C*, then it's a *D*)" that both "if something's an *A*, then it's a *C*" and "if something's a *B*, then it's a *D*" are true (*Hyp. Syll.* 3.9.1).

In *De hypotheticis syllogismis* Boethius gives the basic truth-condition for a conditional proposition, or consequence (*consequential*), which will be accepted throughout the middle ages. To "destroy" such a proposition, that is, to show that it is false, he says, one must show that it is possible for the consequent to be false when the antecedent is true. A conditional is thus true only if the truth of the antecedent is inseparable from that of the consequent. A simple disjunction, "something's an *A* or it's a *B*," is equivalent, according to Boethius, to a simple conditional with a negative antecedent and affirmative consequent and so holds only for terms connected to one another as immediates (*Hyp. Syll.* 1.3.3).

In addition to stating the inseparability condition for their truth Boethius makes a distinction between conditionals which has profound consequences for the development of medieval logic and metaphysics. He claims that a relation of consequence may be indicated with either "*si*" ("if") or equivalently with "*cum*." The latter, however, usually means *when*, or *whenever* in Latin and that is how it is translated here.

The truth of an antecedent, Boethius notes, will be inseparable from that of a consequent if both are necessarily true even if there is no explanatory connection between them as, for example, "whenever (*cum*) fire is hot, then the heavens are spherical." Boethius does not notice, however, nor does any other ancient source available in the twelfth century, that the inseparability requirement is apparently also satisfied by any conditional whose antecedent is impossible, or whose consequent is necessary.

Boethius designates as "accidental consequences" conditionals formed with "whenever" which meet the inseparability requirement merely on account of the truth-value of their components. He contrasts them with "natural consequences," formed with "if," in which the truth of the antecedent is inseparable from that of the consequent in virtue of an explanatory connection between them. For example "if something's a human being, then its an animal" (*Hyp. Syll.* 1.3.6).

Finally, although Boethius correctly observes that Aristotle wrote nothing about hypothetical syllogisms, he takes from *Prior Analytics*, II. 4, as basic for the logic of conditional propositions what has been called Aristotle's Principle: No two conditionals of the form "if something's *A*, then its *B*" and "if something's not *A*, then its *B*" can both be true (*Hyp. Syll.* 1.4.1).

ABELARD AND THE DISCOVERY OF PROPOSITIONALITY

Peter Abelard, the first significant, and arguably the greatest, of all medieval logicians taught in Paris at various times between 1101 and 1140. Although most logical writing which we have from the twelfth century has been transmitted anonymously and with no certainty about its date of production, very fortunately both Abelard's own survey of logic, the *Dialectica*, written probably around 1116, his *Logica*, consisting of commentaries on Porphyry, Aristotle, and Boethius, written around 1120, and his *Glossulae* on Porphyry, written in the 1120s, have survived more or less intact. The following account of logic in the first half of the twelfth century is thus mainly an account of Abelard's work. He was, however, certainly not the only logician active at the time and much of his writing consists of arguments against sophisticated but unnamed opponents.

Most important, Abelard understood the distinction between the propositional content of a sentence and the force with which it is uttered (Martin 2004). The propositional content "that Socrates is running," for example, may be asserted with an assertive utterance of "Socrates is running" or it may contribute to the meaning without itself being asserted in an assertive utterance of the conditional "if Socrates is running, then he is moving." Since Boethius treats "proposition" (*propositio*) and "assertion" (*enuntiatio*) as synonyms, however, it was rather difficult for Abelard to formulate clearly the distinction for an assertion between force and content.

Abelard uses the term "proposition" (*propositio*) to refer to a token propositional sentence. In his early writings he borrows from Priscian the expression "the being of the thing" (*essentia rei*) to speak about propositional content and identifies it with a state-of-affairs. In later writings he refers rather to the *dictum* of a proposition, that is, to "what is said" with it. For Abelard it is *dicta* which are in the first place the bearers of truth and falsity and so, for example, a conditional is true if and only if the truth of the *dictum* of the consequent follows from the truth of the *dictum* of the antecedent.

The distinction between force and content, which Peter Geach has called the *Frege Point* in deference to its supposed discoverer, is crucial for the development of genuinely propositional logics. Abelard saw this and consequently rejected Boethius's views on copulative conjunction. To the contrary, he insists that a copulative conjunction of propositions is itself a single proposition and may thus be subject to a further propositional operation. "It's not the case that (*p* and *q*)" where "*p*" and "*q*" are propositions is just as much a single proposition, he insists, as "it's not the case that (if *p*, then *q*)."

ABELARD'S TWO NEGATIONS. Negation is the simplest propositional operation. If it is defined truth-functionally, it takes any propositional content and produces another, its contradictory, false if the first is true and true if it is false.

The invention of this operation in Latin logic cannot quite be claimed with certainty for Abelard. It is possible that it was used by his predecessors since it appears in very limited way in a discussion of the appropriate way to negate a simple conditional proposition in the *Dialectica* of Garlandus Compotista, apparently written in the second decade of the twelfth century roughly contemporary with Abelard's *Dialectica*.

Abelard, however, is the first Latin writer known to us who discusses propositional negation in general and applies it both to simple and compound propositions (Martin 2004). He distinguishes, indeed, two kinds of negation. First, and principally, propositional negation, which he calls "destructive" negation, and which has the whole of the following propositional content for its

scope. Second, and derivatively, a negation, which he refers to as "separative" which in the case of affirmative categoricals is obtained by negating the predicate (*Dial.* II.2, 173 sq.). Abelard follows Boethius in classifying conditionals as affirmative according the quality of their consequents. The separative negation of a given affirmative conditional is obtained by negating its consequent either destructively or separatively.

A necessary condition for the truth of both an affirmative categorical and its separative negation is that the subject term is not empty. There is no such requirement for the truth of its destructive negation.

With this distinction between negations Abelard constructs an account of the relationships between quantified propositions which results in effect in a rectangle of opposition rather than the famous square of Aristotle as Boethius understood it. Aristotle gives "not every *A* is *B*" as the contradictory opposite of "every *A* is *B*" in *De interpretatione* but in the *Prior Analytics* "some *A* is not *B*" and according to Boethius the meaning is the same.

Abelard, however, argues that "some *A* is not *B*" is not the contradictory of "every *A* is *B*" but rather "it is not the case that every *A* is *B*." He thus avoids the problem typically raised against Aristotle's logic of quantified terms, that since it requires for the truth of a universal affirmation that the subject term is not empty, given there are no chimeras, an affirmation such as "every chimera is conversing" is false. It follows that its contradictory is true. Since "some chimera is not conversing" is true, however, only if the subject term is not empty, there must be some chimeras for it to be true of! For Abelard this is not a problem since on his account both propositions are false (*Log. "Ingred." sup. Perierm.* 7, 408–11).

THE MANIPULATION OF MODALITY. Once the notion of propositional content was available the difference between two different interpretations of modal propositions could be formulated precisely. In his *Dialectica* Abelard notes that a mode may appear in a categorical proposition either as an adverb or an adjective as, for example, in "Socrates is possibly a bishop" and "that Socrates is a bishop is possible" (Knuutila 1993). Abelard holds that though they differ syntactically these two propositions are semantically equivalent and it is the first which properly expresses the intended meaning since possibility is properly attributed to things (*de rebus*) (*Dial.* II.2, 191sq.). The adverb serves to indicate that the inherence of the predicate in the subject is modified in some way. Later medieval logicians will refer to this as the *de re* reading of the modal claim.

In the case of true *de re* claims about possibility there is of course no actual inherence to modify and Abelard holds that such propositions are true just in case the nature of the subject is compatible with the predicate. Human nature is compatible with being a bishop so "Socrates is possibly a bishop" is true even though he never has been nor never will be one (*Dial.* II.2, 193).

Abelard records that one of his masters proposed an alternative account of propositions with adjectival modes. They are to be understood, he held, as claims about the possibility, necessity, etc. of the sense (*de sensu*), that is the propositional content, of the simple propositions from which they "descend." Against this interpretation Abelard, in effect, argues that if we substitute for a given propositional content an equivalent one, the truth-value of the proposition will remain the same. Since universal negatives convert simple, "no blind man is a seeing man" is equivalent to "no seeing man is a blind man." While his opponents accept, however, that "no blind man is possibly a seeing man" is true, since they agree that the blind do not regain their sight, they claim that "no seeing man is possibly a blind man" is false. The *de sensu* reading, however, requires them to have the same truth value (*Dial* II.2, 196).

Although he maintains in the *Dialectica* the *de sensu* reading is in general not the proper way to interpret modal propositions, Abelard does allow that it is correct for the adjectival modes "true" and "false" since these, he argues, they are properly predicated of propositional contents (*Dial.* II.2, 204–6).

Abelard discusses the same questions at length in his *Logica* in commenting on Aristotle's account of the relations between modalities in *De interpretatione*, 12. He notes, in the first twelfth-century reference to the *Sophistical Refutations*, that the distinction he is interested corresponds to that made by Aristotle between reading a proposition such as "a standing man is possibly sitting" in a composite (*per compositionem*), or a divided way (*per divisionem*). Here, however, Abelard does not insist on the reading *de rebus* but rather works out in detail the relations between modal claims of both kinds (Abelard 1958, 13).

ABELARD ON ENTAILMENT. In his logical works Abelard sought to unify into a single theory of inference the disconnected remarks on topics and the consequence relation which he found in Boethius (Martin 2004). To do this he provides a new general definition of a *locus* as the force of, or as we would say, the warrant for an entailment (*vis inferentiae*) (*Dial.* III.1, 253). He then devotes hun-

dreds of pages of his logical works to investigating the role of *loci* thus conceived in proving conditionals and validating the corresponding enthymemes.

According to Abelard, a proposition *p* entails a proposition *q*, just in case the corresponding conditional, or consequence, *if p, then q* expresses a relationship of following, or "consecution" (*consecutio*). For this to be so, he holds, the sense of the antecedent, that is, its propositional content, must contain that of the consequent. Abelard characterizes this kind of connection as necessary but insists that it must be distinguished from the satisfaction of the inseparability condition which it guarantees, and which alone provides only the necessity of what he calls association (*comitatio*) (*Dial*. III.2, 459).

Entailments are divided by Abelard into the perfect and the imperfect. Perfect entailments satisfy the containment requirement in virtue of the form, or structure, of the propositions involved. Imperfect entailments are those in which the sense of the antecedent contains that of the consequent but does not do so in virtue of their form (*Dial*. III.1, 253).

Abelard makes the notion of perfection, and so form, more precise, and anticipates modern definitions of logical truth, by giving as a necessary condition for perfect entailment that consecution is preserved through all uniform substitutions of terms or propositional contents. He does not, however, regard the condition as sufficient and, in particular, although he classifies the conditionalizations of all valid categorical and hypothetical syllogisms as perfect, he holds that instances of the principle of reflexivity, *if p, then, p*, are imperfect, presumably because they fail to have a canonical syllogistic form. Like all other imperfect entailments, according to Abelard, they must thus be warranted as instances of an appropriate maximal proposition (*Dial*. III.1, 255).

By far the greatest part of Abelard's *Dialectica* is concerned with establishing just which conditional propositions express imperfect entailments. Boethius in *De Topicis Differentiis* says that he will explore which *loci* are suited to which syllogisms and according to Abelard this led some logicians to hold that even the canonical syllogistic figures needed topical warrants. He and his mid-twelfth century followers known, probably because of their views on universals, as the *Nominales*, rejected this. They held rather that putative principles cited to support categorical and hypothetical syllogism are simply their metalinguistic formulation as rules. They contain no term indicating a topical relationship, that is no *locus differentia*, upon which the inference in question rests (*Dial*. III.1, 256–263).

Imperfect entailments, according to Abelard, are conditionals and the corresponding enthymemes, which satisfy the two conditions necessary and sufficient for following for a restricted range of terms. The topical difference specifies the relevant substitution class and the maximal proposition warrants the inference for substitutions from that class. For example, the conditional "if Socrates is a human being, then Socrates is an animal" is true and so are all substitutions for "human being" and "animal" which stand in the relationship of species to genus. For example, "if Socrates is a pearl, then Socrates is a stone," warranted by the maximal proposition "of whatever a species is predicated, so is its genus" (*Dial*. III.1, 315).

NECESSITY. Abelard's main task in his discussion of topical inference is to establish just which topical relations and which maximal propositions warrant true conditionals. He argues in the *Dialectica* that since what is being proved are conditional propositions, even though their surface form may be categorical, maximal propositions must in fact be general conditionals "containing" each of the proved conditionals as their instances. His treatment of this question involves a sophisticated discussion of how relative pronouns function in quantified propositions and the rules for logically manipulating them.

Since Boethius had allowed that some *argumenta* are probable but not necessary certain of Abelard's contemporaries had, he tells us, accepted as true any conditionalization of an enthymeme supported by a probable maximal proposition. In particular they took to be true conditionals warranted by maximal propositions which guarantee the inseparability of association but not the following or consecution which Abelard requires for entailment (*Dial*. III.1, 271 sq.).

Against them Abelard invokes the principle from the *Prior Analytics* mentioned above. His opponents accept conditionals warranted by appeal to the *locus* from immediates and the maximal proposition "of that from which one of a pair of immediates is removed the other is predicated." They must thus accept the following argument: [I1] if something does not exist, then it is not well (by the *locus* from part to whole, since "not-well" is predicated of all non-existent things as well as all existing things which are not well); [I2] if something is not well, then it is sick (from immediates); [I3] if something is sick, then it exists (from part to whole); so, by transitivity, [I4] if something does not exist, then it is sick, and thus [I5] if something does not exist, then it exists. [I5], how-

ever, contradicts Aristotle's principle and, Abelard maintains, is obviously impossible (*Dial*. III.1, 276).

Abelard investigates in detail various proposals to modify [I2] to block the embarrassing inference while retaining its warrant from immediates. In particular he considers various ways of adding what he calls a "temporal" qualification, indicated with "when" ("*cum*"), to form propositions such as "if (when something's an animal, it's not well), then it's sick."

Boethius, as noted, claims that "if" and "when" are equivalent as indicators of a conditional connection and in *De hypotheticis syllogismis* he invariably gives the conditional components of compound conditionals with "when." For example, "if (when something's an *A*, it's a *B*), then it's a *C*." This practice allows Abelard to treat the embedded propositions as temporal rather than conditional in interpreting Boethius claims about the hypothetical syllogism (*Dial*. IV.1, 472 sq.).

The problem for Abelard is that having insisted that one destroys a conditional by showing that it is possible for the antecedent to hold without the consequent, Boethius apparently assumes that an affirmative simple conditional and the corresponding negative conditional are contradictory opposites. He thus claims to be valid, for example, syllogisms of the form "if (when something's an *A*, its a *B*), then it's a *C*, but it's not a *C*; therefore when something's an *A*, it's not a *B*."

Abelard in the end rejects Boethius's account of the hypothetical syllogism. In this case, for example, he maintains, contrary to Boethius, that the valid argument is rather an instance of *modus tollens* (*if p, then q, not:q; therefore not:p*) which concludes with the propositional negation of the antecedent: "if (when something's an *A*, it's *B*), then it's a *C*, but it's not a *C*; therefore it is not the case that (when something's an *A*, it's *B*)." Abelard thus, in effect, replaces Boethius's account of the hypothetical syllogism with a genuinely propositional theory which takes *modus ponens* (*if p, then q, p; therefore q*) and transitivity (*if p, then q, if q, then r; therefore if p, then r*) as basic principles and *modus tollens* as a derived principle and holds that all uniform substitution instances, no matter how complex, are valid (*Dial*. IV.1, 498 sq.).

Abelard was unable to save Boethius's account of the hypothetical syllogism and so he replaced it with the correct one. Apparently no one else could to do any better and *De hypotheticis syllogismis* disappeared from the logic curriculum some time in the twelfth century. It is not until Walter Burley (1274–1344) published *De puritate artis logicae* in about 1325 that hypothetical syllogisms

were discussed in any detail again, and there the conditional premises are always simple conditionals.

RELEVANCE. Abelard accepts that the *locus* from immediates and many others guarantee the inseparability of association, but he also requires a relevant connection between antecedent and consequent for the conditional to be true (Martin 2004). He does not, however, insist on relevance for the validity of an argument. So long as it is impossible for the premises to be true and the conclusion at the same time false, true premises will guarantee a true conclusion and that is all that an argument is asked to produce. Abelard thus denies as a general principle what we would now call the Deduction Theorem, that an argument *p; therefore q* is valid if and only if the corresponding conditional *if p, then q* is true (*Dial*. III.2, 455).

Abelard's distinction between association and following or consecution as two kinds of necessary connection is based on the account given in the *Isagoge* of the relationship between substances and their accidents. According to this a substance does not require a particular accident in order to exist and so accidents are separable from their subjects. The problem is that while a given substance may undergo a change with respect to certain of its accidental features there are others, according to Porphyry, which must always be present. Blackness, for example, in the case of crows, and the property of being able to laugh in the case of humans. Neither of these are included in the account of what it is to be a crow or to be human but there is no natural possibility of their subjects existing without them. Such "inseparable" accidents can, however, it is claimed, be removed in the sense that we can conceive of a crow without conceiving its blackness. They are thus contrasted with definitional features which are included as part of its essence, in the definitional account of what it is to be a particular kind of thing (*Log. "Ingred." sup. Porph.* 6, 93).

Abelard's two necessities are a generalization of this distinction between actual and conceptual inseparability. He points out in his own discussion of inseparable accidents that although the antecedent and consequent of "if Socrates is a stone, then Socrates is a pearl," are inseparable, a pearl being classified as a kind of stone, nevertheless the conditional is false. The antecedent and consequent are inseparable, and Abelard is the first medieval logician we know of to make this point, merely because the antecedent is impossible. He goes to point out that if the inseparability of association were sufficient as well as necessary for following, then any conditional with an impos-

sible antecedent would be true. For example, "if Socrates is a stone, then Socrates is a donkey" (*Dial.* III.1, 285).

Abelard does not, however, formulate the famous principles that anything follows from an impossibility and that a necessity follows from anything. He could not be expected to do so, however, since given his definition of following they are false.

Abelard believes that his own account of the semantics of the conditional generates what we would today call a connexive logic, a logic, that is, for which no proposition can entail or be entailed by its contradictory opposite. These principles entail, Abelard recognizes, both the propositional version of Aristotle Principle and what we may call Abelard's Principle: No two conditionals of the form *if p, then q* and *if p, then not:q*, can both be true.

Abelard accepts simplification (*if (p and q), then p* and *if (p and q), then q*), contraposition (*if (if p, then q), then (if not:q, then not:p)*), and transitivity (*if p, then q, if q, then r; therefore if p, then r* is valid). Suppose, then, that Abelard's Principle is false for some p and q, that is both (1) *if p, then q* and (2) *if p, then not: q* are true. But then if (3) *if (p and not:q), then p* is true and likewise (4) *if q, then not:(p and not:q)*, we may infer by transitivity that *if (p and not:q), then not:(p and not:q)*, an instance of *if p, then not:p*, which Abelard insists is a paradigm of impossibility. Abelard's Principle is thus necessarily true and he gives a similar argument to prove Aristotle's Principle (*Dial.* III.1, 290).

From these principles there follows the most characteristic feature of the logical theory advocated by Abelard and the *Nominales*: No conditional can be true of which the antecedent and the consequent differ in quality. For example if *if p, then not: q* were true, for some p and q, then *if (p and q), then not:(p and q)* would true by transitivity and contraposition.

Most famously Abelard argued against the *locus* from opposites in this way. If the *locus* warranted a true conditional then the conditional "if Socrates a human being, then Socrates is not a donkey" would we be true and we could infer the impossibility "if Socrates is a human being and a donkey, then it is not the case that Socrates is a human being and a donkey." He sees too, and explicitly acknowledges, that it follows from the principles of his logic that the conditional principle of double negation (*p if and only if not:not:p*) is false in both directions (*Dial.* II.2, 179).

Unfortunately Abelard's various intuitions about the propositional connectives are inconsistent (Martin 1987). In particular the principles which he holds to govern

negation are incompatible with simplification. This point seems to have been first noticed the 1130s by Alberic of Paris who confronted Abelard with the following argument: The conditional [A0] "if Socrates is a human being, then he is an animal" is a paradigm of entailment according to Abelard. He must also accept each of the following: By simplification [A1] if Socrates is human and Socrates is not an animal, then Socrates is not an animal; by contraposition, [A2] if Socrates is not an animal, then Socrates is not a human being; again by contraposition, [A3] if Socrates is not a human being, then it is not the case that Socrates is human being and Socrates is not an animal; so by transitivity, [A4] if Socrates is human being and Socrates is not an animal, then it is not the case that Socrates is a human being and not an animal—contradicting a fundamental principle of Abelard's logic. Alberic's proof of inconsistency precipitated a crisis in the history of logic.

THE PARISIAN SCHOOLS AND THE CRISIS OVER THE CONDITIONAL

In middle decades of the twelfth century a number famous logicians were active at Paris and with each of was associated a school (Martin 1987). In some cases very substantial treatises have survived from these schools, illustrating that this was a period of intense activity in logic. Unfortunately most of these and certainly the most important are still unpublished. The schools may be distinguished by their response to Alberic's proof of the inconsistency of Abelard's system.

Abelard's own followers, the *Nominales*, continued to maintain the correctness of his account of the conditional and the connexive principles. Their strategy seems to have been to take negation to be a cancellation of content so that nothing follows from *p and not:p* rather than both *p* and *not:p*.

The followers of Alberic, the *Montani*, so-called because their school was located on Mont Ste. Geneviève, held that the argument failed because the conjunction of contraries in [A1] undermined the relationship on which [A0] was based. In a different context Abelard himself anticipates this objection to impossible antecedents and argues at length against it that since the antecedent is not asserted, and the argument is formally valid, the conclusion follows.

The school of Gilbert of Poitiers, the *Porretani*, held that the problem lay in the unrestricted principle of simplification. They required, as do twentieth century connexive logics, that both conjuncts play a role in such an inference. The most surprising response was that of the

followers of Robert of Melun, the *Melidunenses*, who took as their basic principle for the logic of the conditional the rule "nothing follows from the false."

The solution that eventually won the day, however, was that proposed by the followers of Adam of the Little Bridge, the *Parvipontani*, so called again because of the location of their school in Paris. They accepted that the argument was sound because they apparently held that inseparability alone is both necessary and sufficient for the truth of a conditional. Aristotle's Principle thus fails when the consequent is necessary and Abelard's when the antecedent is impossible.

John of Salisbury tells us in his *Metalogicon* (1159) that one of his students, William of Soissons, had gone on to join the *Parvipontani* and discovered the twelfth-century version of one of the twentieth century's most famous arguments, the proof that *ex impossibli quodlibet*, the so-called paradox of strict implication, according to which anything follows from an impossibility (*Metalogicon* II.10).

In his *De naturis rerum* written at the end of the twelfth century Alexander Neckham gives the argument as follows: [S1] if Socrates is a human being and Socrates is not a human being, then Socrates is a human being; [S2] if Socrates is a human being, then Socrates is a human being or Socrates is a stone; [S3] if Socrates is a human being and Socrates is not a human being, then Socrates is not a human being; therefore [S4] if Socrates is a human being and Socrates is not a human being, then Socrates is a stone (*De Naturis Rerum* cixxiii, 288–89).

The outcome of the crisis provoked by Alberic was a complete change in the understanding of the logical connectives. John of Salisbury tells us that he could not conceive why any one would think that anything follows from an impossibility but according to Alexander Neckham nothing was more obvious.

Abelard had insisted that a genuine connection was required for the truth of conditionals and disjunctions. Alexander's argument, on the other hand, assumes only inseparability for the conditional and much less for the disjunction. [S2] is the so-called Principle of Addition characteristic of the disjunction defined as true if one of the disjuncts is true. The disjuncts are no longer required to be related as immediates.

The conditional and disjunction were standardly defined in this way for the rest of the middle ages. Until the end of the thirteenth century, however, a contrast continued to be drawn between an accidental consequence which held wherever the inseparability condition was met and a natural consequence in which the sense of the antecedent contained the consequent. This stronger connection was needed because it was necessary to reason about impossibilities.

THE RECEPTION OF THE *LOGICA VETUS* AND THE DEVELOPMENT OF THE *LOGICA MODERNORUM*

Some time towards the end of the twelfth century the various different schools disappeared as the independent masters formed themselves into the corporation that became the University of Paris. Teaching and research in logic was the preserve there of the Faculty of Arts and its results appear in the introductory textbooks of the *logica modernorum*. To the traditional topics these add extensive discussions of fallacies and the properties of terms.

FALLACIES. Although Abelard had some limited access to the *Sophistical Refutations* it was not until around 1140 that the analysis of fallacies became a major concern for logicians. From the beginning, however, a short list was available in Boethius's discussion of Aristotle's remark in *De interpretatione* 6, that the putative negation of a given proposition may fail to have the required opposite truth value because the subject or predicate terms have different meanings in the two propositions (De Rijk 1962–1967).

Although Boethius's list of the ways in which this might occur ceased to be of much interest once the *Sophistical Refutations* were easily available, one of his fallacies was particularly important for the later development of logic. With no further explanation Boethius gives as an example of what he calls *univocation* the propositions "*homo ambulat*" ("human being walks") and "*homo non ambulat*" ("human being does not walk"). He claims that they are true together when the first is true of an individual, or particular man, and the second is true of "special man."

Abelard notes that univocation arises because the context in which a term is used may affect its meaning. For example, since medieval Latin has no articles or quotation marks it cannot distinguish between the occurrences of "*homo*" in "*homo est albus*," "*homo est vox*," and "*homo est species*," in the way in which we distinguish in their translations between "a human is white," "'human' is a word," and "human is a species" (De Rijk 1962–1967, I, pp. 51–56).

Logicians in the second half of the twelfth century commented at length on and refined Aristotle's account

of fallacy in the *Sophistical Refutations*. By the end of the century the results of their work are clear in theology where the theory of fallacy is frequently invoked to explain and resolve errors in argumentation. In addition to the standard fallacies logicians also developed as a special form of argument the idea of counter instances (*instantiae*) which they found in the *Sophistical Refutations*, *Topics*, and *Prior Analytics*. With these the principles advocated by one or another of the schools were shown to lead to a conclusion which was unacceptable to it.

Once the works of the *logica nova* were available logicians seem to have turned their attention from the theory of consequences and topical inference to issues in philosophical semantics. Here a distinction was made between categorematic words, or terms, that is words which on their own can be the subject or predicate of a categorical proposition, and all other words which can occur in any kind proposition. The latter were called syncategorematic words.

THE PROPERTIES OF TERMS. Termist logic, so called because of its interest in the semantical properties of terms, seems to have developed in rather different ways in Paris and Oxford. The most famous Parisian termist was certainly Peter of Spain (c. 1205–1277), whose *Tractatus*, or *Summulae logicales*, written around 1235, was much commented on and remained the standard introductory text in logic in continental Europe and Scotland for the rest of the middle ages. It seems, however, not to have been greatly used in England, where the University of Oxford had its own textbooks. The *Introductiones in logicam* (c. 1245) by William of Sherwood (c. 1210–c. 1270) perhaps also belongs in the Oxford tradition. Another text belonging to the Parisian tradition is the *Summa Lamberti* (c. 1255) of Lambert of Auxerre (fl. 1250s) on which the following remarks are based.

IMPOSITION AND SIGNIFICATION. Medieval logicians developed their philosophical semantics in the first place from Boethius's commentaries on the first chapter of *De interpretatione*: Spoken words are introduced to bring to mind mental items, understandings (*intellectus*), which are obtained from the things which exist in the extra-mental world and are likenesses of them. For substantial common terms such as "human being" the corresponding understandings are the mental correlates of the forms which in the world make individuals to be the kinds of things that they are. For accidental terms such as "whiteness" they are the forms which cause individuals to have the accidental features that they do.

Words were held to acquire their meaning through acts of baptism, known as *imposition* (*impositio*), or *institution* (*institutio*) (Kretzmann et al. 1982, ch. 9). In the case of individual humans literally so. For general terms the impositor introduces a name in the presence of a paradigmatic sample with the intention that all and only individuals of the kind in question bear the same name. Adam's naming of the beasts of the field and the fowls of the air (Genesis 2:19) provided a suitable example. Although medieval accounts of imposition do not seem to have been very developed there are obvious similarities to modern causal theories of reference.

The immediate and proper signification of a common term is the understanding constituted when it uttered in the mind of a listener who speaks the language. Just what a given philosopher thought about the things understood and their relationship to individuals in the world depended on where he stood on the question of universals. Lambert, for example, was a realist. The term "human being," he claims, signifies immediately the understanding of the form which makes humans to be human and mediately the form itself. It does not signify individual human beings (*Logica*, 206).

SUPPOSITION. "Supposition" is used in the thirteenth century to refer to what earlier writers had called "appellation," it is a property which an already significant term has in virtue of its use. Corresponding to the three different contextual meanings recognized in the fallacy of univocation there are three forms of supposition. With no change in the signification established by its original imposition, the term "*homo*" thus supposits, or stands for three different kinds of things in the propositions "*homo est albus*," "*homo est vox*," and "*homo est species*."

In the first, according to Lambert, "*homo*" has personal supposition because it stands for the individuals "contained under" the form which it indirectly signifies. In the other two, he says, its supposition is simple (*Logica*, 209). In the second it stands for the thing which the term signifies indirectly—a form according to Lambert, and a "universal thing" according to Peter of Spain. In the third proposition the terms stands for itself.

William of Sherwood gives a slightly different classification. According to him in the third proposition "*homo*" has material supposition and in the other two formal supposition. In the first this formal supposition is personal and in the second it is simple (*Introductiones*, 75).

Personal supposition is the semantical property which most interested logicians since their task was to say

in general what determines the truth or falsity of a given proposition and to do so they needed to decide what the terms in the proposition stand for.

THE DIVISIONS OF SUPPOSITION. Treatises on the properties of terms make many distinctions and precisions within personal supposition. Supposition properly speaking is a property of a substantive noun which it has when it stands for something. An adjective in use, on the other hand, couples something and so is said to have the property of copulation.

Supposition in general, according to Lambert, is either natural and accidental. The imposition of a term connects it mediately with a form and, at a second remove, prior to any contextual determination to all the individuals which have done, do, or will share in that form. These are what it naturally supposits for (*Logica*, 208).

Accidental supposition is supposition determined by context and may, as noted, according to Lambert, be simple, or personal. Personal supposition is further divided into discrete supposition, the supposition had by proper names, and common supposition, the supposition of common terms.

The common supposition of a term such as "human being" is further determined by its interaction with the syncategorematic words of quantity and quality, and may be either determinate or confused. Logicians offered various accounts of these forms of supposition but by the fourteenth century typically explained them in terms of their inferential relations (Kretzmann et al. 1982, ch. 9)

Supposition is determinate when the term is the subject of an indefinite or particular affirmative, such as "a human being is running" and "some human being is running." Here we may descend from the particular or indefinite proposition to the propositional disjunction of singulars whose subjects are the *supposita* of the common term and ascend from any one of those singulars to the general proposition. So from "some human being is running" we may infer "Socrates is running or Plato is running or …" and from the truth of any one of the disjuncts we may infer that some human being is running.

In confused supposition, a common term stands for all its *supposita* together. It may do this in one of two ways, either as with the subject of a universal affirmative where the supposition is distributive, and one may descend to, and ascend from, the propositional conjunction of each of the corresponding singulars. For example from "every human being is running" to "Socrates is running and Plato is running and …" and conversely.

The other form of confused supposition, merely confused supposition, is exemplified by a common term occurring as the predicate of a universal affirmative proposition. Here the term again stands for all supposita but taken together in such way that one can descend only to the predicate disjunction but ascend from any singular. For example from "every human being is an animal" to "every human being is (this animal or that animal or …) and from "every human being is this animal" to every man is an animal.

Negation distributes any simple term to which it is applied, so both the subject and predicate of no man is running, that is, every man is not running, have confused and determinate supposition (Lambert, *Logica*, 210).

Historians have puzzled about the relationship between supposition theory and modern quantification theory but this seems to miss the point. Supposition theory does not aim to state truth-conditions for propositions but to determine which of the supposita of a term occurring in a proposition someone uttering it should be understood as referring to and in what way.

AMPLIATION AND RESTRICTION. The propositions given above to illustrate the divisions of supposition all have simple subjects and predicates with the verb in the present tense and not modified in any way. A term is said to appellate those of its *supposita* which actually exist and in the case of all these propositions appellation and supposition coincide. The qualification of a substantive with an adjective restricts its supposition to suitably qualified things. In "a white human being is running," for example, "human being" has determinate supposition only for those of its *appellate* which are white (Lambert, *Logica*, 226).

Tense affects the supposition of terms by ampliating them to stand for *supposita* other than their *appellata*, though these may also be included in the supposition. For example in "an old man was a young man" the predicate term has merely confused supposition for those of its *supposita* which existed in the past but do not now exist. The subject term has determinate supposition for its *appellate* and its past *supposita*.

There is no suggestion in the twelfth century termists named that a term might supposit for possibilia which never exist. Lambert and Peter of Spain hold, for example, that in the modal proposition "some man might be the Antichrist" "man" supposits for past and future men (Lambert, *Logica*, 228). Ampliation to pure possibilia is allowed, however, in the *Summa logicae* (c. 1324) of William of Ockham (c. 1285–1349) and *Summulae de*

dialectica (1330s) of John Buridan (c. 1300– c. 1360). The change in theory of ampliation reflects a radically new conception of possibility introduced in the work of John Duns Scotus (c. 1265–1308) at the beginning of the four-teenth century. Against the assumption that all possibili-ties must be realized in time Scotus famously argued for the logical possibility that things could now be otherwise than they in fact are and so that there are possibilities that are never realized.

SYNCATEGOREMATIC WORDS. Both Peter of Spain and William of Sherwood as well as other termist logi-cians produced treatises entirely devoted to syncategore-matic words (Kretzmann et al. 1982, ch. 11). These treatises do not deal with all words that are not categore-matic but only with a relatively small and fairly standard set. In addition to the definition by exclusion, syncate-gorematic words are further characterized as semantically incomplete in that they acquire a signification only by being combined in some way with categorematic terms. For this reason they are said to be *consignificant*.

It is in the treatises on *syncategoremata* that termist logicians deal with the difficult words whose presence may affect the validity of a principle of inference and allow the construction of sophisms. As, for example, in the proof by Sherwood that no man lectures at Paris unless he is an donkey: "A man lectures at Paris unless he is an donkey" is a false conditional since the antecedent "a man is not a donkey" is necessarily true and consequent may be false. Therefore the contradictory of the condi-tional is true (*Syncategoremata* 82–3). In the fourteenth century such puzzles and their resolutions were collected together in separate works devoted to grammatical, logi-cal, including modal and epistemic, and physical sophisms. Their resolution often required that the inner structure of a syncategorematic term be exposed by what was called *exposition*. "Socrates is beginning to be white," for example, might be expound as 'Socrates is not now white and after now Socrates will be white' leading on to a discussion of tense, change, and the structure of time.

Included among the *syncategoremata* in these trea-tises we find the propositional connectives and confirma-tion the twelfth century insight into their nature had not been lost. William of Sherwood, for example, discusses both negation and the copulative conjunction. He clearly distinguishes, extinctive, or propositional negation and argues that if the conjunction "Socrates is running and Plato is arguing" is negated with a preposed particle the result is true just in case one of the coupled propositions is false (*Syncategoremata* 86).

MODISM

In the last quarter of the thirteenth century the termist semantics of supposition was replaced by what is known as modism, or speculative, that is, theoretical, grammar (Marmo 1994, Kelly 2002). The proponents of this the-ory, the *modisti*, for example Martin of Dacia (d. 1304), Boethius of Dacia (fl. 1275), and Thomas of Erfurt (fl. 1300) were concerned to say something more general about the meaning of both categorematic and syncate-gorematic terms than their termist predecessors. They held that all meaningful words are characterized by cer-tain modes of signifying and that these correspond to the traditional parts of speech. Corresponding to each modes of signifying, is a mode of understanding, and a mode of being.

According to the modists a proper name like "Socrates" as well as signifying Socrates, carries informa-tion about the essential character of what it signifies. It signifies it as a substance, for example, in the *modus sub-stantiae*, though not as an existent, since we use nouns to speak about presently non-existent and fictional items. A verb, on the other hand, signifies what it signifies in the mode of change and becoming. Grammatical features which were regarded as less fundamental, for example, number and tense, were held to correspond to accidental modes of signifying, understanding, and being (Kretz-man et al. 1982, ch. 13).

On the basis of their distinction between modes the *modisti* developed an account of grammatical con-gruity—the modes have to fit together in the right way. They sought to go beneath the surface structure of their language to locate the underlying relationship between the components of propositions. Their idea was that the order required by Latin grammar did not properly repre-sent the real relationships between the things signified. Though twelfth century logicians had already explored some of these ideas especially with regard to pronouns, the *modisti* deserve credit for being the first to attempt to develop a systematic theory of syntax.

Although the modists distinguished between the full signification of a word including its mode of signifying and the things in the world to which it applies, they made no use of the idea of supposition. They seem not to have developed an account of the contextual dependence of reference to compete with that of termists and in the end it was the semantics of termism which won the day (Kret-zman et al. 1982, ch. 13).

OBLIGATIONS

The earliest treatises on what were known as obligations (*obligationes*) date from the second half of the twelfth century (Martin 1993). In obligational disputation one participant, the respondent, is required to agree to a hypothesis and to reply consistently with it in the face of questions put to him by the opponent. The aim of the opponent is make the respondent contradict himself.

The most important form of obligation was the one known as *positio*, in which the opponent posits to be true something which is in fact false. In the twelfth and thirteenth centuries it had two forms depending on whether the *positum* was false but possibly true, possible *positio*, or an impossibility, impossible *position*. The original motivation for the latter seems to have been Boethius's proposal in *De hypotheticis syllogismis* that an impossibility be posited in order to see what follows (*Hyp. Syll.* I.2.6).

The earliest surviving treatise on impossible *positio*, the *Tractatus Emmeranus*, recognizes that no coherent argumentation is available under such an hypothesis if one accepts that anything follows from an impossibility. It stipulates instead that reasoning in impossible *positio* should rely only on consequences in which the consequent is contained in the antecedent and so not employ those with an affirmative antecedent and negative consequent—the theory uniquely characteristic of Abelard and the *Nominales* (De Rijk 1974). Later treatments of *impossible position* require only that they be conducted using consequences satisfying the containment condition.

In accounts of possible *positio* written before 1330s the respondent's answers are required to be consistent with everything that has gone before. He must thus concede a *propositum* which follows from the conjunction of the *positum* with all *proposita* already conceded and the contradictories of those which have been denied and deny a *propositum* whose contradictory follows from this conjunction. A *propositum* is irrelevant if neither it nor its contradictory follows from the conjunction and the respondent is required, if it is true, to concede it and, if it is false, to deny it (Kretzmann et al. 1982, ch. 16A).

A well conducted *positio* thus yields a set of propositions cotenable with the original *positum* and so an account of how the world might be. In treatises on possible *positio* written before the beginning of the fourteenth century we find a rule to the effect that if *n* is the present time, the *propositum* "*n* is the present time" must be denied, since it is not possible for things now to be other than now they are. Duns Scotus rejects this principle in

setting out his new account of possibility and it is no longer found in fourteenth century accounts of *positio*.

Possible *positio* provides a way of testing the respondent's reasoning skills but also of constructing alternative possible world-histories. This application is common in fourteenth century treatments of reconciliation of divine foreknowledge with the possibility that things might be otherwise than they will be.

In the mid-1330s a group of logicians at Oxford proposed modifications to the principles of *position*. Richard Kilvington (c. 1305–1361) in his *Sophismata* required that the respondent answer an irrelevant positum not in accordance with his beliefs about its actual truth-value but rather in accordance with the beliefs he would have if the *positum* were true. Kilvington noticed that these may well differ if the *positum* refers to the respondent's epistemic states (Kretzmann et al. 1982, ch. 16B).

Roger Swineshead (d. 1356) went much further in his *Obligationes* (1340s?) and proposed what became known as the "new response" (Kretzmann et al. 1982, ch. 16B). For reasons which remain obscure he required the respondent simply to concede a *propositum* if it follows from the *positum* alone and to deny it if is incompatible. Everything else is irrelevant. This change, however, undermines the constructive character of *position* since, for example, if some false proposition *p* is posited and *q* is an irrelevant truth, the respondent must concede both *p* and *q* when they are proposed but go on to deny their conjunction *p and q*. Swineshead's account of *position* seems to have enjoyed some limited success but it is not mentioned after the end of the fourteenth century.

INSOLUBLES

The most famous example of what medievals called insolubles, sentences difficult but not impossible to solve, is the Liar: "This sentence is false" (Spade 1988). The difficulty is to assign it a truth-value since it seems that if it is true, then it is false, and if it is false, then it is true. The problem is first noticed the middle ages in the *Ars disserendi* of Adam of the Little Bridge published in 1132 and its medieval origins may well lie in reflection on possible *positio*.

Both the *Tractatus Emmeranus* and another treatise from the second half of the twelfth century, the *Obligationes Parisiensis* (De Rijk 1975), note that if a respondent accepts as a *positum* "the *positum* is false" or an equivalent, then the opponent will be able to force him to contradict himself (Martin 1993). Both works go on to discuss propositions such as "a falsehood is conceded"

which may be certainly be posited but cannot then be conceded as the rules of *position* require since if it is, it becomes a Liar. The appropriate response, they claim, is to reply "You are not saying anything" (*nugaris*).

The earliest known treatise entirely devoted to the Liar, the *Insolubilia Monacensis*, from roughly the same date, adopts the same solution, voiding (*cassatio*): A self-referential utterance of "this sentence is false" fails to assert anything (De Rijk 1966). This solution continued to be invoked in the thirteenth century but is no longer employed in the heyday of insoluble literature, the first half of the fourteenth century.

Many different solutions were proposed to the problem and Thomas Bradwardine (c. 1295–1349) lists eight others besides *cassatio* in his *Insolubilia* (Spade 1988). These include, for example:

1) *Secundum quid et simpliciter* (qualified and unqualified): Distinguish between the qualified and unqualified possession of a property as Aristotle does in the *Sophistical Refutations* discussing the puzzle of a man who takes an oath to break his oath. The Liar is false without qualification, but relatively true.

2) *Transcasus* (change of situation): The claim made in uttering the Liar refers to an instant before the utterance. The Liar is simply false since the speaker said nothing then.

3) *Restrictio* (restriction): The supposition of the term "false" in the Liar is restricted to standing only for sentences other than the Liar or sentences equivalent to it. Since uttering the Liar utters only that sentence, it is simply false.

Bradwardine rejected all the theories in his list and offered a new one which set the agenda for later discussions. He maintained, first, that a proposition is true if it signifies things *only* as they are but is false if it signifies things as other than they are—it may well also signify them as they are. Second, he held, and seems to have been the first to do so that a proposition signifies just what follows from it. Bradwardine concluded that if a proposition signifies itself to be false, then it signifies itself to be true. The Liar thus signifies itself to be both true and false and so is false (Roure 1970).

CONSEQUENCES

Treatises devoted to consequences seem to be product of the fourteenth century and, although one was written by the great Parisian logician John Buridan, they are almost exclusively a British production. The second or third decade of the fourteenth century marks a turning point in the history of consequences as important as the resolution of the twelfth-century crisis (Martin 2005).

Duns Scotus was not a logician but he put logic to the service of metaphysics when he located a formal distinction between any two items which are actually inseparable but conceptually separable. If being *B* follows accidentally but not naturally from being *A*, then being *A* is formally but not existentially distinct from being *A*.

Ockham's rejection of the formal distinction seems to explain his introduction of an entirely new theory of consequences. In his *Summa logicae* rather than distinguishing between natural and accidental consequences by appealing to *loci* which guarantee containment in contrast to those which do not, he takes basic logical distinction to be between what he calls material and formal consequences (*Sum. Log.* III.3.1).

All consequences must satisfy the inseparability requirement. Material consequences satisfy it merely in virtue of truth-values of the antecedent and consequent and so include all the paradoxical consequences. Formal consequences hold in virtue of there being a connection between antecedent and consequent guaranteed by a middle, another name for a *locus*. The middle, however, is required only to guarantee non-trivial inseparability.

There is thus no logical distinction between consequences for Ockham corresponding to that between natural and accidental consequences. It is replaced by an appeal to the epistemological notion of evidence but this does not partition the class of true consequences in the way the natural—accidental distinction does. Nor, more importantly, can it be used to argue for the formal distinction.

In an alternative classification of consequences Ockham invokes a distinction already made the thirteenth century to consequences which satisfy the Inseparability condition in virtue of the necessity of the present. He holds that if the conjunction *p and not:q* is now false but at some time will be true, the truth of the antecedent is now inseparable from that of the consequent and so *if p, then q* is a consequence *ut nunc* (as-of-now). If *p and not:q* is false at all times, past, present, and future, according to Ockham, *if p, then q* is a simple consequence (*Sum. Log.* III.3.1).

Ockham's new theory of consequences seems to have very rapidly supplanted the old one and natural consequences are not mentioned in logic texts after the first quarter of the fourteenth century. Nor for that matter is impossible *positio*.

While Ockham's examples of the middles which provide the guarantee of formal consequence are all what we would classify as formal in that they hold for all uniform substitution instances of terms, his practice indicates that some middles hold only for limited classes of terms. This possibility is absent in later writers such as Buridan who explicitly defines formal consequence in terms of the uniform substitution of any terms satisfying the inseparability conditions.

By the middle of the fourteenth century the logic of consequences is thus fully formal in the modern sense and treatises on the subject contain many of the rules recognized in classical modal propositional logic.

THE LOGIC OF MODALITY

While the *Prior Analytics* offered logicians nothing on categorical syllogisms not already available in Boethius what Aristotle had to say about modal forms was extremely problematic (Lagerlund 2000). The difficulty is that he accepts modal conversion principles such as accidental conversion: *if every A is necessarily a B, then some B is necessarily an A* but also claims that while *every B is necessarily C and every A is B; therefore every A is necessarily C* is valid *every B is a C and every A is necessarily B; therefore every A is necessarily C* is not. The conversion seems only to hold only if the modality is understood in the composite sense while the claim about the syllogisms requires the divided sense.

The first known medieval solution is found in the commentary on the *Prior Analytics* written Robert Kilwardby (1215–1279) in the 1240s (Thom 2003). Aristotle had designated as *per se* predications in which the subject contains the predicate and Kilwardby claims that modality may be uniformly construed in the divided sense if the conversion principles are restricted to those in which the antecedents are per se predications. Thus "every man is necessarily an animal" converts accidentally with "some animal is necessarily human' but "every literate (man) is necessarily a man" does not convert in this way with "every man is necessarily literate." Kilwardby thus makes just the distinction between modal claims that was made between natural and accidental consequences.

Ockham in his *Summa logicae* explores the relationship between divided and composite readings on the basis of his claim that these do not differ in the case singular propositions (Normore 1999). He derives syllogisms for composite modals by applying to categorical syllogism the principles of modal inference, for example "if the premises are all necessary, then so is the conclusion." Ockham goes on to examine syllogisms formed with divided modals and with mixtures of both divided and composite (*Sum. Log.* III.1.20–46). He holds that divided claims are equivocal. Thus in "every A is possibly B," according to Ockham, the predicate is always ampliated by the mode but the supposition of subject may be understood to be only for what are now actually A or as ampliated for what can be A.

The most important development in syllogistics in the middle ages is in the work of Buridan. Buridan goes beyond Ockham in taking the theory of the syllogism to be simply an instance of the general theory of formal consequence (King 1985). He shows how the validity of the moods of the categorical syllogism can be proved from basic principles governing the semantics of general terms. The theory of modal syllogism with composite modality is, as with Ockham, quite straightforward. Buridan's treatment of divided modals is complex and of great interest since it reveals his attitude to the iteration of modalities and seems to commit him to the same principles as that of the modern system of strict implication known as S5.

Treatises on each of the subjects mentioned above continued to be produced through the fourteenth and fifteenth centuries by vast numbers of logicians. None of them, however, were of the stature of Abelard, Ockham, or Buridan, and originality in logic gave way at the end of the period to mere pedantry.

See also Abelard, Peter; Aristotle; Augustine, St.; Avicenna; Boethius, Anicius Manlius Severinus; Boetius of Dacia; Bradwardine, Thomas; Buridan, John; Burley, Walter; Cicero, Marcus Tullius; Conditionals; Duns Scotus, John; Gilbert of Poitiers; John of Salisbury; Kilwardby, Robert; Kilvington, Richard; Liar Paradox, The; Modality and Language; Peter of Spain; Porphyry; Proper Names and Descriptions; Propositions; Swineshead, Richard; Themistius; William of Ockham; William of Sherwood.

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INTERREGNUM (BETWEEN MEDIEVAL AND MODERN)

The interregnum between medieval scholastic logic and modern mathematical logic may be taken as having begun about the middle of the fifteenth century. There is no clear mark of division; the change was a shift away from the characteristic interests of the twelfth to the fifteenth century, with nothing of comparable importance arising to take their place. At the same time, certain less desirable trends in scholastic logic were perpetuated. The result is that formal logic was reduced almost entirely to a very imperfectly presented syllogistic. Medieval influences continued to operate in the early years of the sixteenth century, and medieval authors were still sometimes read in the seventeenth, but by the time that William of Ockham's *Summa Logicae* was printed at Oxford in 1675, no one had written creatively in the idiom of scholastic logic for many years.

The interregnum was characteristically sterile, a cause for despondency when one thinks of the large place logic continued to occupy in the educational curriculum and of the innumerable writers who put manuals of logic on the market. The tendency to publish at all costs was encouraged by the post-Reformation and post-Tridentine growth of universities, colleges, and seminaries.

VALLA

The first author to consider is the humanist Lorenzo Valla (1407–1457), best remembered for his writing on the forged donation of Constantine. In his *Dialecticarum Libri Tres* (1441), Valla gave no definitions of syllogistic figures and moods, evidently assuming that the reader would know about these. His aim was to confine the syllogistic to the first two figures, without the five moods of

Theophrastus and Eudemus. To do this he would have had to reject subalternation, conversion, and reductio ad absurdum. About subalternation he was inconsistent; conversion he rejected as lacking brevity, ease, pleasantness, and utility; reductio ad absurdum he largely neglected. The five offending moods were called "Agrippine births," and of them all the most monstrous was "Frisemomorum, forsooth!"

Here we see the common humanist objection to the barbarity of scholastic terminology, but of course Valla was not objecting merely to comparatively recent Scholastics. His fullest invective was saved for the six moods of the third figure, which he thought insane and never found in use, unlike the first-figure and second-figure moods, which he accepted as dictated by nature to everyone, "even peasants, even women, even children." The standard means of reduction are but "remedies for sick syllogisms." The standing of the third figure would remain a point of dispute for a hundred years, until Ramus undercut Valla's argument by declaring that the figure was in obvious fact very commonly used (*Institutionum Dialecticarum Libri Tres*, Paris, 1554). Thus, Philipp Melanchthon (*Compendiaria Dialectices Ratio*, Basel, 1521) could not make up his mind on the subject.

MELANCHTHON

In Melanchthon (1497–1560), a most influential writer, the rhetorical approach to logic already appeared at a high state of development, although he retained some Aristotelian doctrine. The rhetorical tradition, derived from Cicero and Quintilian, had a place, albeit a very subordinate one, in scholastic logic. We can see it beginning to predominate in the *Dialectica ad Petrum de Medicis* (edited by D. M. Inguanez and D. G. Muller, Monte Cassino, 1943; composed about 1457), by Joannes Argyropoulos, who held that the detail of the theory of *suppositio*, which was the distinctive and most original scholastic contribution to logic, offered almost nothing to oratorical practice.

Thus, scholastic logic, which in its origins had borrowed considerably from grammar, began to yield to the third member of the trivium, rhetoric. Accordingly Melanchthon declared the fruit of dialectic to be the ability to speak with propriety and exactness on any theme, and he expounded the Ciceronian syllogism, with its five parts—*propositio, approbatio, assumptio, assumptionis approbatio*, and *complexio*—before the Aristotelian. (A century later a similar five-part syllogism, with proposition, reason, example, application, and conclusion, came into favor in the New Nyāya school of Indian logic.) In