

mation, and capital across national borders makes it increasingly difficult for contemporary nations to manage their own economies. The liability of some states to the environmental consequences of actions undertaken by other states and the corporations they house implies that there are important parts of a state—the quality of its air and water—that some governments cannot be presumed control.

Even when states are able significantly to control their economies or their environmental quality, they may think it wise to cede a certain amount of control over their economies, their environments, or the pursuit of their national security interests to multinational unions such as NATO and the European Union. Such surrender of control is a surrender of some of the powers of sovereignty. Thus are the increasing importance of non-state actors, globalization, and the emergence of economic, political, and military unions all thought to erode the sovereignty the post-Westphalian model ascribes to states.

Why question whether states should enjoy the sovereignty the post-Westphalian model ascribes to them? The sovereignty of a state is usually taken to imply that it has a very strong presumption of control over the natural resources that lie within its borders. According to this view, a state can extract, consume, or conserve those resources as it sees fit. But it is surely open to question whether states are morally entitled to deplete a resource the rest of the world needs, to control a river on which citizens of another state downstream depend, or to exacerbate global inequalities of wealth by profiting excessively from a resource it happens to possess. Furthermore, it is open to question whether states are morally entitled to control access to its resources and opportunities by forbidding or restricting the movement of people across its borders. So-called “failed states” may lack the capacity to address humanitarian crises that affect their citizens. They can also harbor terrorist and criminal organizations that threaten international order. The incapacities of failed states, and the dangers they pose, are sometimes thought to license foreign intervention even if such intervention entails a violation of state sovereignty.

Perhaps the most profound challenge to the post-Westphalian model is posed by growing international recognition of human rights. These rights are rights that people enjoy simply in virtue of their humanity. While the list and the philosophical foundations of human rights remains disputed, it is increasingly accepted that there are such rights, that they limit what governments may do to their people and that the gross and widespread

violation of such rights by a government may give non-governmental organizations, other states, and international bodies the right to intervene. The easier it is to defeat the presumption of non-intervention in such cases, the greater the challenge a global regime of human rights poses to the post-Westphalian model and to the forms of sovereignty that model implies. With the rejection of the post-Westphalian model as descriptively or normatively inadequate, its displacement by another model of the political world, or the loosening of its hold on the imagination of political theorists, sovereignty would cease to be the central organizing concept it long has been.

*See also* Civil Disobedience; Cosmopolitanism; Multiculturalism; Postcolonialism; Republicanism

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**Paul Weithman (2005)**

## SPACE

When men began to think about the nature of “space,” they thought of it as an all-pervading ether or as some sort of container. Since a thing can move from one part of space to another, it seemed that there was something, a place or a part of space, to be distinguished from the material objects that occupy space. For this reason places might be thought of as different parts of a very subtle jellylike medium within which material bodies are located.

### HISTORY OF THE CONCEPT OF SPACE

Some of the Pythagoreans seem to have identified empty space with air. For more special metaphysical reasons Parmenides and Melissus also denied that there could be truly empty space. They thought that empty space would be nothing at all, and it seemed to them a contradiction to assert that a nothing could exist. On the other hand,

there seems to be something wrong with treating space as though it were a material, which, however subtle, would still itself have to be *in* space. Democritus and the atomists clearly distinguished between the atoms and the void that separated them. However, the temptation to think of space as a material entity persisted, and Lucretius, who held that space was infinite, nevertheless wrote of space as though it were a container. Yet he seems to have been clear on the fact that space is unlike a receptacle in that it is a pure void. Since material bodies, in his view, consist of atoms, there must be chinks of empty space even between the atoms in what appear to be continuous bodies.

Plato's views on space have to be gotten mainly from the obscure metaphors of the *Timaeus*; he, too, appears to have thought of space as a receptacle and of the matter in this receptacle as itself mere empty space, limited by geometrical surfaces. If so, he anticipated the view of René Descartes, where the problem arises of how empty space can be distinguished from nonempty space. Even if, like Lucretius and other atomists, we make a distinction between the atoms and the void, what is this void or empty space? Is it a thing or not a thing?

ARISTOTLE. Aristotle tried to dodge the difficulty by treating the concept of space in terms of place, which he defined as the adjacent boundary of the containing body. For two things to interchange places *exactly*, they would have to be identical in volume and shape. Consider two exactly similar apples that are interchanged in this way. The *places* are not interchanged; rather, the first apple is now at the very same place at which the second apple was and vice versa. We seem, therefore, to be back at the notion of space as a substratum or ether, but it is probable that Aristotle was trying to avoid this and that he meant to define place by reference to the cosmos as a whole. Aristotle thought of the cosmos as a system of concentric spheres, and the outermost sphere of the cosmos would, on his view, define all other places in relation to itself. In the Aristotelian cosmology each of the various "elements" tends toward its own place. Thus, heavy bodies tend toward the center of Earth, and fire goes away from it. This is not, however, for any other reason than that the center of Earth happens to be the center of the universe; the places toward which the elements tend are independent of what particular bodies occupy what places. In more recent times we view these as two different and seemingly irreconcilable ways of thought—the notions of space as a stuff and of space as a system of relations between bodies.

DESCARTES AND LEIBNIZ. Descartes held that the essence of matter is extension, and so, on his view, space and stuff are identical, for if the essence of matter is to be extended, then any volume of space must be a portion of matter, and there can be no such thing as a vacuum. This raises the question of how we can distinguish one material object (in the ordinary sense of these words) from another. How, on Descartes's view, can we elucidate such a statement as that one bit of matter has moved relative to another one? In what sense, if matter just *is* extension, can one part of space be more densely occupied by matter than another? Descartes considered these objections but lacked the mathematical concepts necessary to answer them satisfactorily. We shall see that a reply to these objections can be made by denying that space is the same everywhere, and this can be done by introducing the Riemannian concept of a space of variable curvature.

As against Descartes, Gottfried Wilhelm Leibniz held a relational theory of space, whereby space is in no sense a stuff or substance but is merely a system of relations in which indivisible substances, or "monads," stand to one another. Few philosophers have followed Leibniz in his theory of monads, but in a slightly different form the relational theory of space has continued to rival the Cartesian, or "absolute," theory. The issue between the two theories has by no means been decisively settled, at least if we consider not space but space-time. It is still doubtful whether the general theory of relativity can be stated in such a way that it does not require absolute space-time.

KANT. In his *Prolegomena*, Immanuel Kant produced a curious argument in favor of an absolute theory of space. Suppose that the universe consisted of only one human hand. Would it be a left hand or a right hand? According to Kant it must be one or the other, yet if the relational theory is correct it cannot be either. The relations between the parts of a left hand are exactly the same as those between corresponding parts of a right hand, so if there were nothing else to introduce an asymmetry, there could be no distinction between the case of a universe consisting only of a left hand and that of a universe consisting only of a right hand. Kant, however, begged the question; in order to define "left" and "right" we need the notions of clockwise and counterclockwise rotations or of the bodily asymmetry which is expressed by saying that one's heart is on the left side of one's body. If there were only one hand in the world, there would be no way of applying such a concept as left or clockwise. The relationist could therefore quite consistently reply to Kant that if there were only one thing in the universe, a human hand,

it could not *meaningfully* be described as either a right one or a left one. (The discovery in physics that parity is not conserved suggests that the universe is not symmetrical with respect to mirror reflection, so there is probably, in fact, something significant in nature analogous to the difference between a left and a right hand.)

Later, in his *Critique of Pure Reason*, Kant argued against both a naive absolute theory of space and a relational view. He held that space is something merely subjective (or “phenomenal”) wherein in thought we arrange nonspatial “things-in-themselves.” He was led to this view partly by the thought that certain antinomies or contradictions are unavoidable as long as we think of space and time as objectively real. However, since the work of such mathematicians as Karl Theodor Wilhelm Weierstrass, Augustin-Louis Cauchy, Julius Wilhelm Richard Dedekind, and Georg Cantor, we possess concepts of the infinite which should enable us to deal with Kant’s antinomies and, indeed, also to resolve the much earlier, yet more subtle, paradoxes of Zeno of Elea.

#### NEWTON’S CONCEPTION OF SPACE

Isaac Newton held absolute theories of space and time—metaphysical views that are strictly irrelevant to his dynamical theory. What is important in Newtonian dynamics is not the notion of absolute space but that of an inertial system. Consider a system of particles acting on one another with certain forces, such as those of gravitational or electrostatic attraction, together with a system of coordinate axes. This is called an inertial system if the various accelerations of the particles can be resolved in such a way that they all occur in pairs whose members are equal and lie in opposite directions in the same straight line. Finding an inertial system thus comes down to finding the right set of coordinate axes. This notion of an inertial system, not the metaphysical notion of absolute space, is what is essential in Newtonian dynamics, and as Ernst Mach and others were able to show, we can analyze the notion of an inertial system from the point of view of a relational theory of space. Psychologically, no doubt, it was convenient for Newton to think of inertial axes as though they were embedded in some sort of ethereal jelly—absolute space. Nevertheless, much of the charm of this vanishes when we reflect that, as Newton well knew, any system of axes that is moving with uniform velocity relative to some inertial system is also an inertial system. There is reason to suppose, however, that in postulating absolute space Newton may have been partly influenced by theological considerations that go back to Henry More and, through More, to cabalistic doctrines.

We can remove the metaphysical trappings with which Newton clothed his idea of an inertial system if we consider how in mechanics we determine such a system. But even before we consider how we can define an inertial system of axes, it is interesting to consider how it is possible for us to define any system of axes and spatial positions at all. As Émile Borel has remarked, how hard it would be for a fish, however intelligent, which never perceived the shore or the bottom of the sea to develop a system of geometrical concepts. The fish might perceive other fish in the shoal, for example, but the mutual spatial relations of these would be continually shifting in a haphazard manner. It is obviously of great assistance to us to live on the surface of an earth that, if not quite rigid, is rigid to a first order of approximation. Geometry arose after a system of land surveying had been developed by the Egyptians, who every year needed to survey the land boundaries obliterated by the flooding of the Nile. That such systems of surveying were possible depended on certain physical facts, such as the properties of matter (the nonextensibility of chains, for example) and the rectilinear propagation of light. They also depended on certain geodetic facts, such as that the tides, which affect even the solid crust of Earth, were negligible. The snags that arise when we go beyond a certain order of approximation were unknown to the Egyptians, who were therefore able to get started in a fairly simple way.

It might be tempting to say that it was fortunate that the Egyptians were unaware of these snags, but of course in their rudimentary state of knowledge they could not have ascertained these awkward facts anyway. When, however, we consider geodetic measurements over a wide area of the globe we need to be more sophisticated. For example, the exact shape of Earth, which is not quite spherical, needs to be taken into account. Moreover, in determining the relative positions of points that are far apart from one another it is useful to make observations of the heavenly bodies as seen simultaneously from the different points. This involves us at once in chronometry. There is thus a continual feedback from physics and astronomy. Increasingly accurate geodetic measurements result in more accurate astronomy and physics, and more accurate astronomy and physics result in a more accurate geodesy.

Such a geodetic system of references is, however, by no means an inertial one. An inertial system is one in which there are no accelerations of the heavenly bodies except those which can be accounted for by the mutual gravitational attractions of these bodies. It follows, therefore, that the directions of the fixed stars must not be

rotating with respect to these axes. In principle we should be able to determine a set of inertial axes from dynamical considerations, even if we lived in a dense cloud, as on Venus, and were unaware of the existence of the fixed stars. This may have influenced Newton to think of space as absolute. However, Newton was not on Venus, and he could see the fixed stars. It is therefore a little surprising that he did not take the less metaphysical course of supposing an inertial system to be determined by the general distribution of matter in the universe. This was the line taken in the nineteenth century by Mach and is referred to (after Albert Einstein) as Mach's principle. It is still a controversial issue in cosmology and general relativity.

Mach's principle clearly invites, though it does not compel, a relational theory of space, such as Mach held. The origin of the axes of an inertial system in Newtonian mechanics was naturally taken to be the center of gravity of the solar system, which is nearly, but not quite, at the center of the sun. In fact, it is continually changing its position with reference to the center of the sun. Now that the rotation of the galaxy has been discovered, we have to consider the sun as moving around a distant center. We shall here neglect the possibility that our galaxy is accelerating relative to other galaxies. In any case, once we pass to cosmological considerations on this scale we need to abandon Newtonian theory in favor of the general theory of relativity.

The philosophical significance of the foregoing discussion is as follows: When we look to see how inertial axes are in fact determined we find no need to suppose any absolute space. Because such a space would be unobservable, it could never be of assistance in defining a set of inertial axes. On the other hand, the complexities in the determination of inertial axes are such that it is perhaps psychologically comforting to think of inertial axes, or rather some one preferred set of such axes, as embedded in an absolute space. But Newton could equally have taken up the position, later adopted by Mach, that inertial systems are determined not by absolute space but by the large-scale distribution of matter in the universe.

#### SPACE AND TIME IN THE SPECIAL THEORY OF RELATIVITY

We have already noticed the dependence of space measurements on time measurements which sometimes obtains in geodesy. This situation is accentuated in astronomy because of the finite velocity of light. In order to determine the position of a heavenly body we have to make allowance for the fact that we see it in the position it was in some time ago. For example, an observation of a

star that is ten light-years away is the observation of it in its position years ago. Indeed, it was the discrepancy between the predicted and observed times at which eclipses of the satellites of Jupiter should occur that led Olaus Rømer to assign a finite, and approximately correct, value to the velocity of light. The correction of position and time on account of the finite velocity of light presupposes in any particular case our knowing what this velocity is, relative to Earth. This would seem to depend not only on the velocity of light relative to absolute space (or to some preferred set of inertial axes) but also on Earth's velocity relative to absolute space (or to the preferred set of inertial axes). The experiment of Albert Abraham Michelson and Edward Williams Morley showed, however, that the velocity of light relative to an observer is independent of the velocity of the observer. This led to the special theory of relativity, which brings space and time into intimate relation with one another. For present purposes it is necessary to recall only that according to the special theory of relativity events that are simultaneous with reference to one inertial set of axes are not simultaneous with reference to another inertial frame. The total set of point-instants can be arranged in a four-dimensional space-time. Observers in different inertial frames will partition this four-dimensional space-time into a "space" and a "time," but they will do so in different ways.

Before proceeding further it is necessary to clear up a certain ambiguity in the word *space*. So far in this entry space has been thought of as a continuant. In this sense of the word *space* it is possible for things to continue to occupy space and to move from one point of space to another and for regions of space to begin or cease to be occupied or to stay occupied or unoccupied. Here space is something that endures through time. On the other hand, there is a different, timeless use of the word *space*. In solid geometry a three-dimensional space is thought of as timeless. Thus, if a geometer said that a sphere had changed into a cube, he would no longer be thinking within the conceptual scheme of solid geometry. In geometry all verbs must be tenseless. In this tenseless way let us conceive of a four-dimensional space-time, three of whose dimensions correspond roughly to the space of our ordinary thought whereas the other corresponds to what we ordinarily call time. What we commonly think of as the state of space at an instant of time is a three-dimensional cross section of this four-dimensional space-time.

Taking one second to be equivalent to 186,300 miles, which is the distance light travels in that time, any physical object, such as a man or a star, would be rather like a

four-dimensional worm—its length in a timelike direction would be very much greater than its spacelike cross section. Thinking in terms of space-time, then, two stars that are in uniform velocity with respect to each other and also with respect to our frame of reference will appear as two straight worms, each at a small angle to the other. An observer on either star will regard himself as at rest, so he will take his own world line—the line in space-time along which his star lies—as the time axis. He will take his space axes as (in a certain sense) perpendicular to the time axis. It follows that observers on stars that move relative to one another will slice space-time into spacelike cross sections at different angles. This makes the relativity of simultaneity look very plausible and no longer paradoxical. As Hermann Minkowski observed, the relativity of simultaneity could almost have been predicted from considerations of mathematical elegance even before the experimental observations that led to the special theory of relativity. Indeed, Minkowski showed that the Lorentz transformations of the theory of relativity can be understood as simply a rotation of axes in space-time. (In trying to picture such a rotation of axes it is important to remember that Minkowski space-time is not Euclidean but semi-Euclidean.) In Minkowski's words, "Henceforth space by itself, and time by itself, are doomed to fade away into mere shadows, and only a kind of union of the two will preserve an independent reality." We must not forget that space-time is a space in the mathematical sense of the word *space*, not in the sense in which space is a continuant. Thus, certain objectionable locutions are often used in popular expositions. For example, we sometimes hear it said that a light signal is propagated from one part of space-time to another. The correct way to put the matter is to say that the light signal *lies* (tenselessly) along a line between these two parts of space-time. Space-time is not a continuant and is not susceptible of change or of staying the same.

#### EUCLIDEAN AND NON-EUCLIDEAN SPACE

Geometry, as we observed earlier, developed out of experiences of surveying, such as those of the ancient Egyptians. The assumptions underlying the surveying operations were codified by Greek mathematicians, whose interests were mainly theoretical. This codification was developed by Euclid in the form of an axiomatic system. Euclid's presentation of geometry shows a high degree of sophistication, though it falls considerably short of modern standards of rigor. Euclid's geometry was a metrical one. There are, of course, geometries that

are more abstract than metrical geometry. The most abstract of all is topology, which deals with those properties of a space that remain unchanged when the space is distorted, as by stretching. Thus, from the point of view of topology a sphere, an ellipsoid, and a parallelepiped are identical with one another and are different from a torus. Metrical geometry uses a bigger battery of concepts—not only such notions as those of betweenness and of being longer than (which itself goes beyond topology) but also those of being, say, twice or three and a half times as long as.

Euclid regarded one of his axioms as more doubtful than the others. This is the axiom that is equivalent to the so-called axiom of parallels. It will be more convenient to discuss the axiom of parallels than Euclid's own axiom. The axiom of parallels states that if  $C$  is a point not on an infinite straight line  $AB$ , then there is one and only one straight line through  $C$  and in the plane of  $AB$  that does not intersect  $AB$ . Geometers made many efforts to deduce the axiom of parallels from the other, more evident ones. In the seventeenth and eighteenth centuries Gerolamo Saccheri and J. H. Lambert each tried to prove the axiom by means of a *reductio ad absurdum* proof. By assuming the falsity of the axiom of parallels they hoped to derive a contradiction. They did not succeed; in fact, Saccheri and Lambert proved a number of perfectly valid theorems of non-Euclidean geometry, though they were not bold enough to assert that this was what they were doing.

János Bolyai and N. I. Lobachevski replaced the axiom of parallels with the postulate that *more than one* parallel can be drawn. The type of geometry that results is called hyperbolic. Another way to deny the axiom of parallels is to say that *no* parallel can be drawn. This yields elliptic geometry. (Some adjustments have to be made in the other axioms. For instance, straight lines become finite, and two points do not necessarily determine a straight line.) It is easy to prove (by giving a non-Euclidean geometry an interpretation within Euclidean geometry) that both hyperbolic and elliptic geometries are consistent if Euclidean geometry is. (And all can easily be shown to be consistent if the theory of the real-number continuum is.) A priori, therefore, there is nothing objectionable about non-Euclidean geometries. Unfortunately, many philosophers followed Kant in supposing that they had an intuition that space was Euclidean, and mathematicians had to free themselves from this conservative climate of opinion.

The question then arose whether our actual space is Euclidean or non-Euclidean. In order to give sense to this question we must give a physical interpretation to our

geometric notions, such as that of a straight line. One way of defining a straight line is as follows: Suppose that rigid bodies  $A, B, C$  have surfaces  $S_A, S_B, S_C$ , such that when  $A$  is applied to  $B$ , then  $S_A$  and  $S_B$  fit; when  $B$  is applied to  $C$ , then  $S_B$  and  $S_C$  fit; and when  $C$  is applied to  $A$ , then  $S_C$  and  $S_A$  fit. Suppose also that  $S_A, S_B, S_C$  can all be slid and twisted over one another—that is, that they are not like cogged gears, for example. Then  $S_A, S_B, S_C$  are all by definition plane surfaces. The intersection of two planes is a straight line. (In the above we have used the notion of a rigid body, but this can easily be defined without circularity.) With the above definition of a straight line and the like we can make measurements to tell whether the angles of a triangle add up to two right angles. If they make more than two right angles, space is elliptic; if less than two right angles, space is hyperbolic; and if exactly two right angles, space is Euclidean. However, such experiments could not determine the question to any high degree of accuracy. All that this method shows is that, as every schoolchild knows, physical space is *approximately* Euclidean.

To make measurements that could settle the question to any high degree of accuracy we should have to make them on an astronomical scale. On this scale, however, it is not physically possible to define straight lines by means of the application of rigid bodies to one another. An obvious suggestion is that we should define a straight line as the path of a light ray in empty space. One test of the geometry of space might then come from observations of stellar parallax. On the assumption that space is Euclidean, the directions of a not very distant star observed from two diametrically opposite points on Earth's journey round the sun will be at a small but observable angle. If space is hyperbolic, this angle, which is called the parallax, will be somewhat greater. If space is elliptic, the parallax will be less or even negative. If we knew the distance of the star, we could compare the observed parallax with the theoretical parallax, on various assumptions about the geometry. But we cannot know the distances of the stars except from parallax measurements. However, if space were markedly non-Euclidean, we might get some hint of this because the distribution of stars in space, calculated from parallax observations on Euclidean assumptions, would be an improbable one. Indeed, at the beginning of the twentieth century Karl Schwarzschild made a statistical analysis of parallaxes of stars and was able to assign an upper limit to the extent to which physical space deviates from the Euclidean.

A good indication that space, on the scale of the solar system at least, is very nearly Euclidean is the fact that

geometrical calculations based on Euclidean assumptions are used to make those predictions of the positions of the planets that have so strongly confirmed Newtonian mechanics. This consideration points an important moral, which is that it is impossible to test geometry apart from physics; we must regard geometry as a part of physics. In 1903, Jules Henri Poincaré remarked that Euclidean geometry would never be given up no matter what the observational evidence was; he thought that the greater simplicity of Euclidean, as against non-Euclidean, geometry would ensure our always adopting some physical hypothesis, such as that light does not always travel in straight lines, to account for our observations. We shall not consider whether—and if so, in what sense—non-Euclidean geometry is necessarily less simple than Euclidean geometry. Let us concede this point to Poincaré. What he failed to notice was that the greater simplicity of the geometry might be bought at the expense of the greater complexity of the physics. The total theory, geometry plus physics, might be made more simple even though the geometrical part of it was more complicated. It is ironical that not many years after Poincaré made his remark about the relations between geometry and physics he was proved wrong by the adoption of Einstein's general theory of relativity, in which overall theoretical simplicity is achieved by means of a rather complicated space-time geometry.

In three-dimensional Euclidean space let us have three mutually perpendicular axes,  $Ox_1, Ox_2, Ox_3$ . Let  $P$  be the point with coordinates  $(x_1, x_2, x_3)$ , and let  $Q$  be a nearby point with coordinates  $(x_1 + dx_1, x_2 + dx_2, x_3 + dx_3)$ . Then if  $ds$  is the distance  $PQ$ , the Pythagorean theorem

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2$$

holds. In a "curved," or non-Euclidean, region of space this Pythagorean equation has to be replaced by a more general one. But before considering this let us move to four dimensions, so that we have an additional axis,  $Ox_4$ . This four-dimensional space would be Euclidean if

$$ds^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2.$$

In the general case

$$\begin{aligned} ds^2 = & g_{11}dx_1^2 + g_{22}dx_2^2 + g_{33}dx_3^2 + g_{44}dx_4^2 \\ & + 2g_{12}dx_1dx_2 + 2g_{13}dx_1dx_3 + 2g_{14}dx_1dx_4 \\ & + 2g_{23}dx_2dx_3 + 2g_{24}dx_2dx_4 + 2g_{34}dx_3dx_4. \end{aligned}$$

The  $g$ 's are not necessarily constants but may be functions of  $x_1, x_2, x_3, x_4$ . That it is impossible to choose a coordinate system such that for a certain region  $g_{12}, g_{13}, g_{14}, g_{23},$

$g_{24}$ ,  $g_{34}$  are all zero is what is meant by saying that the region of space in question is curved. That a region of space is curved can therefore in principle always be ascertained by making physical measurements in that region—for instance, by testing whether the Pythagorean theorem holds. There is, therefore, nothing obscure or metaphysical about the concept of curvature of space. The space-time of special relativity, it is worth mentioning, is semi-Euclidean and of zero curvature. In it we have

$$g_{11} = g_{22} = g_{33} = -1, \quad g_{44} = +1,$$

and  $g_{12}$ ,  $g_{13}$ ,  $g_{14}$ ,  $g_{23}$ ,  $g_{24}$ ,  $g_{34}$  are all zero.

According to the general theory of relativity, space-time is curved in the neighborhood of matter. (More precisely, it has a curvature over and above the very small curvature that, for cosmological reasons, is postulated for empty space.) A light wave or any free body, such as a space satellite, is assumed in the general theory to lie along a geodesic in space-time. A geodesic is either the longest or the shortest distance between two points. In Euclidean plane geometry it is the shortest, whereas in the geometry of space-time it happens to be the longest. Owing to the appreciable curvature of space-time near any heavy body, a light ray that passes near the sun should appear to us to be slightly bent—that is, there should be an apparent displacement of the direction of a star whose light passes very near the sun. During an eclipse of the sun it is possible to observe stars very near to the sun's disk, since the glare of the sun is blacked out by the moon. In the solar eclipse of 1919, Sir Arthur Stanley Eddington and his colleagues carried out such an observation that gave results in good quantitative accord with the predictions of relativity. In a similar way, also, the general theory of relativity accounted for the anomalous motion of the perihelion of Mercury, the one planetary phenomenon that had defied Newtonian dynamics. In other cases the predictions of Newtonian theory and of general relativity are identical, and general relativity is, on the whole, important only in cosmology (unlike the special theory, which has countless verifications and is an indispensable tool of theoretical physics).

### IS SPACE ABSOLUTE OR RELATIVE?

The theory of relativity certainly forces us to reject an absolute theory of space, if by this is meant one in which space is taken as quite separate from time. Observers in relative motion to one another will take their space and time axes at different angles to one another; they will, so to speak, slice space-time at different angles. The *special* theory of relativity, at least, is quite consistent with either

an absolute or a relational philosophical account of space-time, for the fact that space-time can be sliced at different angles does not imply that it is not something on its own account.

It might be thought that the *general* theory of relativity forces us to a relational theory of space-time, on the grounds that according to it the curvature of any portion of space-time is produced by the matter in it. But if anything the reverse would seem to be the case. If we accept a relational theory of space-time, we have to suppose that the inertia of any given portion of matter is determined wholly by the total matter in the universe. Consider a rotating body. If we suppose it to be fixed and everything else rotating, then we must say that some distant bodies are moving with transitional velocities greater than that of light, contrary to the assumptions of relativity. Hence, it is hard to avoid the conclusion that the inertia of a body is partly determined by the local metrical field, not by the total mass in the universe. But if we think of the local metrical field as efficacious in this way, we are back to an absolute theory of space-time. Furthermore, most forms of general relativity predict that there would be a curvature (and hence a structure) of space-time even if there were a total absence of matter. Indeed, relativistic cosmology often gives a picture of matter as consisting simply of regions of special curvature of space-time. (Whether this curvature is the cause of the existence of matter or whether the occurrence of matter produces the curvature of space-time is unclear in the general theory itself.) The variations of curvature of space-time enable us to rebut the objection to Descartes's theory that it cannot differentiate between more and less densely occupied regions of space.

Nevertheless, there are difficulties about accepting such a neo-Cartesianism. We must remember that quantum mechanics is essentially a particle physics, and it is not easy to see how to harmonize it with the field theory of general relativity. One day we may know whether a particle theory will have absorbed a geometrical field theory or vice versa. Until this issue is decided we cannot decide the question whether space (or space-time) is absolute or relational—in other words, whether particles are to be thought of as singularities (perhaps like the ends of J. A. Wheeler's "wormholes" in a multiply connected space) or whether space-time is to be understood as a system of relations between particles. This issue can be put neatly if we accept W. V. Quine's criterion of ontological commitment. Should our scientific theory quantify over point-instants of space-time, or should we, on the other hand, quantify over material particles, classes of them,

classes of classes of them, and so on? The latter involves a commitment to particle physics, but if a unified field theory is successful, our ontology may consist simply of point-instants, classes of them, classes of classes of them, and so on, and physical objects will be definable in terms of all of these. So far neither Descartes nor Leibniz has won an enduring victory.

**See also** Aristotle; Atomism; Cantor, Georg; Cartesianism; Descartes, René; Eddington, Arthur Stanley; Einstein, Albert; Geometry; Kant, Immanuel; Lambert, Johann Heinrich; Leibniz, Gottfried Wilhelm; Leucippus and Democritus; Logical Paradoxes; Lucretius; Mach, Ernst; Melissus of Samos; More, Henry; Newton, Isaac; Parmenides of Elea; Philosophy of Physics; Plato; Poincaré, Jules Henri; Pythagoras and Pythagoreanism; Quantum Mechanics; Quine, Willard Van Orman; Relativity Theory; Time; Zeno of Elea.

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J. J. C. Smart (1967)