certain real, intrinsic, and nonrelative MODE of a thing by which it is said to be somewhere (*DM* 51.1.13).

Beyond its character as one of the predicaments, *ubi* is also employed to address divine ubiquity. In this usage, Thomas explains that GOD is in every place, which is to be everywhere (*ubique*), by giving BEING to the creatures that fill every place (cf. *ST* 1, q. 8, a. 2). Relatedly, Suárez thinks that beginning from corporeal things, which are better known, "place" can apply to other, non-corporeal things, for example, the SOUL's presence in the body and God's presence to CREATION, by proportion or analogy (*DM* 51.3).

SEE ALSO Categories of Being.

BIBLIOGRAPHY

Albertus Magnus. *Alberti Magni opera omnia*. T. 1.P.2, *De sex principiis*. Edited by Ruth Meyer and Hannes Möhle. Cologne, Germany: Aschendorff, 2006.

Aristotle. *The Categories. On Interpretation. Prior Analytics*. Translated by H.P. Cooke and Hugh Tredennick. Cambridge, MA: Harvard University Press, 1938.

Aristotle. *Physics*. Vol. 1. Translated by Philip Henry Wicksteed and Francis Macdonald Cornford. Cambridge, MA: Harvard University Press, 1957.

Boethius, Manlius Severinus. "Commentarium in categorias Aristotelis" (ICA). In *Patrologia Latina*, edited by J.-P. Migne, 64: 159–294. Paris: Garnier Fratres, 1847.

Boethius, Manlius Severinus. "De hebdomadibus" (DH). In *Patrologia Latina*, edited by J.-P. Migne, 64. Paris: Garnier Fratres, 1847.

Hoenen, Petrus. *Cosmologia*, 5th ed. Rome: Universitas Gregoriana, 1956.

Masi, Roberto. *Cosmologia*. Rome: Desclée, 1961.

Poinsot, John. *Cursus philosophicus Thomisticus*, *Ars logica (CPT)*. Edited by B. Reiser. Turin: Marietti, 1930.

Suárez, Francisco, SJ. *Disputationes metaphysicae (DM)*. Paris: L. Vivès, 1856–1866.

Thomas Aquinas. *Commentaria in octo libros physicorum Aristotelis (Phys.)*. Rome: Ex Typographia Polyglotta de S. C. de Propaganda Fide, 1884.

Thomas Aquinas. *In duodecim libros metaphysicorum Aristotelis expositio (Metaph.)*. Edited by M. R. Cathala and Raymundi M. Spiazzi. 2nd ed. Turin-Rome: Marietti, 1971.

Thomas Aquinas. *Summae theologiae (ST)*. Rome: Ex Typographia Polyglotta de S. C. de Propaganda Fide, 1888–1889.

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LOGIC, HISTORY OF

ARISTOTLE (384–322 BC) provided the first systematic study of the principles and inferences of logical reasoning. Before him, he reports, "there was nothing at all." He focused primarily on a version of predicate logic. The Stoics then developed a version of propositional logic, investigating truth-conditional connections between propositions. Medieval logicians developed, systematized, and streamlined these approaches. In the fourteenth century they developed highly sophisticated semantic theories and began to investigate modality, opaque contexts, and anaphora. Most of their contributions were forgotten during the early modern period, only to be rediscovered during the twentieth century. Gottfried Wilhelm von Leibniz, George Boole, and Gottlob Frege applied mathematical techniques to logic, transforming it from a liberal art into a branch of mathematics and making possible the deep results proved by Kurt Gödel and others in the twentieth century.

Aristotle. The logical works of Aristotle, known as the *Organon*, include the *Categories*, *On Interpretation*, the *Prior Analytics*, the *Posterior Analytics*, the *Topics*, and *Sophistical Refutations*. They include much that in the twenty-first century would be classified as RHETORIC, philosophy of LANGUAGE, philosophy of SCIENCE, and METAPHYSICS. The last two works devote themselves to the construction rather than the evaluation of arguments.

The *Prior Analytics* contains Aristotle's theory of syllogisms. This is essentially a theory of the quantifiers "all," "some," and "no." Aristotle recognized that validity is a matter of FORM and introduced variables for terms or predicates. He characterized a realm of arguments and identified all the valid arguments within it, showing how to prove their validity by deriving them from others directly or by reduction. He thus devised the first natural deduction system, demonstrated its completeness, and proved various metatheorems about it. In some respects his theory is more powerful than modern first-order logic, for it treats quantifiers as relational rather than monadic. Aristotle's theory is in other ways less expressive than modern logic, for he excludes relational predicates; he had no theory of propositional connectives.

Aristotle's classification of syllogisms into figures was confusing, and he gave no general definition of major or minor terms. His square of opposition maintained (1) that All F are G and No F are G are contraries (i.e., cannot both be true); (2) that Some F are G and Some F are not G are subcontraries (i.e., cannot both be false); (3) that All F are G and Some F are not G are contradictories (i.e., exactly one is true), as are No F are G and Some F are G; and (4) that All F are G and No F are G, respectively, imply Some F are G and Some F are not G. These relations can all hold, however, only if all terms are true of at least some objects. His theory of modal syllogisms fails to note scope ambiguities.

Theophrastus (c. 372–c. 288 BC) succeeded Aristotle as head of the Peripatetic school. He made explicit five syllogistic forms Aristotle had failed to mention and introduced a non-Aristotelian modal syllogistic in which the modality of the conclusion follows that of the weakest premise. He also elaborated a theory of hypothetical syllogisms, that is, CONDITIONALS, by way of analogy to universal affirmative propositions. In other words, he interpreted If A, then B, as All cases in which A are cases in which B—a strategy for analyzing conditionals that persists in contemporary logic.

Stoic Logic. Stoic logicians, working independently of Aristotle and his followers, made three important contributions to the history of logic. Eubulides (fourth century BC) discovered the liar paradox: "If I say that I speak falsely, do I speak truly?" If "This sentence is false" is true, then it must be false; but if it is false, then, because that is precisely what it says, it must be true. This was the first of a series of PARADOXES that Greek logicians developed. Philetas of Cos (fourth century BC) blamed the liar paradox for his death, and Diodorus Cronos (fourth–third century BC) reportedly died, heartbroken, when he could not solve a logical puzzle the king posed at a banquet. Diodorus himself left his successors the "master argument" purporting to show the necessity of everything based on the historical necessity of the past.

The Stoics developed the first propositional logic, with modern notions of argument and PROPOSITION. They gave truth conditions for negation (not p is true if and only if p is false, and false if and only if p is true) as well as for multigrade versions of conjunction and disjunction (a conjunction is true if and only if all conjuncts are true, and false if and only if at least one conjunct is false; a disjunction is true if and only if at least one disjunct is true, and false if and only if every disjunct is false). Their preferred notion of disjunction, however, is a generalization of exclusive disjunction: it is true if and only if exactly one disjunct is true, and false if and only if every disjunct is false or more than one disjunct is true. They developed a natural deduction system of propositional logic and incorrectly claimed completeness for it.

The Stoics debated the nature of the conditional, intensely enough that Callimachus (third century BC) proclaimed, "Even the crows on the rooftops caw about which conditionals are true." The Stoics had four competing theories:

1. Philo: If A, then B is true if and only if not both A and not B. This is the material conditional of modern first-order logic.
2. Diodorus: If A, then B is true if and only if, at all times, not both A and not B.
3. Chrysippus: If A, then B is true if and only if, necessarily, not both A and not B. This is C. I. Lewis's strict conditional.
4. Anonymous: If A, then B is true if and only if A includes B. This, the option about which later logicians have the least information, seems to be intended to be stronger than Chrysippus's (c. 280–c. 207 BC) account, requiring something such as analytic or conceptual necessity.

The Stoics associate conditionals with arguments; some, at least, hold that an argument is valid if and only if the conditional with the conjunction of its premises as antecedent and its conclusion as consequent is true. This makes sense on the last two views of conditionals.

Medieval Logic: The Old Logic. Ancient logicians after Theophrastus and the Stoics preserved and systematized earlier contributions. CICERO (106–43 BC) serves as an important source for Stoic logic; he wrote a treatise on topics intended as a manual on argument construction for lawyers. GALEN (129–c. 199), too, commented on Stoic logic. Greek commentators on Aristotle, especially Alexander of Aphrodisias (b. c. 200), SIMPLICIUS (d. c. 560), and John PHILOPONOUS (c. 490–c. 570), made theoretical contributions, improving Aristotle's account of immediate inference as well as his definitions of figure, major term, and minor term. PORPHYRY (c. 234–c. 305) contributed his "tree," a scheme of genera and species, and his Isagoge set the stage for much medieval work on logic, metaphysics, and the philosophy of language.

In the sixth century, the works of Aristotle, as well as those of most of his commentators, disappeared, and so BOETHIUS (480–524) was the great transmitter of ancient logic to the medieval world. He translated and commented on Porphyry's Isagoge as well as Aristotle's Prior Analytics and composed works on syllogisms and on argument construction. He reformulated categorical propositions to put quantifiers in initial position, introduced infinite terms such as nonF (thus introducing a negation as a term-forming operator), and introduced immediate inferences of contraposition, obversion, and conversion per accidens. He also developed an extensive and evidently original theory of conditionals. His translations of the *Categories* and *On Interpretation* were the only Aristotle texts available to early medieval thinkers. The logic they developed with Porphyry and Boethius as sources became known as the Old Logic.

The Old Logic, following Boethius, defines a proposition as a statement signifying truth or falsehood. It distinguishes seven kinds of compound propositions—conjunctions ("and"), disjunctions ("or"), conditionals ("if"), locals ("where"), causals ("because"), temporals ("when," "while," "as often as"), and adjuncts ("in order that")—and notes that compounds may be embedded in other compounds. It follows the Stoics in giving truth conditions for conjunction and disjunction—the latter understood in the modern, inclusive sense. Old Logic texts generally adopted a strict conditional, following Chrysippus, though some advocated an inclusion account, and one, the *Dialectica Monacensis*, maintained that If A then B is true if and only if the truth of A makes the truth of B probable. Old Logic texts generally followed Boethius in presenting the theory of syllogisms with infinite terms and contraposition. Peter ABELARD (1079–1142) and the monks of Mount St. Geneviève saw the trouble this produced for the square of opposition; the monks used Aristotle's theses to show that one could wrongly deduce the existence of stones from the proposition that every man is an animal, thus showing that the square requires that no term in the language be either empty or universally applicable.

Medieval Logic: The New Logic. The rediscovery of Aristotle's works in the thirteenth century inspired the development of the New Logic, a logic of terms that introduced the doctrine of distribution, rules for determining the validity of syllogisms, and the theory of supposition. The New Logic found expression in innovative thirteenth-century textbooks by WILLIAM OF SHERWOOD (Shyreswood; c. 1200–1266/1271), Lambert of Auxerre (fl. 1250s), ROBERT KILWARDBY (d. 1279), and Peter of Spain (probably, Pope JOHN XXI; c. 1215–1277), whose *Summulae logicales* (also known as the *Tractatus*) was used throughout Europe for the next several centuries. Peter followed the Old Logic in his truth conditions for conjunctions, disjunctions, and conditionals, which he interpreted as strict. He followed Abelard in concluding that all conditionals are necessarily true or necessarily false, and he followed Kilwardby in presenting the mnemonic verses:

Barbara Celarent Darii Ferio Baralipton
Celantes Dabitis Fapesmo Frisesomorum
Cesare Cambestres Festino Barocho Darapti
Felapto Disamis Datisi Bocardo Ferison

These summarize the valid syllogistic forms as well as the method of deriving them in Aristotle's deduction system. Peter also introduced rules for syllogistic validity. His set is incomplete, but it inspired the search for a complete set that occupied logicians for the next century. Peter and Lambert introduced the concepts of distribution and supposition, but did not use them to state rules or analyze opaque contexts.

During the fourteenth century, logic attained a level of theoretical sophistication it would not see again for five hundred years. JOHN BURIDAN (c. 1300–1362), WILLIAM OF OCKHAM (c. 1287–1347), and WALTER BURLEY (Burleigh; c. 1275–1344) built on thirteenth-century texts and made several advances whose significance would not be recognized until the later twentieth century. Burley, thinking about immediate inference, realized that one can move from a distributed term to an undistributed term, but not the reverse. He used this insight to devise a rule to add to Peter of Spain's list: Any term distributed in the conclusion must be distributed in the premises. Buridan added the rule that the middle term must be distributed at least once. The result is a complete set of rules that constitutes a decision procedure for syllogistic inferences. Buridan also, for the first time, expanded the theory of syllogisms to give an adequate account of infinite terms. Burley and Buridan recognized that there are many more quantifiers in natural language than "all," "some," and "no" and began the task of identifying and including them.

Burley and Buridan renewed interest in paradoxes (in their terms, *sophismata*). They discussed relational predicates, thinking about inferences such as "If someone is father of a daughter, then someone is daughter of a father." They worried about scope ambiguities ("I owe you a horse"), opaque contexts ("I think of a rose"), and anaphoric connections ("If a farmer owns a donkey, he loves it"), seeking a semantic theory that accounts for them.

Fourteenth-century logicians also extended Abelard's attempt to devise a theory of consequences. The Stoics and both the Old and the New Logic held that an argument is valid if and only if its associated conditional is true. That encouraged a confusion between the conditional and the entailment relation. Boethius used *consequentia* for the conditional, translating a term Aristotle used for entailment. Abelard distinguished these, using *consequentia* strictly for conditionals and *consecutio* for entailment. Abelard intended his theory of consequences as a theory of conditionals. Even he held, however, that conditionals are true if and only if the antecedent entails the consequent. Burley proposed four rules for consequences, and Buridan and ALBERT OF SAXONY (c. 1316–1390) offered a more comprehensive set.

The last great medieval logician was Paul of Venice (c. 1369–1429), who recognized that conjunction and disjunction apply to terms as well as propositions, and developed a theory observing the distinction between distributed and collective readings of noun phrases. He defined a proposition as a mental sentence and gave a novel treatment of the liar paradox by distinguishing between a proposition and its significate. Paul presented a theory of consequences that includes necessity, possibility, knowledge, belief, understanding, denial, and doubt.

Early Modern Logic. Antoine ARNAULD (1612–1694) and Pierre NICOLE(1625–1695) wrote *Logic, or, The Art of Thinking*, also known as the *Port-Royal Logic*, which revived logic after centuries of neglect. They combined Aristotelian logic with René DESCARTES's (1596–1650) new way of ideas and drew their examples from the BIBLE and classical literature. Their chief innovation is the distinction between intension and extension. Gottfried Wilhelm von LEIBNIZ (1646–1716), diplomat, philosopher, and inventor of the calculus, developed a number of highly original logical ideas, including a characteristica universalis that would allow the resolution of disputes through computation and an algebra of concepts, which he presented as a logic of terms but later reinterpreted as an algebra of propositions. Leibniz introduced two operations on concepts that yield other concepts—negation and conjunction—as well as two primitive relations among concepts: containment and possibility. Later Leibniz interpreted propositions as concepts on the space of possible worlds, using his calculus as a propositional logic. But he omitted inclusive disjunction as well as some medieval principles from the theory of consequences.

The Nineteenth Century. *Elements of Logic* by Richard WHATELY (1787–1863) introduced an organizational pattern—language, deductive logic, fallacies, inductive reasoning, questions of method—that continues in twenty-first-century textbooks. Whately defined a proposition as an indicative sentence. He maintained that logic is the study of relations of classes, an idea that profoundly influenced Augustus De Morgan (1806–1871), who developed numerical quantifiers and investigated the logic of relations, and George Boole (1815–1864), the founder of modern logic. Boole's algebra takes 1 to represent the universe of discourse, 0 the null set, $1 - x$ the class of objects not in x, and xy the intersection of x and y. The equation $x(1 - x) = 0$ expresses the principle of noncontradiction. Boole intended his algebra not only as a theory of classes covering the field of Aristotle's theory of syllogisms but also as a theory of propositional logic. He relied on an elective symbol v standing for an indeterminate part of a class; he rendered All X are Y as $x = vy$ and Some X are Y as $vx = v'y$. Boole's treatment of elective symbols, addition, and subtraction lacks rigor, and the system's reliance on equations limits its power. But his system made possible the profound advances to come. Charles Sanders PEIRCE (1839–1914) augmented Boolean algebra with the now customary symbol of inclusion. He devised the truth-table test for the necessary truth of a formula, and, by introducing quantifiers, turned Boole's system into full first-order, relational predicate logic. He showed that all truth-functional connectives can be defined by joint exclusion (neither . . . nor . . .), which was rediscovered more than thirty years later by Henry M. Sheffer (1882–1964). Boole provided a sound and complete axiomatization of first-order logic and devised a proof system that converts formulas to prenex normal form.

In 1879 German mathematician Gottlob FREGE (1848–1925) published the *Begriffsschrift*, perhaps the most important logical work ever published. Frege formalized a very powerful logic, using only the rules of *modus ponens* and substitution for variables to derive valid propositional formulas. Like Peirce, Frege incorporated a logic of relations. Unlike Peirce, he allowed quantification over predicates. His logic is thus a full impredicative second-order logic, which Bertrand RUSSELL (1872–1970), with his celebrated PARADOX, showed to be inconsistent.

The Twentieth Century. Russell and Alfred North Whitehead (1861–1947) used a restricted version of Frege's logic known as the theory of types in their famous work, *Principia Mathematica* (1910–1913), which constructs mathematics rigorously from logic. Leopold Löwenheim (1878–1957) and Thoralf Skolem (1887–1963) showed that any satisfiable proposition is satisfiable in a countable domain. Emil Post (1897–1954) demonstrated that propositional logic is decidable and developed a theory of degrees of undecidability. Stanisław Jaśkowski (1906–1965) and Gerhard Gentzen (1909–1945) devised natural deduction systems. Kurt Gödel (1906–1978) proved the completeness of predicate logic and then, one year later, the incompleteness of arithmetic. Alfred Tarski (1901–1983) formalized the concept of a model and used it to demonstrate that truth is indefinable. Alonzo Church (1903–1995) showed that the predicate calculus is undecidable. Because propositional logic is decidable, this implies that the Leibniz/Boole dream of a theory that is at once a theory of propositional and predicate reasoning cannot be realized.

Twentieth-century logicians developed many alternatives to and extensions of first-order logic. Jan Łukasiewicz (1878–1956) began work on many-valued logic, continued by Post, Stephen Cole Kleene (1909–1994), and others, in which propositions may be neither true nor false. Alan Anderson (1925–1973) and Nuel Belnap developed relevance logics in which propositions may be both true and false. Clarence I. Lewis (1883–1964) revived the study of MODAL LOGIC, the logic of possibility and necessity, and Saul Kripke provided a Leibnizian SEMANTICS that revolutionized PHILOSOPHY as well as logic. Robert Stalnaker and David Lewis (1941–2001) built on that semantics to offer new theories of the conditional.

As the century closed, logicians increasingly realized that first-order logic had limited expressive power. Peter Geach, a scholar of fourteenth-century logic, drew attention to problems from Burley and Buridan that escape first-order predicate logic: determiners such as "most," and sentences such as "Some critics admire only each other," which cannot be defined in first-order logic; "donkey sentences," such as "Every farmer who owns a donkey feeds it," which have first-order equivalents that cannot be derived from the representations of their parts; and other sentences whose anaphoric connections seem to require unwanted existence assertions. Andrzej Mostowski (1913–1975), Per Lindström (1936–2009), and others developed a theory of generalized quantifiers to remove some of these limitations, and Hans Kamp developed discourse representation theory, a dynamic semantics, to express anaphoric phenomena in a fully compositional way.

SEE ALSO Logic, Symbolic; Modal Logic; Reasoning.

BIBLIOGRAPHY

Bocheński, I. M. *A History of Formal Logic*. Translated and edited by Ivo Thomas. Notre Dame, IN: University of Notre Dame Press, 1961.

Enderton, Herbert B. *A Mathematical Introduction to Logic*. San Diego, CA: Harcourt Academic Press, 2001.

Gabbay, Dov M., Francis Jeffry Pelletier, and John Woods, eds. *Handbook of the History of Logic*. 11 vols. Amsterdam: North Holland, 2004-2012.

Gabbay, Dov M., and F. Guenthner. *Handbook of Philosophical Logic*. 14 vols. Dordrecht, Netherlands: Springer, 1983-2010.

Geach, Peter. *Reference and Generality: An Examination of Some Medieval and Modern Theories* (1962). Ithaca, NY: Cornell University Press, 1980.

Kamp, Hans, and Uwe Reyle. *From Discourse to Logic*. Dordrecht, Netherlands: Kluwer, 1993.

Kneale, William C., and Martha Kneale. *The Development of Logic*. Oxford, UK: Oxford University Press, 1985.

Lewis, Clarence I. *A Survey of Symbolic Logic* (1918). Bristol, UK: Thoemmes Press, 2001.

Łukasiewicz, Jan. *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*. 2nd ed. Oxford, UK: Clarendon, 1998.

Mates, Benson. *Stoic Logic*. Berkeley: University of California Press, 1989.

Moody, Ernest Addison. *Truth and Consequence in Mediaeval Logic*. Westport, CT: Greenwood, 1976.

Priest, Graham. *An Introduction to Non-Classical Logic: From If to Is*. Cambridge: Cambridge University Press, 2008.

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LOGIC, SYMBOLIC

A modern version of formal logic, referred to variously as logistic, mathematical logic, and the algebra of logic, symbolic logic may be described generally as the set of logical theories elaborated since the mid-nineteenth century with the aid of symbolic notation and a rigorous method of DEDUCTION. Symbolic logic differs from traditional logic in its extensive use of symbols similar to those used in mathematics, in its lack of concern with the psychology and EPISTEMOLOGY of KNOWLEDGE, and in its FORMALISM. It is concerned mainly with the analysis of the correctness of logical laws, such as the law of contradiction, that of the hypothetical syllogism, and so on. Symbolic logicians attempt to deduce logical laws from the smallest possible number of principles, that is, axioms and rules of inference, and to do this with no hidden assumptions or unexpressed steps in the deductive process.

History. Gottfried Wilhelm von LEIBNIZ (1646–1716) is usually regarded as the forerunner of symbolic logic, largely for his attempt to formulate a *mathesis universalis* and for his discovery of several theorems that later assumed importance. Historians of symbolic logic, mainly of the Polish school (Jan Łukasiewicz [1878–1956], Jan Salamucha [1903–1944], I. M. Bocheński [1902–1995]), have pointed out that the principal concepts utilized in the new logic are to be found in the works of ARISTOTLE (384–322 BC), who introduced variables and the idea of the deductive system. Similarly, they have shown that the logic of propositions was extensively treated by the Stoics and by the later scholastics, and that even some aspects of the problem of antinomies had their counterparts in the medieval concern with *insolubilia*. Yet it was not until the mid-nineteenth century, with the work of George Boole (1815–1864) and Augutus DE MORGAN (1806–1871), that systems of symbolic logic similar to those used in the twentieth century were developed. The history of this development may be conveniently divided into three periods: the first (1847–1890) dominated by the work of Boole; the second (1890–1930) principally under the influence of Gottlob FREGE (1848–1925); and the third (1930–1960s) devoted largely to metalogical considerations.

Boolean logic had two characteristics: it was a logic of classes, and it was developed using a rigorous mathematical method. It was Boole's intention, in fact, to apply the method of algebra to logic, hence the designation of his system as "the algebra of logic." De Morgan furthered the development, discovering some new laws, doing work on the SYLLOGISM, and making a pioneering study of the logic of relations. Charles Sanders PEIRCE (1839–1914) also belongs to this period. The