LOCUTIONS

Affirmations or statements supernaturally effected in the external sense, internal senses, or directly in the intellect. They often accompany visions and they are divided in the same manner: corporeal (auricular), imaginative, and intellectual. Auricular locutions are words perceived by the bodily sense of hearing, and are generally caused by supernaturally produced acoustical vibrations. They sometimes seem to emanate from a vision or a religious object such as a statue or crucifix. As extraordinary phenomena they could be caused by God or the devil or proceed from natural causes. Imaginative locutions are words perceived in the imagination during sleep or in waking hours. Since they, too, could be supernatural, diabolical, or natural in origin, the rule for discernment is to study the effects produced in the individual. Locutions of supernatural origin cannot be produced at will; they are distinct, causing fervor, peace, humility, and obedience. Intellectual locutions are words or statements perceived immediately by the intellect without the aid of the external senses or imagination. Sometimes they are directly infused; at other times they are a supernatural coordination of naturally acquired ideas. It is beyond the power of the devil to produce truly intellectual locutions. St. John of the Cross divides intellectual locutions into successive, formal, and substantial.

Successive intellectual locations are a kind of dialogue or conversation between the Holy Spirit and the soul. It is a discursive reasoning rather than an instantaneous intuition, and although it is under the direction of the Holy Spirit, the human intellect plays its part. Therefore the actual functioning of the human intellect in this type of locution requires the operation of the imagination, with the result that error can proceed from the human side of the dialogue. The devil can indirectly affect successive locutions by influencing the imagination. Similar locutions occur in the natural phenomenon of the dual personality, although the effects are noticeably different from the effects of truly supernatural successive locutions.

Formal intellectual locutions are those words or statements which come to the mind from without and do not involve the activity of the intellect itself, except to receive them. Unlike the successive locutions, they may be infused into the mind when it is thinking of something entirely different. When they are truly supernatural, they produce virtuous effects in the soul and impart great illumination and certitude. Although the devil cannot directly influence the intellect, an individual may be deceived by the devil, so that the phenomenon itself cannot easily be distinguished by its effects. St John of the Cross advises that souls should never act according to their own opinions or accept the locutions without much reflection and the counsel of others.

Substantial intellectual locutions are basically the same as formal locutions, but with this difference: what is stated is effected immediately. They are similar to the creative word of God. According to St. John of the Cross, there is no possibility of deception or the influence of the devil in substantial locutions.

Since locutions are often closely associated with visions, the same rules applied to locution (*see* VISIONS). Locutions are unmerited and freely given graces in the sense that they do not proceed from the normal development of the spiritual life; they differ somewhat from the usual charismatic gifts given for the benefit of others in the sense that they can bring much consolation and many blessings to the soul that receives them. They should not be desired, except for the substantial locutions, of which St. John of the Cross says: ''Blessed is the soul to whom the Lord speaks the substantial locution.''

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[J. AUMANN]

LOGIC, HISTORY OF

Western formal logic began among the Greeks of the 5th and 4th centuries B.C., who developed syllogistic and prepositional systems. The Greeks of the Hellenistic age and the Romans did nothing to advance these beginnings, but injected a stream of rhetoric that was to plague the subject until quite recent times. It also began a long sequence of sketchy textbooks. After the Dark Ages logic began to revive in the 12th century, and by the middle of the 13th century scholastic logic was well developed. While borrowing much from Aristotle and a little from Roman hints about Stoicism, it developed original methods in propositional and quantificational logic and in regard to logical antinomies. It borrowed rather little from rhetoric but was a good deal influenced by grammar. About mid-15th century the impetus failed and within 100 years had died completely, giving place to a centuries-long crop of incompetent handbooks, often infected with rhetoric, entirely lacking in originality or serious investigation. Only occasionally was the monotonous de-

sert interrupted by something of interest, notably, by the great genius of G. W. LEIBNIZ. In mid-19th century the modern period began with G. Boole and with renewed authority through the immense analytical acumen of G. Frege. Their work brought new understanding of the past and a huge increase in doctrine, presented with an altogether new completeness, strictness, and critical control. One thus has four periods to consider: (1) the Greco-Roman, (2) the medieval, (3) the post-Renaissance, and (4) the modern.

Greco-Roman Period

Aristotle claimed to be the founder of logic, saying that he could find nothing like what he had done among his philosophic predecessors. The claim seems to be justified. One can, of course, find a climate of intense discussion that favored such a development. Both in the school founded by Euclid of Megara, who was a pupil of SOCRATES, and in the Platonic Academy descended from the same source, as also in the tradition of the 5th-century SOPHISTS, discussion was so strongly cultivated that it is not surprising that people should have begun to reflect on the processes of argument, to notice patterns of recurrence, and to generalize in a reflective way about conclusive and inconclusive methods.

Already in PLATO one can see intimations of what would become, in the hands of Aristotle, the syllogism, and, in the hands of the Megarians and Stoics, propositional logic. Roughly speaking, Athens gave birth to the former, Megara to the latter. Plato was surely influential in that he developed the notion of universal law, already in evidence among the pre-Socratics, but it was left for Aristotle to achieve the first conscious, general, explicit system of formal logic, so that Leibniz could say of him that he was the first to write mathematically outside of mathematics.

Aristotelian Logic. The logical works of ARISTOTLE, known as the *Organon,* have been handed down in a systematic order: *Categories,* dealing with the TERM; *On Interpretation,* the PROPOSITION; *Prior Analytics*, the SYLLOGISM in general; *Posterior Analytics, Topics,* and *Sophistical Refutations,* apodictic, dialectical, and sophistical syllogisms, respectively. Surely this list does not represent the order of composition, but attempts to ascertain this through the varying complexity of doctrine are somewhat uncertain, since a thinker's development may not be continuous and homogeneous. Thus the *Topics* and *Sophistical Refutations*, though lacking the doctrine of the syllogism, contain some insights that belong to a more advanced area.

Syllogistic is a theory of whole or partial inclusion between classes, its laws being presented in schematic form, the use of letters instead of words from ordinary language being a brilliant device to secure generality and isolate form. (The device would not be fully exploited until Frege.) Aristotle begins by presenting his syllogisms listwise, classified by patterns called ''figures,'' those that are valid being alternated in each figure with those that are inconclusive, the latter being rejected by counterexample. His incomplete definitions of the figures would give much trouble to later writers, and those who paid more attention to the letter than the spirit would be troubled by the incompleteness of the explicit list. Aristotle reworked his system in several ways, propounding alternative methods of deduction from axioms (thus showing that there is nothing inflexible about a given set of axioms) and making some metalogical statements. The deductions are either direct, by laws of conversion, or indirect, by *reductio ad absurdum.* They are carried out in an intuitive, not in a formalized way, for Aristotle states only two or three laws of propositional inference, though it is noteworthy that he does there consciously use propositional variables.

Especially to be distinguished from the nonsyllogistic laws are some belonging to the logic of relations, e.g., ''if knowledge be conceiving, then an object of knowledge is an object of conceiving,'' a principle that A. De Morgan in the 19th century would adduce against contemporary would-be Aristotelians as unprovable syllogistically. Also from the *Topics* and *Sophistical Refutations* come laws about identity that add up to the ''principle of the identity of indiscernibles'' commonly ascribed to Leibniz. The presence of such things in Aristotle has been more often ignored than noticed, and they are fragmentary in character. Even the assertoric syllogistic is not treated with the thoroughness and generality currently accorded to the systematic investigation of logical ideas. Aristotle's modal syllogistic is even less fully elaborated and still awaits definitive investigation and assessment. But he got logic off to an astonishingly good start, and in spite of the undoubted merits of some medieval treatises, there is no extant work (in the absence of full Stoic texts) of comparable promise until Leibniz.

Theophrastus. Aristotle was succeeded as head of the Peripatetic school by Theophrastus of Eresos. He is known chiefly for having made explicit the five syllogistic moods later known as *Baralipton, Celantes, Dabitis, Fapesmo,* and *Frisesomorum.* He introduced a non-Aristotelian modal syllogistic in which the assertoric law that the conclusion follows the weakest premise holds; and he offered an extensional proof, perhaps with a spatial model before him, of the convertibility of universal negative propositions. Only fragments of his work remain. They contain references to his work on syllogisms ''from hypotheses,'' i.e., with conditional premises, initi-

ated by his predecessor. It is possible that Theophrastus stimulated the Megarian-Stoic work on propositional logic.

Megarian-Stoic School. Materials exist only in fragmentary and often hostile reports. Among the Megarians, Eubulides of Miletus is credited with the discovery of the PARADOX called ''the Liar,'' or the ''Epimenides,'' noted by Aristotle and much pondered over by Theophrastus and Chrysippus. A new form was claimed as late as 1937, but it has been found to have existed in the Middle Ages. One early version goes: ''If you say that you lie, and in this say true, do you lie or speak the truth?'' Eubulides is reported to have been hostile to Aristotelian doctrine, thus depriving later Aristotelians of a progressive and complementary influence.

Diodorus Cronus of Iasus (end of 4th century B.C.) held views on modality, the accounts of which have proved difficult for modern interpreters. His definition of the necessary introduced a time variable, ''that which neither is nor will be false.'' Although it is tempting to think that his definition of implication was that at no time ever is its antecedent true and its consequence false, the text does not certainly justify this. He was the author of a ''master argument'' about the incompatibility of three modal propositions, which it has proved impossible to reconstruct satisfactorily. Stilpo of Megara was influential in drawing new adherents, including Zeno of Citium, who founded the Stoa (*c.* 300 B.C.). Philo of Megara was the first to formulate the truth conditions for the material conditional, true except when its antecedent is true and its consequent false.

The Megaric school seems to have disappeared with the rise of STOICISM, and the logical history of the latter is overshadowed by Chrysippus of Soli, its second founder, who died shortly before 200 B.C. The most important contribution to logic made by the Stoics was a deductive system of propositional logic. It was based on five ''indemonstrable moods'' (one should not say ''axioms,'' for this word is kept by them for the objective meanings of declarative sentences) and four ''themes'' or rules, only two of which have been preserved. Instead of letters they used ordinal terms as variables. W. Kneale has suggested a convincing reconstruction of the system, for which the Stoics claimed completeness, but it is not clear what they could have intended by such a claim.

Later Developments. For the remainder of ancient logic, one should mention CICERO—no logician indeed, but his rhetorical syllogism influenced logic in the Renaissance and after; the handbooks of Galen and Apuleius of Madaura (2d century A.D.); the Greek commentators on Aristotle, especially Alexander of Aphrodisias (3d century) and JOHN PHILOPONUS (6th century). GALEN was later credited with the invention of a fourth figure of syllogism, but J. Lukasiewicz has shown that this was a mistake. Apuleius gave the square of OPPOSITION, which has become traditional. Alexander showed how to derive a law of conversion from a syllogism and a law of identity by *reductio ad absurdum,* which offered to medievals and to Leibniz new possibilities in syllogistic axiomatics. Philoponus suggested resolving doubts about how to define syllogistic figures by calling the subject (predicate) of the conclusion the minor (major) term and denominating the premises thence. This is the most economical method, but it did not come into general use until the end of the 17th century. Porphyry of Tyre (3d century) contributed his ''tree'' or scheme of genera and species, of which he took an extensional view, the species being contained in the genus predicated of it (*see* PORPHYRIAN TREE).

Boethius was the great transmitter of ancient logic to the medieval world. He was a peripatetic but preserved some Stoic doctrines, translated most of the *Organon,* and composed works on *Topics* or *Loci,* as had been done in the domain of rhetoric by Cicero and Marius Victorinus (4th century). His translations of the *Categories* and *On Interpretation* constituted the *logica vetus* of the early medievals, the other parts of the *Organon* being the *Logica nova.* The variables in his treatise on hypothetical syllogisms have been taken as propositional, but since the doctrine is basically Theophrastan, and in view of Boethius's Aristotelian convictions, they are probably term variables.

Medieval Period

Study in the field of logic began to revive toward the end of the 11th century, amid a great deal of fruitful activity, of which much remains to be learned through the publication of further texts. The full logic of Aristotle, notably the *Prior Analytics,* became available only in the course of the 12th century. Boethius was influential, as was Cicero, but the grammarians seem to have been more influential than the rhetoricians.

Twelfth Century. ABELARD, remembered by his contemporaries as ''the Aristotle of our time, the equal or superior of all logicians there have been,'' noted that logic is not a science of using arguments but of discerning their validity. In his *Dialectica* he distinguishes ''antecedent'' and ''consequent'' as referring both to subject and predicate within simple propositions and to the parts of hypothetical propositions. This and other passages show the emergence of medieval propositional logic in distinction from a logic of terms. Abelard knew that these were different and that there are analogies between them—he reports a view that propositional connectives

and their term analogues have the same sense, and he rejects it. The statement that a hypothetical proposition is called both a ''consequence'' and a ''conditional'' may raise a doubt whether relations of implication and inference were yet clearly distinguished, and the fact that ALBERT OF SAXONY (14th century) distinguishes *si* and *ergo* only by their positioning should engender caution in viewing the theory of ''consequences'' in one or the other light. Abelard already has a number of valid consequences, and some are even deduced from others, but the Middle Ages never attained an axiomatized system of propositional logic. One of Abelard's most elaborate consequences is ''of whatever hypotheticals the antecedents are concomitant, the consequents are concomitant.'' This is the theorem that Leibniz would rediscover and call *praeclarum.* One should note the metalogical formulation, a style that would remain standard and that is perhaps derived from the *De differentiis topicis* of Boethius, who distinguished the maxim or metalogical formulation of a class of truths from the instances.

In the 12th century, Adam of Balsham also wrote a highly original work, *Ars disserendi,* in which one sees the rise of a concern with *sophismata* or logical puzzles, which became very characteristic of the period. While, under an inventive hand, *sophismata* could produce a rich body of doctrine, the medium favored the perpetuation of a fragmented treatment rather than a genuinely systematic one. Adam made a rare attempt to begin a logic of questions, in the course of which he reached the conclusion that an infinite set could be equinumerous with a proper part of itself.

Thirteenth Century. The best-known works of the 13th century are the *Introductiones in logicam* of WILLIAM OF SHERWOOD (Shyreswood), the *Summulae logicales* of Peter of Spain (Pope JOHN XXI), and the commentaries on the *Prior Analytics* by St. ALBERT THE GREAT and ROBERT KILWARDBY. This last shows that consequences were already a normal part of logical teaching. Peter of Spain became a standard author throughout the 15th century. Curiously his summary handbook does not have a chapter on consequences, but it does have a well-developed doctrine of *proprietates terminorum,* as does the earlier and similar book of Shyreswood. The origins of this can be faintly detected in the previous century, where more can surely be found. The property that came to be chiefly discussed is SUPPOSITION, the reference that the subject (and later also the predicate) has in a proposition. The *De suppositionibus dialecticis* (1372) of St. VINCENT FERRER shows a wide selection of disparate logical material discussed in this connection, including some points of quantification theory. Once again the necessity of considering numerous examples from ordinary speech favored the fragmented approach.

Later Centuries. WILLIAM OF OCKHAM sparked an intensification of activity, partly because of the very comprehensiveness of his *Summa totius logicae.* His influence can be seen even in those who repudiated his epistemology. WALTER BURLEY, JOHN BURIDAN, Albert of Saxony, MARSILIUS OF INGHEN, the Mertonians, WILLIAM OF HEYTESBURY (HENTISBER) and RALPH STRODE, and Richard Ferabrich were some of the notable writers. Besides the areas already mentioned, they paid much attention to *insolubilia,* or logical paradoxes, developing many versions of the Epimenides, which was already known to Adam of Balsham. Numerous solutions were proposed, including the outlawing of self-referring propositions from meaningful language (*see* ANTIMONY).

The 15th century was unoriginal; toward its end there was the encyclopedic *Logica magna* of Paul of Venice (Paolo VENETO), who with Peter of Mantua and Paul of Pergolae formed a school known to their contemporaries as the *Sorticolae.*

From the 13th century on, syllogistic was considered as a special department and even rather a small one. The supposedly Aristotelian idea that developed in the next period—that valid arguments are always syllogistic—was quite foreign to the medievals. The subject was of course treated at length in the Aristotelian commentaries and required detailed treatment in commentaries on the *Summulae,* but in the more general treatises, syllogisms are just one kind of consequence. The usual method of defining terms was a generalization of that of Boethius, the first premise stated being the major premise by definition, and the extreme term therein the major term. This is quite different from the method of Philoponus, and there are signs that some people could work out its consequences correctly, but again a unified and systematic presentation was lacking. Mnemonics of various kinds were experimented with in the 13th century, and the familiar ''*Barbara, Celarent,* etc.'' occurs in Shyreswood.

Post-Renaissance Period

It was about 1440 that the first recorded voice of the new age, or non-age, in logic made itself heard. L. VALLA, a renowned humanist scholar, then rejected the third figure of the syllogism on the grounds that women, children, and nonlogicians generally, do not argue that way. Perhaps this is the first time that ordinary language was claimed as the standard of logical doctrine. Evidently all sense of syllogistic as a deductive system had been lost; indeed Valla said that conversion, Aristotle's chief means of deduction, is only a ''remedy for sick syllogisms.'' R. Agricola's *De dialectica inventione* swung the ambivalent ''topical'' tradition firmly into the path of rhetoric, in contrast with Abelard. P. MELANCHTHON,

writing in 1521, expounded Cicero's syllogism before Aristotle's. Older doctrines were quickly dropped or ridiculed. G. SAVONAROLA kept telling the 16th century in numerous reeditions that anyone arguing from a conjunction to one of its parts was *dignus explosione.*

Ramist Controversy. In the mid-16th century, vernacular logics began to appear, for example, T. Wilson's *The Rule of Reason* (1551) and the *Dialectique* (1555) of Peter RAMUS. This last writer's views on logical reform provoked widespread and long-lasting controversy. His simplified syllogistic and novel terminology occasioned long commentaries on very little and a new technical scholasticism. Aristotelians found little to discuss besides the iniquities of Ramism and the fourth figure of the syllogism, few recognizing that this was a matter to be settled by definition. Sextus Empiricus appeared in Latin in 1569, but led to no rediscovery of Stoic logic.

There was an occasional break in the clouds. J. Hospinianus (1515–75) thoroughly investigated syllogistic on a combinatory basis, and G. Cardano illustrated his *Dialectica* with geometrical arguments. J. Junge (*Logica Hamburgensis,* 1638) showed a deductive interest in the syllogism and some appreciation of Aristotle's logic of relations. In 1662 A. GEULINCX pleaded for the restoration of medieval doctrines. In that year the ''Port Royal Logic'' of A. ARNAULD and P. NICOLE was published. Anti-rhetorical and anti-Ramist, the authors idolized geometry and did much to tighten up syllogistic theory. At the same time they opened the way to introducing epistemological and psychological discussions into books of logic. H. Aldrich, in his *Artis logicae compendium* (1691), correctly tabled 24 moods of syllogism in four figures and methodically proved all others invalid.

Leibniz and After. Meanwhile G. W. LEIBNIZ had begun to develop quite new ideas. A polymath famous in philosophy for his *Monadology,* and in mathematics for his invention of the infinitesimal calculus, he was not yet 20 years old when he began to be haunted by the idea that logic might be developed in a mathematical way. Others before him had discerned a kinship (e.g., ROGER BACON), but mathematical notations had not been used. Leibniz experimented with various versions of a logical calculus that he wanted used in association with a rationally constructed universal language. He also envisioned an encyclopedia that would be progressively perfected as the sciences advanced and at any one stage would unify the whole body of achieved human knowledge. In forming these projects Leibniz found his interest caught by J. Wilkins, G. Dalgarno, and other contemporaries for the language, T. Zwinger and J. H. Alsted for the encyclopedia, and Raymond LULL for the calculus. But his own ideals went beyond any of theirs, especially in regard to the analysis of ideas into their simplest parts; this the language would mirror, the encyclopedia present, and the calculus reverse so as to be effective for the discovery of new combinations. Leibniz's efforts with his calculus of logic were frustrated by difficulties with empty terms (which the medievals had also noticed) and by doubts about the relationship between extensions and intensions. He anticipated the circular diagrams of L. EULER and the ruled or dotted lines of J. H. Lambert (1728–77).

After Leibniz a number of attempts were made to construct a satisfactory symbolic calculus, e.g., by Lambert, his contemporary G. J. Holland, and G. F. Castillon. Sir William Hamilton claimed priority in quantifying the predicate, but this had been done by Leibniz and those just mentioned. A real breakthrough was achieved by A. DE MORGAN, whom C. S. Peirce called ''unquestionably the father of the logic of relatives.''

Modern Period

In the same year (1847) that De Morgan's *Formal Logic* appeared, George Boole published *The Mathematical Analysis of Logic,* which was followed in 1854 by *An Investigation of the Laws of Thought.* From this time on there was a steady clarification of ideas interdependent with the perfecting of a calculus. Thorough systematization and investigation of logical notions became possible as never before.

Boolean Algebra. Boole's algebra, in which $1 - x$ represents the class of objects in the universe of discourse, 1, which are not in the class x, and in which the equation $x(1 - x) = 0$ expresses the principle of noncontradiction, is rich enough for all the traditional modes of class reasoning, though some (e.g., subalternation) require statement that the classes involved are not empty. The system can be interpreted as well in the domain of truth functions or that of probabilities. W. S. Jevons (1835–82) showed that inclusive alternation offered some advantages over the exclusive used by Boole; it gives the law $x + x = x$, getting rid of coefficients. In 1869 he used the new methods to make a logical machine; the logical diagrams proposed by J. Venn in 1881 also mirror the new methods. C. S. PEIRCE, a very original and inventive thinker, augmented the Boolean algebra with the now customary symbol of inclusion (similar ones had been used by Lambert and J. D. Gergonne), which he also interpreted prepositionally as material implication. In 1885 he devised the truth-table test for the necessary truth of a formula, and by the introduction of essentially new notions, ''expanded'' the Boolean system into a logic of relations; here he also developed De Morgan's work, with the help of O. H. Mitchell. Peirce also showed how all truth-functional connectives can be defined by joint ex-

clusion (neither . . . nor . . .), which was rediscovered more than 30 years later by H. M. Sheffer. The *Vorlesungen über die Algebra der Logik* of E. Schröder incorporated the various improvements made in Boole's system in the interval and further developed Peirce's ideas about relations. Since this represents the peak of the Boolean line of thought, the resulting system is now known as the Boole-Schröder algebra.

Frege and After. Meanwhile, in 1879 there appeared the *Begriffsschrift* of G. Frege, perhaps the most penetrating and original logical work ever published. Frege was explicitly concerned with banishing all rhetorical and even traditional grammatical influence, on the one hand, and, on the other, providing for an accurate analysis of reasoning in a more thorough way than was possible by means of an equational system such as Boole's. The Boole-Schröder system utilized an unexpressed intuitive logic, as Aristotle's syllogistic had done. This fundamental logic was successfully formalized by Frege, with the use only of the rules of *modus ponens* and substitution for variables to derive valid propositional formulas from axioms (which later were seen to be unduly lavish). Frege's connectives were built out of vertical and horizontal lines; and while his expressions can be read quite mechanically in terms of negation and conjunction, the space they occupy has prohibited their general use. There are more compact notations, for example, the ''wheels'' of S. Lesniewski, which are diagrammatically closer to the intended meaning and serve calculation more readily. Applying his propositional system to propositional functions, and analyzing such functions, Frege gave rules for the use of quantifiers and discussed the differing nature of variables according to whether they are governed by quantifiers or not. In these systems logic at last reached its maturity.

Frege's aim was to analyze and codify mathematical reasoning in a deductive way. G. Peano actually brought the new methods to bear on mathematics and introduced improvements in symbolism. B. RUSSELL and A. N. WHITEHEAD joined the ideas of Frege and Peano to produce *Principia Mathematica* (1910–13), the most comprehensive exposition of logical and mathematical thought ever effected. In 1917 J. Lukasiewicz announced his first views on many-valued logic (inspired by Aristotle, and published in 1920, when E. Post's independent investigation in the same field also appeared). The natural deduction systems of S. Jaskowski and G. Gentzen, and K. Gödel's proof of the completeness of predicate logic, appeared in 1930. Gödel's epoch-making adaptation of the *Epimenides* in 1931 to show that the system of *Principia Mathematica* is undecidable continues to be adapted to show the same for many other systems, especially by A. Tarski. In 1936 A. Church showed that the predicate calculus has this property.

See Also: LOGIC, SYMBOLIC; AXIOMATIC SYSTEM.

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[I. THOMAS]

LOGIC, SYMBOLIC

A modern version of formal logic, referred to variously as logistic, mathematical logic, and the algebra of logic; it may be described generally as the set of logical theories elaborated since the mid-19th century with the aid of symbolic notation and a rigorous method of DEDUCTION. Symbolic logic differs from traditional logic in its extensive use of symbols similar to those used in mathematics, in its lack of concern with the psychology and epistemology of knowledge, and in its FORMALISM. It is concerned mainly with the analysis of the correctness of logical laws, such as the law of contradiction, that of the hypothetical syllogism, and so on. Symbolic logicians attempt to deduce logical laws from the smallest possible number of principles, i.e., axioms and rules of inference, and to do this with no hidden assumptions or unexpressed steps in the deductive process (*see* AXIOMATIC SYSTEM).