clusion (neither . . . nor . . .), which was rediscovered more than 30 years later by H. M. Sheffer. The *Vorlesungen über die Algebra der Logik* of E. Schröder incorporated the various improvements made in Boole's system in the interval and further developed Peirce's ideas about relations. Since this represents the peak of the Boolean line of thought, the resulting system is now known as the Boole-Schröder algebra.

Frege and After. Meanwhile, in 1879 there appeared the *Begriffsschrift* of G. Frege, perhaps the most penetrating and original logical work ever published. Frege was explicitly concerned with banishing all rhetorical and even traditional grammatical influence, on the one hand, and, on the other, providing for an accurate analysis of reasoning in a more thorough way than was possible by means of an equational system such as Boole's. The Boole-Schröder system utilized an unexpressed intuitive logic, as Aristotle's syllogistic had done. This fundamental logic was successfully formalized by Frege, with the use only of the rules of *modus ponens* and substitution for variables to derive valid propositional formulas from axioms (which later were seen to be unduly lavish). Frege's connectives were built out of vertical and horizontal lines; and while his expressions can be read quite mechanically in terms of negation and conjunction, the space they occupy has prohibited their general use. There are more compact notations, for example, the ''wheels'' of S. Lesniewski, which are diagrammatically closer to the intended meaning and serve calculation more readily. Applying his propositional system to propositional functions, and analyzing such functions, Frege gave rules for the use of quantifiers and discussed the differing nature of variables according to whether they are governed by quantifiers or not. In these systems logic at last reached its maturity.

Frege's aim was to analyze and codify mathematical reasoning in a deductive way. G. Peano actually brought the new methods to bear on mathematics and introduced improvements in symbolism. B. RUSSELL and A. N. WHITEHEAD joined the ideas of Frege and Peano to produce *Principia Mathematica* (1910–13), the most comprehensive exposition of logical and mathematical thought ever effected. In 1917 J. Lukasiewicz announced his first views on many-valued logic (inspired by Aristotle, and published in 1920, when E. Post's independent investigation in the same field also appeared). The natural deduction systems of S. Jaskowski and G. Gentzen, and K. Gödel's proof of the completeness of predicate logic, appeared in 1930. Gödel's epoch-making adaptation of the *Epimenides* in 1931 to show that the system of *Principia Mathematica* is undecidable continues to be adapted to show the same for many other systems, especially by A. Tarski. In 1936 A. Church showed that the predicate calculus has this property.

See Also: LOGIC, SYMBOLIC; AXIOMATIC SYSTEM.

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[I. THOMAS]

LOGIC, SYMBOLIC

A modern version of formal logic, referred to variously as logistic, mathematical logic, and the algebra of logic; it may be described generally as the set of logical theories elaborated since the mid-19th century with the aid of symbolic notation and a rigorous method of DEDUCTION. Symbolic logic differs from traditional logic in its extensive use of symbols similar to those used in mathematics, in its lack of concern with the psychology and epistemology of knowledge, and in its FORMALISM. It is concerned mainly with the analysis of the correctness of logical laws, such as the law of contradiction, that of the hypothetical syllogism, and so on. Symbolic logicians attempt to deduce logical laws from the smallest possible number of principles, i.e., axioms and rules of inference, and to do this with no hidden assumptions or unexpressed steps in the deductive process (*see* AXIOMATIC SYSTEM).

This article provides a brief survey of the history of the discipline and discusses its basic concepts and principal divisions, viz, propositional logic, the logic of predicates and of classes, and the 1ogic of relations.

History. G. W. LEIBNIZ is usually regarded as the forerunner of symbolic logic, largely for his attempt to formulate a *mathesis universalis* and for his discovery of several theorems that later assumed importance. Historians of symbolic logic, mainly of the Polish school (J. Lukasiewicz, J. Salamucha, I. M. Bocheński), have pointed out that the principal concepts utilized in the new logic are to be found in the works of ARISTOTLE, who introduced variables and the idea of the deductive system. Similarly, they have shown that the logic of propositions was extensively treated by the Stoics and by the later scholastics, and that even some aspects of the problem of antinomies had their counterparts in the medieval concern with *insolubilia.* Yet it was not until the mid-19th century, with the work of G. Boole and A. DE MORGAN, that systems of symbolic logic similar to those used in the 20th century were developed. The history of this development may be conveniently divided into three periods, the first (1847–90) dominated by the work of Boole, the second (1890–1930) principally under the influence of G. Frege, and the third (1930–60s) devoted largely to metalogical considerations.

Boolean logic had two characteristics: it was a logic of classes and it was developed using a rigorous mathematical method. It was Boole's intention, in fact, to apply the method of algebra to logic—whence the designation of his system as ''the algebra of logic.'' De Morgan furthered the development, discovering some new laws, doing work on the SYLLOGISM, and making a pioneer study of the logic of relations. C. S. PEIRCE likewise belongs to this period. The most ample development of logic according to Boole's method, however, is to be found in the work of E. Schröder, *Vorlesungen über die Algebra der Logik* (3 v. Leipzig 1890–1905).

The Fregean period was characterized by a more formal development of the new discipline. Frege himself discovered a new logic of propositions and developed the first axiomatic system for such a logic; this has been regarded as a fundamental work on the foundations of mathematics. Improving on Frege's symbolism, G. Peano invented a form of symbolic writing that was later adopted by B. RUSSELL and A. N. WHITEHEAD in their *Principia Mathematica* (3 v. Cambridge, England 1910–13). Another notational advance was made by the Polish logician J. Lukasiewicz, who also invented polyvalent or many-valued logics and did research in the history of formal logic. Also worthy of note, although extending somewhat beyond this period, is the work of the German logicians D. Hilbert and P. Bernays on the foundations of mathematics (*Grundlagen der Mathematik*, 2 v. Berlin 1934–39).

The metalogical period was inaugurated by K. Gödel, who showed that many propositions in the *Principia Mathematica* and in equivalent systems were formally undecidable, i.e., that their truth or falsity could not be proved within the formal structure of the system. Noteworthy in this period is the work of A. Tarski on the semantic definition of truth and that of K. Popper and R. Carnap on the methodology of the exact sciences. Additional applications of the methods of mathematical logic have been made in theology (Bocheński, I. Thomas), in analytical philosophy (A. Church, N. Goodman, W. V. O. Quine, C. G. Hempel), in physics (H. Reichenbach, C. E. Shannon), in biology (J. H. Woodger), and in economics (J. von Neumann, O. Morgenstern). *See* LOGIC, HISTORY OF.

Basic Concepts. A fundamental distinction in symbolic logic is that between constants and variables. Variables are symbols (usually the letters *x, y, z*) that can be replaced by constants (usually the letters *a, b, c*) or by complex formulas. If a constant is replaced by a variable in a sentence, or proposition, the result is a function; this is a schema for a sentence, or proposition, and in itself is neither true nor false. Thus, ''*x* is a student'' is a function and is neither true nor false, whereas ''*a* is a student'' and ''John is a student'' are sentences and may be true or false. Functions may be transformed back into sentences, or propositions, by prefixing a quantifier to them. There are two types of quantifiers: universal quantifiers, of which an example would be ''for all *x*,. . .'' [written (x)]; and existential quantifiers, of which an example would be ''there is at least one *x* such that . . .'' [written $(\exists x)$].

Symbols are generally divided into basic categories and functor, or predicate, categories. The basic categories are either names (substantives) or sentences. Functors, or predicates, are symbols (usually designated by the Greek letters φ, ψ, χ, or by specially invented characters) that determine other symbols, which are referred to as arguments. Thus, ''Peter'' is the argument of the functor ''walks'' in the sentence ''Peter walks,'' which may be written ''φa,'' where ''*a*'' stands for ''Peter'' and ''φ'' stands for ''walks.'' Functors are divided in three different ways, each based on a different principle of division. (1) First there is the division into sentence-forming and name-forming functors. Thus, ''walks'' is sentence forming because ''Peter walks'' is a sentence, whereas ''brilliant'' is name forming because ''brilliant student'' is a name. (2) A second division is that into name-determining and sentence-determining functors. Thus, ''walks'' is a name-determining functor, as in the example ''Peter walks''; on the other hand, ''it is not the case that'' is a sentence-determining functor, as in the example ''It is not the case that Peter walks.'' (3) Finally, functors are distinguished according to the number of arguments that they determine into one-place, two-place, three-place, or, in general, *n*-place functors. An example of a one-place functor is ''walks'' in the sentence ''Peter walks''—''walks'' here determines only one argument, viz, ''Peter.'' An example of a two-place functor is ''loves'' in the sentence ''Paul loves Joan''—here ''loves'' determines two arguments, viz, ''Paul'' and ''Joan.'' An example of a three-place functor is ''gives'' in the sentence ''Paul gives Joan a ring''—here ''gives'' determines three arguments, viz, ''Paul,'' ''Joan,'' and ''ring.'' And so on.

In accordance with these principles of division, symbolic logic may be seen as divided into three main parts: (1) propositional logic, in which all functors are sentence-determining; (2) the logic of predicates and of classes, which treats of name-determining functors; and (3) the logic of relations, which is concerned with special properties of functors that determine two or more arguments.

Propositional Logic. Propositional logic is concerned exclusively with sentences, or propositions, that may be constructed by means of so-called truth functors. Truth functors are sentence-forming, sentence-determining, generally one- and two-place functors that can be used to form sentences whose truth value depends exclusively on the truth value of their arguments and not upon their meanings. Truth value in propositional logic—which is a two-valued logic—is twofold: it may be either the value of truth (usually written *T* or 1) or the value of falsity (usually written *F* or 0). An example of a truth functor is negation, since the value of a negated true sentence is falsity and the value of a negated false sentence is truth, and this independently of the sentences' meanings. The most widely employed truth functors are negation (''it is not the case that . . . ,'' usually written ~), the logical sum (''either . . . or . . .'' in the sense of ''either or both''), the logical product (''. . . and . . . ,'' usually symbolized by a period or dot), material implication (''if . . . , then. . . ,'' usually written ⊃), equivalence (''if and only if . . . , then . . . ,'' usually written ≡), and disjunction (''either . . . or . . .'' in the sense of ''not both. . . and . . . ,'' usually written |).

The truth functor known as material implication is most important for understanding how symbolic logic differs from traditional formal logic. Although material implication is taken to mean ''if . . . then . . . ,'' it has a different significance from the conditional compound of ordinary discourse. Because of its ordination to a truth-value type of VERIFICATION, material implication abstracts from, ignores, or leaves behind some of the ordinary elements of meaning of the conditional compound. Some authors (e.g., H. Veatch) make this abstraction the central point of their evaluation of material implication, arguing that it cannot express the intentional character of the conditional, which must lie in the relation of meaning between the component propositions, viz, the antecedent and the consequent. Other authors, while recognizing differences between the ordinary conditional compound and material implication, attempt to point out an element common to both. Thus I. M. Copi argues that material implication expresses a partial meaning of the conditional. Every conditional whose antecedent is true and whose consequent is false must be considered a false proposition; it is this element of the conditional that is expressed by material implication. Since material implication has a ''weaker'' meaning than the conditional compound, material implication can always be asserted when a strict conditional obtains, although the converse is not true. The essential value of material implication appears to lie in its permitting one to state that if the antecedent proposition has been assigned the value of truth, the consequent proposition must also be assigned the same value; this makes possible a purely mechanical operation that resembles a deductive process based on the recognition of meanings of what is stated in the antecedent and the consequent.

Using the concept of deduction thus associated with material implication, one may derive all the sentences, or propositions, of propositional logic from very few axioms and rules. Propositional logic is the most completely developed part of symbolic logic; it is regarded by mathematical logicians as the simplest and most basic part of their science, which provides the framework, so to speak, for all other types of logical analysis and deduction.

Logic of Predicates and of Classes. The second branch of symbolic logic falls into two divisions: the logic of predicates, which gives an intensional interpretation of its formulas, and the logic of classes, which gives an extensional interpretation.

In the logic of predicates the sentence is analyzed into a sentence-forming, name-determining functor (usually written φ, ψ, or χ) and a name (usually written as a variable or as a constant). An example of the basic formula would be φx. Formulas of this type are combined by means of sentence-determining functors, i.e., truth functors, and are transformed into sentences by means of quantifiers. Thus the universal proposition ''All φ is ψ'' may be replaced by the expression ''$(x). \varphi x \supset \psi x$,'' and the particular proposition ''Some φ is ψ,'' or ''There is a φ that is ψ,'' may be replaced by the expression ''(∃

x). φx. ψx.'' Use of these modes of writing and the deductive methods of the logic of propositions has led to a considerable extension of Aristotelian syllogistics.

The logic of classes is the extensional counterpart of the logic of one-place functors or predicates. A class or set (generally designated by the Greek letters α, β, or γ) is always defined by a predicate; it is the set of all objects that possess a given property. For example, the class of human beings consists of all objects to which the predicate ''is a man'' can be attributed. The most important concept of the logic of classes is that of class membership, ''$x \varepsilon \alpha$,'' which is usually read ''x is a member of α'' or ''x belongs to α.'' Another concept—one that has caused considerable controversy among philosophers—is that of the null class, i.e., the class that contains no elements. On the basis of the definition of class and the theorems of the logic of predicates, as well as those of propositional logic, various combinations of classes can be effected and the relationships between them ascertained.

Logic of Relations. The logic of relations may be described as an extensional counterpart of the logic of predicates (or functors) that determine two or more arguments, just as the logic of classes may be regarded as an extensional counterpart of the logic of predicates that determine one argument. The reason for this is that relations can hold only between two or more arguments. In this branch of symbolic logic, relations are conceived extensionally, i.e., as relating to groups of objects. A relation, in a manner completely analogous to the defining procedure for a class, may be defined by a two-place predicate. Thus one may define the relation ''in love with'' as ''the set of pairs of persons who love each other.'' The symbol usually employed is R, which is generally written between the two variables it relates, e.g., xRy. Every relation may be conceived as having a converse; thus ''to the right of'' is the converse of ''to the left of,'' and ''the author of'' is the converse of ''the work of.'' It is common also to distinguish various relational descriptions: (1) individual, e.g., the husband of the Queen of England; (2) plural, e.g., the authors of the *New Catholic Encyclopedia*; (3) double plural, e.g., the authors of English poems; and (4) the domain, which is the most general type of relational description, e.g., all authors. Of considerable importance are the concepts used for the purposes of compounding several relations, such as the relative product (e.g., the square of the half, the brother of the mother) and the relative power (e.g., the father of the father, or father ''squared''). Another group of useful concepts is provided by the properties of relations: some are reflexive, i.e., xRx; others are symmetrical, i.e., if xRy then yRx; and still others are transitive, i.e., if xRy and yRz, then xRz. A concept of great use in the investigation of series is that of ancestral relation (R or R^2 or R^3, etc.).

See Also: ANTINOMY; MATHEMATICS, PHILOSOPHY OF; SEMANTICS.

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[W. A. WALLACE]

LOGICAL POSITIVISM

A contemporary philosophical movement that aims to establish an all-embracing, thoroughly consistent empiricism based solely on the logical analysis of language. Because of its anti-metaphysical bias, militantly propagated by its founders and some prominent adherents, the movement constitutes a serious challenge to traditional philosophy and religion. In what follows, consideration is given to its historical development, its principal proponents and some of their antecedents, its philosophical tenets and how these evolved, and a critical evaluation.

Origins with the Vienna Circle. The logical positivist movement began with a small group of philosophers and scientists later known as the Vienna Circle (Wiener Kreis). The group had formed itself around Moritz Schlick, a former physicist who was appointed to the chair of philosophy of the inductive sciences at the University of Vienna in 1922. Meetings to discuss logical and epistemological problems were held regularly. Among those who joined Schlick were Rudolf Carnap, Hans Hahn, Otto Neurath, Herbert Feigl, Philipp Frank, Friedrich Waismann, and Edgar Zilsel. Most of these men had developed an interest in philosophy as an out-