ALBERT THE GREAT AND MATHEMATICS

Anthony Lo Bello

The late James A. Weisheipl edited the volume *Albertus Magnus and the Sciences: Commemorative Essays 1980* to celebrate the 700th anniversary of the death of the Universal Doctor. This volume, published by the Pontifical Institute of Mediaeval Studies in Toronto, contained the articles "Mathematics in the Thought of Albert the Great" by A.G. Molland[1] and "The Commentary of Albert on Euclid's Elements of Geometry" by Paul M.J.E. Tummers.[2] Here, I will discuss the advances that have been made in this area during the last few decades.

1. The most important current topic of debate among those historians of mathematics who study the work of Albert the Great is whether or not the manuscript Vienna, Dominikanerkloster 80/45, contains, on folio pages 105r–145r, the commentary of Albert on Euclid's *Elements of Geometry*. Ostlender,[3] Geyer,[4] Bessel-Hagen, Hofmann,[5] Hossfeld,[6] Ineichen,[7] Anzulewicz,[8] and Tummers[9] hypothesized that the Viennese manuscript is indeed the only surviving witness of that work. The text perhaps extended further than the manuscript, which stops at the end of Book 4. Busard, the preeminent living authority on the transmission of Euclid's *Elements* in the Middle Ages, has not accepted that the commentary is Albert's.[10]

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[1] See 463–478.

[2] See 479–499.

[3] H. Ostlender, "Die Autographe Alberts des Grossen," in *Studia Albertina, Festschrift für Bernhard Geyer zum 70. Geburtstage*, (BGPTM) Supplementband 4 (Münster in Westfalen: 1952), 3–21.

[4] B. Geyer, "Die mathematischen Schriften des Albertus Magnus," *Angelicum* 35 (1958), 159–175.

[5] J.E. Hofmann, "Über eine Euklid-Bearbeitung, die dem Albertus Magnus zugeschrieben wird," *Proceedings of the International Congress of Mathematicians, 14 August 1958*, ed. J.A. Todd (Cambridge: 1960), 554–566.

[6] Paul Hossfeld, "Zum Euklidkommentar des Albertus Magnus," *Archivum Fratrum Praedicatorum* 52 (1982), 115–133.

[7] Robert Ineichen, "Zur Mathematik in den Werken von Albertus Magnus," *Freiburger Zeitschrift für Philosophie und Theologie* 40 (1993), 55–87.

[8] Henryk Anzulewicz, "Neuere Forschung zu Albertus Magnus: Bestandsaufnahme und Problemstellungen," *Recherches de théologie et philosophie médiévales* 66 (1999), 163–206.

[9] Paul M.J.E. Tummers, *Albertus (Magnus)' Commentaar op Euclides' Elementen der Geometrie*, 2 vols (Nijmegen: 1984).

[10] See below, notes 34–35.

The commentary preserved in MS Vienna Dominikanerkloster 80/45, which is the first original commentary on Euclid's *Geometry* in the Latin West, has for its two main sources a 13th-century adaptation of the Robert of Chester version of Euclid's *Elements*,[11] called by Busard MS Bonn et al. and by Tummers *V-B* (for Vatican-Bonn, the locations of the two main manuscripts), and the Gerard of Cremona translation of the commentary of al-Nayrizi ("Anaritius" to the Latins) on Euclid's *Elements*.[12] The former source Albert consulted for what belongs to Euclid proper, and the latter for the additions made by previous commentators, such as Simplicius and Heron. The commentary begins with an exquisite philosophical introduction (Fig. 1). The treatment of the mathematics is competent throughout, though with many mistakes. As is to be expected, the most deficient section is that which deals with the axioms and postulates, for the problems there were not fixed until 1899 when David Hilbert did so. Tummers edited the Latin Text of Book 1 in 1984;[13] the text of the remaining three books has not yet appeared. I published an annotated English translation of Book 1 in 2003;[14] the translation of the remaining books has not yet appeared. An unpublished excerpt from Book 4 will be discussed below.

It is evident from the following passages from Albert's undisputed works that he produced a commentary on Euclid's *Elements*:

> And therefore we too, as we treat the various parts of philosophy, will, with God's help, first complete natural science. Then we shall talk about all of mathematics, and we shall finish our program in divine science.[15]

> But how a chord is turned into an arc in such a way that a line is afterwards produced equal to that arc would take a long time to prove, but, God willing, will be explained in our study of geometry and astronomy.[16]

> All these things, however, must be accepted for now; they are to be proven, though, in the books on sight in the study of Perspective, which science

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11 H.L.L. Busard, *A Thirteenth-Century Adaptation of Robert of Chester's Version of Euclid's Elements*, 2 vols (Munich: 1996).

12 Paul M.J.E. Tummers (ed.), *Anaritius' Commentary on Euclid, The Latin Translation, I–IV*, Artistarium Supplementa 9 (Nijmegen: 1994).

13 Tummers, *Albertus (Magnus)' Commentaar.*

14 Anthony Lo Bello, *The Commentary of Albertus Magnus on Book I of Euclid's Elements of Geometry* (Boston: 2003).

15 "Et ideo etiam nos, tractando de partibus philosophiae, primo complebimus, Deo adiuvante, scientiam naturalem, et deinde loquemur de mathematicis omnibus, et intentionem nostram finiemus in scientia divina." *Phys.* 1.1.1, Ed. Colon. 4/1, 3, lns. 37–41.

16 "Qualiter autem corda convertatur in arcum, ita quod linea postea aequalis arcui accipiatur, longum esset hic demonstrare, sed in geometria hoc docebitur et in astronomia, Domino concedente." *Phys.* 1.2.1, Ed. Colon. 4/1, 17, lns. 53–56.

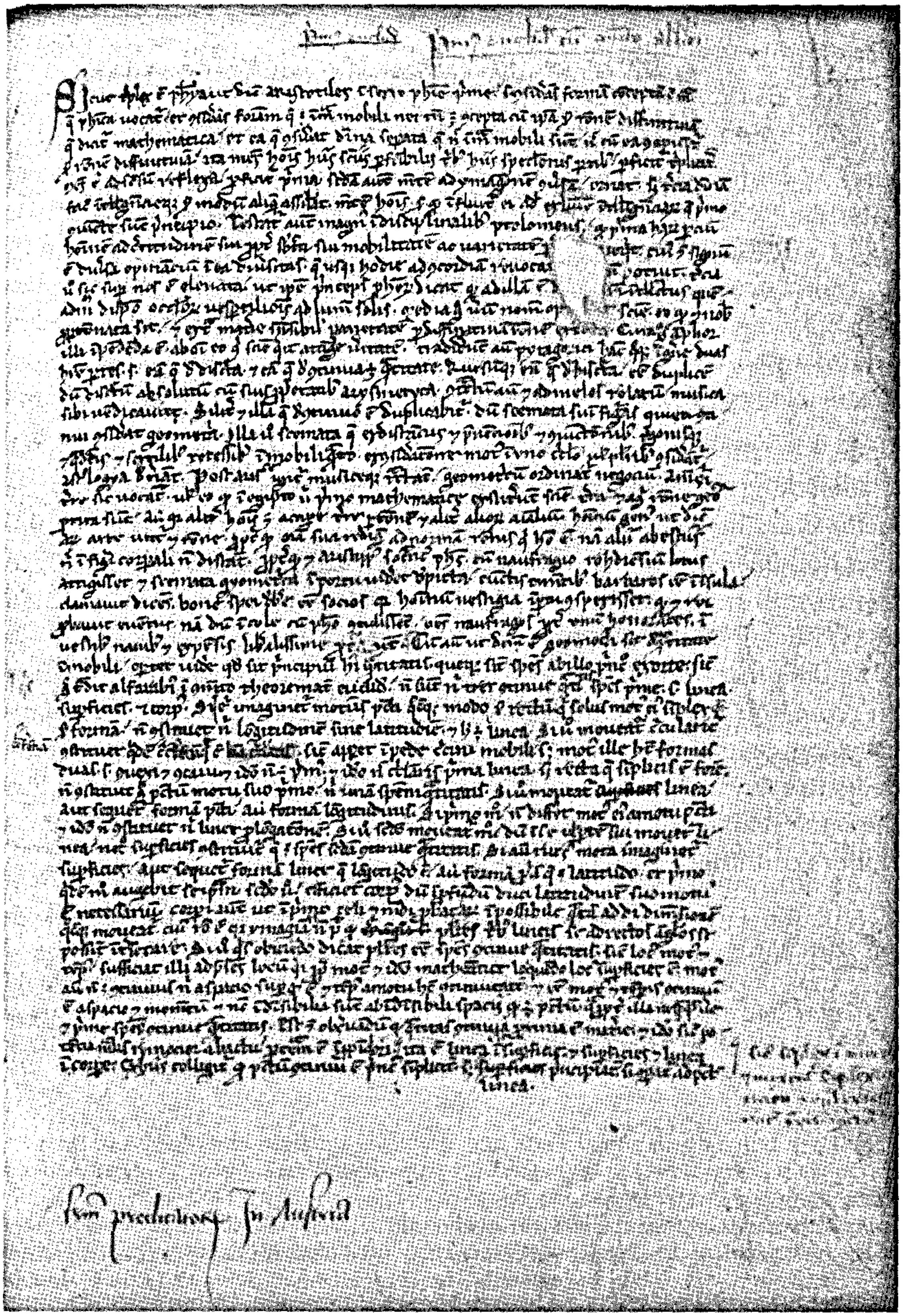

Figure 1. MS Vienna Dominikanerkloster 80/45, fol. 105r, the philosophical introduction to the commentary, reproduced by permission of the Dominikaner-Konvent, Vienna. Photo by Sonja Reisner.

> cannot be adequately treated unless we first consider those matters that pertain to geometry.[17]

> Now that the natural and mathematical sciences have been elucidated as much as was possible, we finally arrive at the true wisdom of philosophy.[18]

> For this [sc. that the diameter and side of a square are incommensurable] has already been proven by us in our book on geometry.[19]

> Just as has been proven in the fifteenth and sixteenth [propositions] of the third [book] of our *Geometry* [sc. namely, that a tangent line to a circle intersects it at only one point].[20]

> As we showed in the first [book] of our *Geometry* [sc. that two straight lines do not enclose a surface].[21]

Since, when he wrote the *Physics*, Albert spoke of his geometric enterprise as something still to be accomplished in the future, but when he wrote the *Metaphysics* he referred to it as already having been accomplished, it follows that the commentary on Euclid's *Elements* was written between 1250 and 1267, if we accept the dates for the *Physics* and *Metaphysics* assigned by Weisheipl[22] and the rest of the learned world.

Before proceeding further, we may ask: what motivated Albert to write on geometry? Mathematics has a special fascination for those who appreciate consecutive thought. Both theology and mathematics share the same deductive method, and I suspect that the Universal Doctor was moved by the same guarantee of certainty that made such an impression on Thomas Hobbes centuries later:

> He was 40 years old before he looked on Geometry, which happened accidentally. Being in a Gentleman's Library, Euclid's *Elements* lay open, and 'twas the 47 *El. Libri* I. He read the proposition. By G-, says he (he would now and then sweare an emphaticall Oath by way of emphasis), this is impossible! So he reads the demonstration of it, which referred him back

17 "Haec autem omnia supponenda sunt, probanda autem in libris de visu in Perspectivis, quae scientia compleri non potest, nisi primum consideremus ea quae pertinent ad geometriam." *De sens. et sensato* 1.14, Borgn. 9, 35 b, lns. 5–10.

18 "Naturalibus et doctrinalibus iam, quantum licuit, scientiis elucidatis, iam ad veram philosophiae sapientiam accedimus." *Metaph.* 1.1.1, Ed. Colon. 16/1, 1, lns. 9–11.

19 "Hoc autem iam a nobis in geometricis est demonstratum", *Metaph.* 1.2.10, Ed. Colon. 16/1, 27, lns. 73–74.

20 "Sicut in XV et XVI tertii geometriae nostrae demonstratum est", *Metaph.* 3.2.3, Ed. Colon. 16/1, 118, lns. 37–38.

21 "Sicut nos in I nostrae geometriae ostendimus", *Metaph.* 5.3.1, Ed. Colon. 16/1, 256, lns. 69–70.

22 James A. Weisheipl, "Albert's Works on Natural Sciences (*libri naturales*) in Probable Chronological Order," *Albertus Magnus and the Sciences: Commemorative Essays 1980*, ed. J.A. Weisheipl (Toronto: 1980), Appendix 1, 565, 576.

> to such a Proposition; which Proposition he read. That referred him back to another, which he also read. *Et sic deinceps*, so that he was demonstratively convinced of that trueth. This made him in love with Geometry.[23]

The question remains: does the Vienna Manuscript contain Albert's commentary? I believe so. I gave a detailed account of the arguments of my predecessors in 2003;[24] I will summarize them here. In 1952, Ostlender wrote that the handwriting in the manuscript was Albert's;[25] Geyer, in 1958, announced that the opinion of Ostlender was *sehr wahrscheinlich*,[26] and he was followed by Hofmann in 1960,[27] who agreed to edit the work for the Cologne edition of the *Opera Omnia*.[28] In 1982, Hossfeld presented a modification of these views when he concluded that the manuscript was an autograph in the sense of its having been taken down by dictation from Albert to a scribe.[29] This view was accepted in 1984 by Tummers, the most knowledgeable authority in the matter:

> Hossfeld's hypothesis (1982) that the manuscript is an autograph in the sense that the text was dictated by the author is correct. The writing seems to indicate that the manuscript was written in the thirteenth century... Albert, the author of the Commentary, can indeed be identified as Albert the Great;... this Commentary should be dated shortly after 1260, and... it gives a representative picture of the geometrical knowledge of a thirteenth century philosopher-theologian who was no mathematician by profession but had an interest in geometry.[30]

Anzulewicz addressed the subject in his learned paper published in 1999, in which he endorsed the verdict of Tummers.[31] At this time, the main objection to Albert's authorship concerned the script, which some said was from the 14th century. I therefore obtained the scientific opinion of an expert. In my English translation of Book 1 of Albert's commentary, I quoted the report of the Chicago paleographer M.I. Allen, whose attitude settled the matter for me, that the commentary is truly Albert's work:

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23 John Aubrey, *Brief Lives*, ed. Richard Barber (Woodbridge, UK: 1997), 151–152.

24 Anthony Lo Bello, *The Commentary of al-Nayrizi on Book I of Euclid's Elements of Geometry, with an Introduction on the Transmission of Euclid's Elements in the Middle Ages* (Boston: 2003), 62–74. Read also Lo Bello, *Albertus Magnus*, xi–xxiii. In both books, the relevant passages from the German and Dutch scholars are translated into English.

25 Ostlender, "Die Autographe," 15–16.

26 Geyer, "Die mathematischen Schriften," 169.

27 Hofmann, "Über eine Euklid-Bearbeitung," 556, 561.

28 He resigned the task, which still remains to be done. On this matter, read Anzulewicz, "Neuere Forschung," 180–181.

29 Hossfeld, "Zum Euklidkommentar," 118–119, 129, 133.

30 Tummers, *Albertus (Magnus)' Commentaar* 1, 327, 329.

31 Anzulewicz, "Neuere Forschung".

> The script (perhaps by several hands) is a clear, informal book hand (*textualis currens*) of a German sort, not later than the middle of the thirteenth century. I would say that it is of the late second quarter of the thirteenth century. That is my conclusion based on looking through everything. As for details, the German origin is indicated by the lightning-bolt shape of an abbreviation that is sometimes, but not uniformly, used for *est.* (The form is a give-away marker for Germany.) The script is not very late, because the bows of various letters (*b, o, d, p, e, etc.*) virtually never touch in the space-saving unions increasingly characteristic of copy-work as the thirteenth century advanced. This textbook is also not heavily abbreviated, although one would very much expect that [kind of thing] in the jargon-heavy idiom of this sort of text; the situation, again, speaks for an early, rather than a later, dating. The 7-shaped Tironian abbreviation for *et* is also still uncrossed, as also starts to appear after the mid-century north of the Alps... There is [thus] no doubt about the basic region and date. Do not mind that the leaves are now part of a composite volume with later materials [the work of Peter of Alvernia]. That sort of binding was a space and resource saver, and means nothing for the handwritten text... I am quite certain of the things that I observed. I was, in fact, rather surprised to have to conclude that the manuscript is so early.[32]

Audiatur altera pars. In a recent work Busard explains the reasons that lead him to reject the hypothesis that Albert the Great is the author of the commentary in the Vienna manuscript:[33]

> In an article completed in 1944 and published in 1958, Geyer claimed to have discovered a commentary on the first four books of Euclid's *Elements* in the manuscript Vienna, Dominikanerkloster 80/45, fols. 105r–145r. On the top folio of the beginning of this text (fol. 105r), there is inscribed, probably in another hand: *Primus Euclidis cum commento Alberti.* According to Geyer, the commentary has to be attributed to Albert the Great. This part of the manuscript is dated by Geyer thirteenth century; Folkerts, however, fourteenth century. If the latter is true, the commentary cannot be composed by Albert the Great. That the editor of the Euclid commentary would be named at all is of itself unusual, since anonymity is the rule. As far as I know, in this case only the names of Adelard and Campanus occur. For the composition of his text, the author of the Euclid commentary employed mainly the two following sources: the text preserved in the manuscripts Bonn et al. and Anaritius' commentary on Euclid's *Elements*. According to Tummers, who assumes the authorship of Albert the Great, the commentary was written between 1235 and 1260. If this should be true, the text of Bonn et al. was written before that time...[34]

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32 Lo Bello, *Albertus Magnus*, xxii.

33 H.L.L. Busard, "Some Thirteenth Century Redactions of Euclid's *Elements*, with Special Emphasis on the Books I–V," *Archives Internationales d'Histoire des Sciences* 51 (2001), 225–256.

34 Busard, "Some Thirteenth Century Redactions of Euclid's *Elements*," 235.

At this point, Busard starts to discuss the idiosyncrasies of the Campanus edition of the *Elements* and those found in the manuscript Oxford, Bodleian Library, Savile 19, which he believes to be later than Campanus. He then proceeds to examine the V-B edition and the second proofs and additional propositions found therein:

> All these additions with proof are also preserved in MS Dresden, Sächs. Landesbibl. Db 86: fol 183r: *Diameter est assimeter coste*; fols. 226–228r: Jordanus, *De proportionibus*; fols. 213r–v: three problems about circles. It is therefore possible that the author of the Bonn et al. text has found the additions in a manuscript. I cannot explain why the author has inserted these propositions. It is clear that the author has used several sources among which [are] Campanus and Savile 19. The author of the Euclid commentary attributed to Albert the Great employed mainly the two following sources for the composition of his text: the text preserved in the MSS Bonn et al. and Anaritius' commentary on Euclid's *Elements*. Tummers has said, that the commentary was written between 1235 and 1260, but if I am right, then the attribution to Albert the Great is questionable.[35]

Since Busard himself agrees that MS Savile 19 could have been written as early as 1251 and the Campanus edition of the *Elements* as early as 1255, it is possible to harmonize all views by holding that the V-B edition of the *Elements* (i.e. what Busard calls the edition Bonn et al.) was written in the second half of the same decade, and that then the well-informed scholar Albert the Great was able to compose his own commentary almost immediately thereafter, before 1260.

Furthermore, and most importantly, Albert quotes from the V-B version of the *Elements* in his undisputed work *De causis proprietatum elementorum*, which Weisheipl has dated to the period 1250–57 by the following chain of reasoning:[36] *De causis* is frequently cited in *Meteora*, Book 2, which was composed before *De mineralibus*, as the opening words of the latter attest. But *De mineralibus* was composed before Albert became provincial of the Dominicans in Germany, and is cited by Albert in *De anima*, which was written during his term in that office (June 1254–June 1257). Therefore, Busard's argument against assigning the commentary on Euclid to the period 1235–60 would also weigh in against assigning *De causis* to the fifties of the 13th century, which is an established fact.

2. That Albert handled mathematics competently outside the *Geometry* may be established by the following passage from the *De causis*

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35 Busard, "Some Thirteenth Century Redactions of Euclid's *Elements*," 241–242.

36 James A. Weisheipl, "Albert's Works on Natural Sciences (*libri naturales*) in Probable Chronological Order," Appendix 1, 567–569.

proprietatum elementorum, which I translate from the edition edited by Hossfeld. This passage is the most significant of all those in which Albert treats mathematics because it is critical for settling the question of the authorship of the commentary on Euclid's *Elements*. In the words of Tummers (which I translate from his Dutch): "This passage is of very great importance because here the formulation of I 4 is reproduced in full, and... this formulation appears only in Albert's Geometry and in V-B, and is not to be derived from any other source."[37] It is a superb passage entirely in the manner of the commentary on Euclid. The style and diction are the same, and the Euclidean propositions are quoted from the V-B version of the *Elements*, one of the two major sources of the commentary.

> From all that has been said so far, it is obvious and clear that the celestial bodies do not make sounds. For if those things that have been said about the aforementioned assumptions are truly understood, the argument of those who say that the stars make sounds when they revolve in their orbits is demolished.
>
> Let us introduce the following geometrical figure, which shows both visibly and reasonably that what they say cannot be true. Let me draw a circle with center E to represent the mass of the whole earth, and let me indicate by the point B the place of our habitation on the surface of the circle of the earth. Then let me draw another circle, either with a different center or with the same center, in whichever way I should deign to draw it, which is to represent the circular trajectory of the sun for one day, indicating its motion from east to directly overhead to west, and I shall mark three points on this circle, the sunrise by G, the sunset by A, and midday, when it is at its zenith directly above our heads, by point D. I shall then draw the straight lines between these points, first drawing AG, which is the diameter of the circle and passes through the center E of each circle; I shall also draw line AB and line GB, which are the lines that indicate the distance that is between the sun and the zenith-point directly above our heads at the times when the sun is rising and setting respectively. I shall also draw line EB from the center, which line is half the diameter of the earth, and I shall extend the same line continuously all the way to point D. Then line DB will be the measure of the distance from the top of our heads to the sun when the sun is at midday above us. Here is how the figure should look [See Fig. 2]:
>
> Now that this drawing had been so laid out, I confidently declare, that because the sun is bigger than the other stars and nearer to us than some of the so-called planets, it must make more of a sound than the other stars, and, furthermore, that this sound is greater at noon, when it is above our heads, than it is at sunrise or sunset, for it is clear to us that lines GB and AB are longer than line BD. The sun should daily and audibly manifest these

37 Tummers, *Albertus (Magnus)' Commentaar* 2, 214.

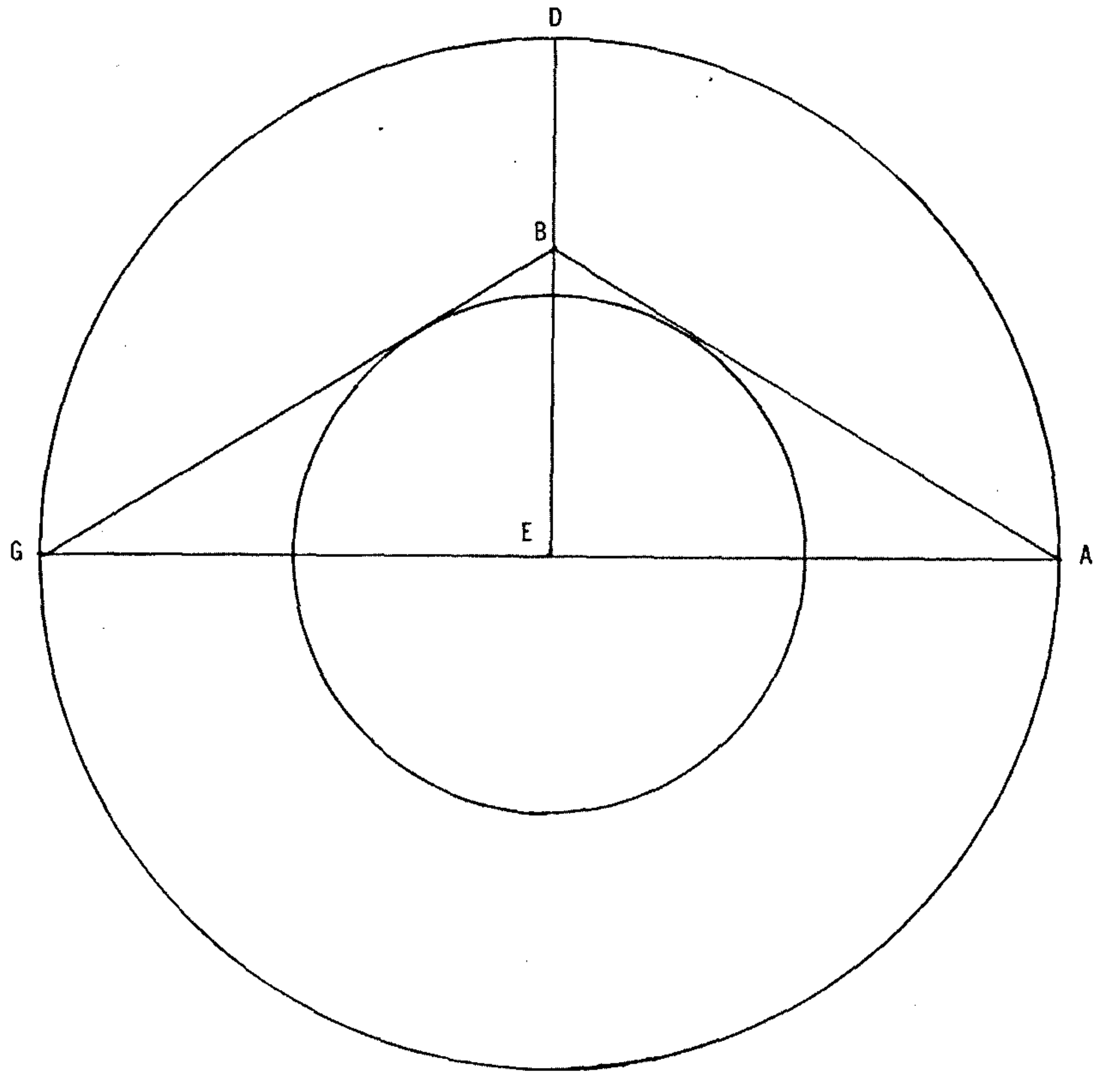

Figure 2. The geometrical diagram accompanying the mathematical demonstration in *De Causis Proprietatum Elementorum.*

differences in the sounds that it makes, but what we actually see and experience contradicts this.

In the same way the moon too and the other stars make a greater or a lesser sound according to their greater or lesser proximity, and they will therefore sound less loud at points G and A, which are where they are rising and setting, because they are further away from us there than at point D when they are directly above our heads and less distant from us, and this must be perceptible to our hearing. But mathematical argument by means of the figure that we have introduced shows that the contrary is true. Now we have already stated those things that are necessary for the drawing of the figure, and we now say that line AB and line GB are equal by Proposition 4 of book 1 of Euclid's *Elements*, which says that *In any two triangles, two sides and the included angle of one of which are equal respectively to two sides and the included angle of the other, the base will also be equal to the base and the remaining angles to the remaining angles, and the whole triangle to the whole triangle.* For it is agreed that the two lines GE and EA are equal, since they are drawn from the same center to the same circumference, and each of them is a radius of the same circle. Furthermore, line EB, which is common to both triangles, GEB, namely, and EBA, is equal to itself. Therefore, two sides of triangle GEB are equal to two sides of triangle EBA. What is more, angle BEA, which is at the center, is equal to angle BEG of the other triangle, which is also at the center, for each of them is right, which is proved by the fact that line BE stands perpendicularly at the center and makes a right angle on each side, for all right angles are equal. Therefore, by the aforementioned proposition of Euclid, base BA of triangle EBA is equal to base GB of triangle GEB, and that is what we said, that GB and BA are equal.

Once this has been established, we say that line GB is longer than line GE. This, though, is proved in the next to last proposition of book 1 of Euclid, which reads: *In any triangle, the square of the side opposite the right angle is equal to the sum of the squares of the two remaining sides.* For GEB is the right triangle, and its right angle is angle GEB; furthermore, line GB is its opposite side. Therefore, if it is squared, its square is equal to the sum of the squares of the two lines GE and EB. Therefore, its square is greater than the square of the line GE alone. But if a square is greater than a square, then the square root too is greater than the square root. But the square root is line GB; therefore, line GB is greater than line GE. And that is what we wanted to demonstrate.

We next say that lines GE and ED are equal, because they extend from the same center to the same circumference. Now since line GB is longer than line GE, it will be longer than line ED too. But line ED is longer than line BD since line BD is a part of the whole line ED. Therefore, line GB, which is longer than the whole line ED, will be much longer than line BD. But line GB is the distance between us and the sun at sunrise, and line BD is the distance between us and the sun at midday. Therefore, the distance between us and the sun at sunrise is much greater than the distance between us and the sun at midday.

The same sort of proof enables us to establish that the sun is further from us at sunset than it is at midday; therefore, it will sound noticeably louder at midday than it does at sunrise or sunset. But this is not true. It therefore remains that it is false that the sun or any other star in the heavens emits sounds. And that is what we wanted to demonstrate.[38]

From the modern point of view, sound is propagated by waves through matter, and since there is no matter in outer space, which is essentially a vacuum, there is no sound. Therefore, the heavenly bodies make no sounds as they describe their orbits. Albert cannot be blamed for not being modern enough to know this. As in most cases of error in medieval

[38] *De causis propr. elem.* 1.2.1, Ed. Colon. 5/2, 61, ln. 13–p. 62, ln. 35. For a complete English translation, see *On the Causes of the Properties of the Elements (Liber de causis proprietatum elementorum)*, trans. Irven M. Resnick (Milwaukee, Wis.: 2010).

thought, his mistakes were with his assumptions, not with his chain of reasoning from those assumptions.

Most of the other references to mathematics in the works of Albert the Great are citations of definitions or propositions that illustrate some point that the Universal Doctor wanted to make, often with regard to the technical terminology of Aristotelian or Scholastic philosophy, or to the physics (if so it may be called) of his time. They are not examples of his doing mathematics as much as proof that he had studied Euclid. The rest are long passages, like the one just translated above, in which he makes a geometrical demonstration to establish some point in the natural sciences: for example, that two solids cannot occupy the same space;[39] or that the surface of the water on the face of the earth is spherical.[40] They show that he understood that physics could not be done without mathematics, a truth that led eventually to the *Principia Mathematica* of Newton.

3. We next proceed to consider an as yet unpublished passage from Albert's commentary, namely, his treatment of Proposition 10 of Book 4 on folio pages 143r and 143v of the Vienna manuscript. It is convenient to illustrate the idiosyncrasies of the commentary from this passage. The enunciation is the same as that of Robert of Chester,[41] V-B,[42] and John of Tynemouth ("Adelard III").[43] As far as I can tell, the proof is Albert's own and is very detailed; it is a nice, alternate, original proof, the presentation of which could be a little clearer. (For example, the phrase "a common angle DBC having been added to each" should read "since, by the axiom, when equals are added to equals, the sums are equal.") It differs, with one important exception, from all the other extant proofs and proof-sketches in that it makes no use of Proposition 32 of Book 1, that an exterior angle of a triangle is equal to the sum of the two opposite interior angles; the exception is V-B, which also contains no mention of the exterior angle equaling the sum of the two opposite interior angles. Albert's amanuensis was *in angustiis* in making the diagram, which is full of erasures. The picture in the manuscript is incorrect, since the smaller circle is not supposed to be internally tangent to the larger circle; in this the scribe erred in the same way as did the fellow who drew the diagrams for the

39 *Phys.*, 4.2.8, Ed. Colon. 4/1, 251, ln. 78–p. 253, ln. 20.

40 *De caelo et mundo* 2.2.3, Ed. Colon. 5/1, 131–132.

41 H.L.L. Busard and Menso Folkerts, *Robert of Chester's (?) Redaction of Euclid's Elements, the so-called Adelard II Version*, vol. 1 (Basel: 1992), 156.

42 H.L.L. Busard, *A Thirteenth-Century Adaptation*, 163.

43 H.L.L. Busard, *Johannes de Tinamue's Redaction of Euclid's Elements, the so-called Adelard III Version*, vol. 1 (Stuttgart: 2001), 120.

Leiden manuscript of al-Nayrizi.[44] Like all those in the Arabic–Latin tradition of the transmission of Euclid's *Elements*, Albert's demonstration cites Proposition 31 of Book 3 (32 of the Greek text) with the unnecessary condition, universal in that tradition, that the line DC must not fall upon the center of the smaller circle; this assumption was no doubt the result of an early misunderstanding of Euclid's proof of 3.32, in which the chord AB, without any loss of generality, is taken to be a diameter. The references to previously proven propositions (e.g. "*undecimum secundi*") are in the style of Robert of Chester.

Proposition 10 of Book 4:

To draw a triangle with two equal sides, such that each one of the two angles that are at the base is double the remaining angle.

Let line AB be drawn, and with the immobile foot of the compass placed at A, let a circle be described with its length as radius, and let line AB be divided as the eleventh proposition of the second book shows how, namely, so that the rectangle that is contained by the whole and one segment is equal to the square that is made from the other segment, and let the symbol C indicate the point of division.

Then, by the first proposition of this book, let there be drawn in the circle a line BD equal to line AC, which is less than the diameter of the circle, and let line AD and line DC be joined.

Next, by the fifth proposition of this book, let a circle be circumscribed around triangle ACD. Once this is done, I say that triangle ABD has two sides equal, and that each one of its angles at the base DB is double the angle DAB at the center.

Proof: By hypothesis the rectangle that is contained by the two lines AB and CB is equal to the square of AC. But, by the same hypothesis, AC is equal to DB. Therefore the aforementioned rectangle is equal to the square of DB. Thus, by the last proposition of the third book, BD is tangent to the smaller circle at point D. What is more, triangle ABD has the two sides AB and AD equal, because they are both from the center to the circumference. Therefore, by the fifth proposition of the first book, the angles above the base are equal. Furthermore, line DC, which is away from the center, is drawn from the point of tangency of the line to the smaller circle. Therefore, by the thirty-first proposition of the third book, angle CDB, which it makes with the tangent, is equal to angle BAD, which is in the alternate portion of the

44 R.O. Besthorn and J.L. Heiberg, *Codex Leidensis 399,1, Euclidis Elementa ex Interpretatione al-Hadschdschadschii cum Commentariis al-Nairizii, arabice et latine ediderunt notisque instruxerunt R.O. Besthorn et J.L. Heiberg, ad finem perduxerunt G. Junge, J. Raeder, W. Thomson* (Copenhagen: 1899–1932), repr. as volumes 14 and 15 of the series *Islamic Mathematics and Astronomy*, ed. Fuat Sezgin, (Publications of the Institute for the History of Arabic-Islamic Studies) 15 (Frankfurt: 1997), 49. (There are two pages numbered 49 in this volume; reference is to the first. The page numbers are consecutive within chapters, but each chapter starts on a page numbered 1.)

smaller circle. Therefore the sum of the two angles ABD and BDC is as the sum of the two angles ADB and DAB, a common angle DBC having been added to each.

Therefore the two triangles ABD and CDB are such that two angles of one are equal to two angles of the other, because angle CDB of the smaller triangle is like angle DAB of the bigger triangle, and angle ADB of the bigger triangle is like angle ABD, which is the angle common to each.

Therefore, by the thirty-second proposition of the first book, the third angle, namely DCB of the smaller triangle, is like the third angle of the bigger triangle, namely angle ABD (in its capacity as an angle of the bigger triangle), since otherwise a triangle would have more or less than two right angles. Therefore, by the sixth proposition of the first book, sides DC and DB of the smaller triangle are equal, for they are opposite equal angles.

Furthermore, it was assumed above that AC and DB are equal, and now it has been proved that DC is equal to BD. Therefore, DC is equal to AC. Therefore triangle ACD has two equal sides, and the angles above the base are thus equal, namely, angle CAD and angle ADC. The angles ADC and CDB are consequently equal.

Since, then, angle CDB is equal to angle BAD, and angle CDA is similarly equal to the same angle, and each angle is thus half of the whole angle ADB, the whole angle ADB is double angle DAB.

But angle ADB is equal to angle ABD; therefore angle ABD too is double angle BAD.

And that is what we wanted to demonstrate. Here is the picture [See Fig. 3].[45]

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[45] I am indebted to Herr Ing. H. Förster for having sent me his transcription of this passage, which I was then able to compare with my own. I have written out the words that are abbreviated in the manuscript and modernized the spelling by writing, for example, *obtinebit* for *optinebit, aequalis* for *equalis, lineae* for *linee*, and *i* for *y* where the latter letter occurs in *diameter* and *perigraphetur*.

Proposition 10 of Book 4:

Duum aequalium laterum triangulum designare, cuius uterque duorum angulorum, quos basis obtinet, reliquo duplus existat.

Ducatur enim linea AB, et posito immobili pede circini in A, ad quantitem eius circumducatur circulus; dividatur etiam linea AB prout praecipit undecimum secundi, scilicet ut quod sub tota et una portione rectangulum continetur aequum sit ei, quod fit ex reliqua sectione quadrato, sitque signum divisionis C punctus.

Deinde per primam huius applicetur in circulo linea BD, quae sit equalis AC lineae, quae diametro circuli minor existit, et producatur linea AD et linea DC.

Postea ACD triangulo perigraphetur circulus per quintum huius. Hoc facto, dico triangulum ABD esse duorum aequalium laterum, cuius uterque angulorum super basim DB duplus est angulo DAB in centro existenti.

Cuius demonstratio haec est: Rectangulum, quod continetur sub duabus lineis AB et CB, valet quadratum AC ex hypothesi. Sed AC est aequalis DB per eandem hypothesim. Ergo rectangulum praedictum valet quadratum DB. Ergo per ultimam tertii linea BD est contingens minorem circulum in puncto D. Amplius triangulus ABD duo latera AB et AD habet aequalia, quia sunt ab eodem centro ad circumferentiam. Ergo per quintum primi, anguli supra basim sunt aequales. Adhuc linea DC

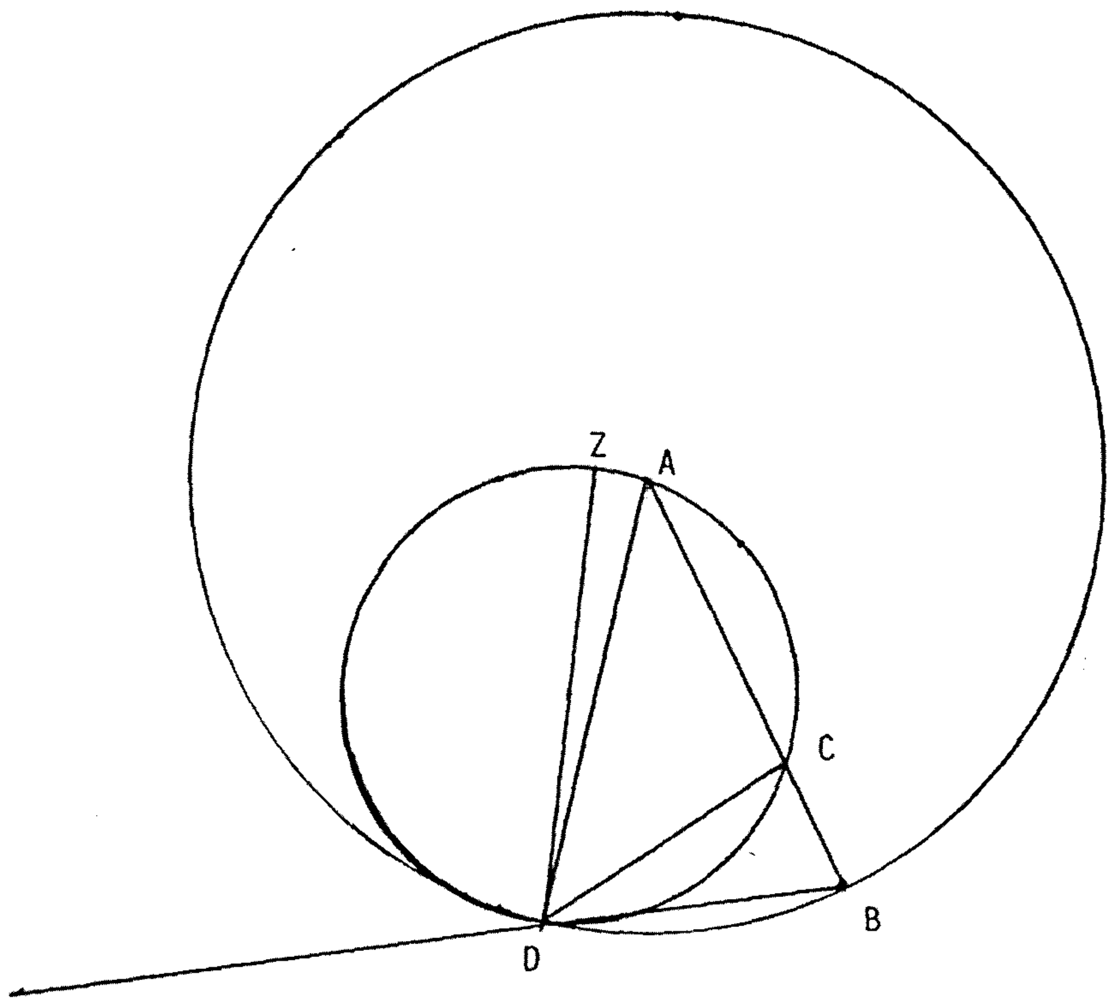

Figure 3. The geometrical diagram accompanying Albert's demonstration of Proposition 10 of Book 4 of Euclid's *Elements*.

praeter centrum a contactu contingentis lineae ducitur in minorem circulum; ergo per tricesimum primum tertii, angulus CDB, quem facit cum contingente, aequalis est angulo BAD, qui est in alternata circuli minoris portione. Ergo coniunctio duorum angulorum ABD et BDC est sicut coniunctio duorum angulorum ADB et DAB, DBC angulo communiter coniuncto.

Ergo duo trianguli ABD et CDB tales sunt, quod duo anguli unius aequales sunt duobus angulis alterius, quia angulus CDB minoris trianguli est sicut angulus DAB maioris trianguli, et angulus ADB maioris est sicut angulus ABD, qui est angulus communis utrique.

Ergo per tricesimum secundum primi, tertius angulus, scilicet DCB minoris, est sicut tertius maioris, scilicet angulo ABD, prout est angulus maioris trianguli, quia aliter triangulus plus vel minus haberet quam duos rectos, Ergo, per sextum primi, latera DC et DB minoris trianguli sunt aequalia, quia respiciunt aequales angulos.

Adhuc supra positum est AC et DB esse aequales, et nunc probatum est, DC esse aequalem BD. Ergo DC est aequalis AC. Ergo triangulus ACD habet duo latera aequalia; ergo anguli supra basim sunt aequales, scilicet angulus CAD et angulus ADC. Ergo per consequens anguli ADC et CDB sunt aequales.

Cum igitur angulus CDB sit aequalis angulo BAD, et iterum angulus CDA aequalis sit eidem, et angulus uterque sit medietas totius anguli ADB, totus angulus ADB est duplus angulo DAB.

At this point, the proof of Euclid's proposition is complete, and Albert now turns to his second source, Gerard's Latin version of al-Nayrizi ("Anaritius"), to supply some additional pertinent information.[46] Very strangely he omits the first paragraph from this source, in which al-Nayrizi shows, first, that the case of 4.5 to be used here is that of an obtuse-angled triangle being circumscribed by a circle, and, second, that circle ACD is not tangent to the bigger circle but intersects it at a second point different from D. The passage that Albert does rework and present is that in which al-Nayrizi explains how, he believes, Euclid must have gotten the idea for the proof. He proceeds by analysis, that is, he assumes the problem solved and examines the consequences, from which one is able to argue backwards. Tummers comments on this passage (I translate his Dutch), "Albert... follows with a *resolutio*... which depends on Anaritius, and even almost word for word. Nevertheless, Albert has not entirely understood it; the deviations from the text of Anaritius give proof of that."[47] The principal deviation of which Tummers speaks is the last sentence of the second paragraph below, where the equality of the rectangle and the square is treated as a fact that follows from 2.11, whereas it is just a claim of al-Nayrizi and is something to be proved. Also, in the next to last paragraph below, Albert refers to the *last* proposition of Book 3 when he needs to mention the *penultimate* proposition. Due to Albert are the many references to previous propositions made in the course of the demonstration; they indicate the critical manner in which he makes use of his sources.

> By the method of resolution you will arrive in the following manner at the point whence Euclid began:
>
> Let me assume that triangle ABD has been constructed and that each one of the two angles ABD and ADB is double angle BAD. I shall then divide angle ADB into two halves by the ninth proposition of the first book, and let the dividing line be DC. Each section of the divided angle is then equal to angle DAB by hypothesis. The rectangle, then, that is contained by the two lines AB and CB is equal to the square of AC by the eleventh proposition of the second book.
>
> Therefore, since angle BAD is equal to angle ADC, line AC will be equal to line DC because they are subtended by equal angles, as the sixth proposition of the first book says. And because angle BCD is equal to the two angles CAD, ADC, since it is external to the triangle ADC, as the first part of the

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Sed angulus ADB aequalis est angulo ABD; ergo etiam angulus ABD duplus est angulo BAD.

Et hoc est, quod voluimus demonstrare. Figura autem haec.

46 Tummers, *Anaritius*, 119, ln. 25–p. 120, ln. 34.

47 Tummers, *Albertus (Magnus)' Commemtaar* 1, 46.

thirty-second proposition of the first book says, and these two angles are equal, therefore angle BCD is double angle CAD. Thus, angle BCD is equal to each one of the two angles ABD and ADB. Therefore, line CD is equal to line BD.

But it was previously shown that line CD was equal to line AC, therefore line AC is equal to line BD. But angle ACD is bigger than angle BCD; therefore, it is obtuse by the fourteenth proposition of the first book, for angles standing around the line CD and falling upon the line AB are either equal (and therefore will be right, which is false) or unequal, in which case the bigger is obtuse and the smaller acute.

And so I shall erect at point D of line BD a perpendicular DZ by the eleventh proposition of the first book. When, therefore, we shall have circumscribed a circle around triangle ACD by the fifth proposition of this book, line DZ will be perpendicular to this circle by the eighteenth proposition of the third book, and line BD will be tangent to the circle, and point B will be outside the circle, and from it the line BA has been joined, a secant to the circle, and line BD is tangent to it. Therefore, the rectangle that is contained by the two lines AB and BC is equal to the square of BD by the last proposition of the third book. But BD is equal to AC. Therefore, the aforementioned rectangle is equal to the square of AC.

Euclid then began his proof at this point. And that is what we wanted to find by resolution. And the first version of the diagram suffices once you add the diameter DZ.[48]

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[48] Secundum solutionis modum hoc modo pervenies, in id unde incepit Euclides: Ponam enim ut triangulus ABD sit constitutus et quod uterque duorum angulorum ABD et ADB sit duplus ad angulum BAD. Dividam ergo angulum ADB in duo media per nonum primi, et sit linea dividens DC. Ergo utraque sectio divisi est aequalis angulo DAB per hypothesim. Rectangulum autem, quod continetur a duabus lineis AB et CB, est aequale quadrato AC per undecimum secundi.

Ergo quia angulus BAD est aequalis angulo ADC, erit linea AC aequalis lineae DC quia respiciuntur ab aequalibus angulis, sicut dicit sextum primi. Et quia angulus BCD est aequalis duobus angulis CAD, ADC, cum sit exterior triangulo ADC, sicut dicit tricesimum secundum primi, pars prima, qui duo anguli sunt aequales, ergo angulus BCD est duplus ad angulum CAD. Angulus igitur BCD est aequalis unicuique duorum angulorum ABD et ADB. Ergo linea CD est aequalis lineae BD.

Sed dudum habitum est quod linea CD fuit aequalis lineae AC; ergo linea AC est aequalis lineae BD. Sed angulus ACD est maior angulo BCD; ergo, ipse est amplus per decimum quartum primi, quia anguli circumstantes lineam DC cadentes super lineam AB aut sunt aequales et tunc erunt recti, (quod falsum est) aut inaequales, et tunc maior est amplus et minor acutus.

Erigam itaque supra punctum D lineae BD perpendicularem DZ per undecimum primi. Cum ergo constituerimus circa triangulum ACD circulum per quintum huius, erit linea DZ perpendicularis illius circuli per duodevicesimum tertii, et linea BD erit contingens circulum, et punctum B erit extra circulum, a quo protracta est linea BA secans circulum et linea BD contingens ipsum. Ergo rectangulum, quod continetur sub duabus lineis AB et BC, est aequale quadrato BD per ultimum tertii. Sed BD est aequalis AC. Ergo, rectangulum praedictum est aequale quadrato AC.

Hic ergo incepit Euclides probationem suam. Et hoc est, quod resolvendo voluimus invenire. Et prior forma sufficit addita diametro DZ.

INTERPRETING ALBERT THE GREAT ON ASTRONOMY

B.B. Price

Pursuant to the goal of this collection of essays, the present chapter will aim to provide scholars and advanced students with a general overview, in English, of the ideas of Albert the Great on astronomy. Emphasizing the contribution of Albert the Great specifically to the body of earlier and contemporary ideas on astronomy, it is designed to complement the volume's other chapters, particularly those dedicated to Albert's interest in natural science. In the context of such a collection, individual essays are afforded the luxury of allowing the proximate resources the others provide to enhance their offering. While this one, like the others, is focused on the thought of Albert himself, at the very outset it must be appreciated that the study of the astronomy of Albert the Great is but a small subfield, nested under the apron of other areas of scholarly investigation, increasingly larger and wider in scope: from astronomy in the Middle Ages, medieval science, and medieval mathematics and numeracy, to 13th-century intellectual history (its combination of philosophical and theological ideas), and medieval thought in general. Only as these pertain to Albert's astronomy do they become a part of this investigation, but they are each one extremely pertinent.

It must be said that virtually all the major issues, controversies, and debates among those scholars interested in Albert's astronomical ideas have had their source outside the strict study of his specific contributions. While these polemics touch on the thought of Albert and sometimes even use him as an example to support one side or the other, his ideas about astronomy are not the reason for their being. Only a handful of scholars have felt it worthwhile to undertake study dedicated to Albert's ideas on astronomy for their own sake, and for the most part their interpretations and understanding of his contribution have yet to be pursued in a sustained and thorough enough way to have an impact on the discussion of his work more generally.